ELEMENTS
OF
PHILOSOPHY
CONCERNING
BODY.

HOBBS.







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ELEMENTS

OF

PHILOSOPHY,

THE

FIRST SECTION,

CONCERNING

BODY.

Written in Latine by

THOMAS HOBBES

OF

MALMESBURY.

And now translated into

ENGLISH.

To which are added Six Lessons to the
Professors of Mathematicks of the Institution of
Sr. *HENRY SAVILE*, in the
University of *OXFORD*.

LONDON,
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TRANSLATOR

To the Reader.

IF when I had finiſhed my Tranſlation of this firſt Section of the Elements of Philoſophy, I had preſently committed the ſame to the Preſs, it might have come to your hands ſooner then now it doth. But as I undertook it with much diffidence of my own ability to perform it well; ſo I thought fit before I publiſhed it, to pray Mr. *Hobbes* to view, correct and order it according to his own minde and pleaſure. Wherefore, though you find ſome places enlarged, others altered, and two Chapters (the 18th and 20th) almoſt wholly changed, you may nevertheleſs remain aſſured, that as now I preſent it to you,

it doth not at all vary from the Authours own ſenſe and meaning. As for the ſix Leſſons to the *Savilian* Profeſſors at *Oxford*, they are not of my tranſlation, but were written, as here you have them in Engliſh, by Mr. *Hobbes* himſelf; and are joyned to this Book, becauſe they are chiefly in defence of the ſame.

The Authors Epistle Dedicatory,

TO THE

RIGHT HONORABLE,

My most Honored LORD,

WILLIAM

Earl of Devonshire.

THis *first Section of the* Elements *of* Philosophy, *the Monument of my Service, & your Lordships bounty, though (after the third Section published) long deferred, yet at last finished, I now present (my most Excellent Lord) and dedicate to your Lordship. A little Book, but full; and great enough, if men count* well *for* great; *and to an attentive Reader versed in the Demonstrations of* Mathematicians, *that is, to your Lordship, clear & easy to understand; and almost new throughout, without any offensive Novelty. I know that that part of Philosophy wherein are considered Lines and Figures, has been delivered to us notably improved by the Ancients*

ents; and withall a most perfect pattern of the Logique by which they were enabled to finde out and demonstrate such excellent Theoremes as they have done. I know also that the Hypothesis of the Earths Diurnal Motion was the invention of the Ancients; but that both it, and Astronomy (that is, Cœlestial Physiques) springing up together with it, were by succeeding Philosophers strangled with the snares of Words. And therefore the beginning of Astronomy (except Observations) I think is not to be derived from farther time then from Nicolaus Copernicus; *who in the Age next preceding the present, revived the opinion of* Pythagoras, Aristarchus & Philolaus. *After him, the Doctrine of the Motion of the Earth being now received, & a difficult Question thereupon arising concerning the Descent of Heavy Bodies,* Galilæus *in our time striving with that difficulty, was the first that opened to us the gate of Natural Philosophy Universal, which is the knowledge of the Nature of* Motion. *So that neither can the Age of Natural Philosophy be reckoned higher then to him. Lastly, the Science of* Mans Body, *the most profitable part of Natu-*

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ral Science, was first discovered with admirable sagacity by our Countryman Doctor Harvey, *principal Physician to King* James *and King* Charles, *in his Books of the* Motion of the Blood *and of the* Generation of Living Creatures; *who is the onely man I know that conquering envy hath established a new Doctrine in his life time. Before these, there was nothing certain in Natural Philosophy but every mans Experiments to himself, and the Natural Histories, if they may be called certain, that are no certainer then Civil Histories. But since these, Astronomy & Natural Philosophy in general have for so little time been extraordinarily advanced by* Joannes Keplerus, Petrus Gassendus, & Marinus Mersennus; *& the Science of Humane Bodies in special by the wit & industry of Physicians (the onely true Natural Philosophers) (specially of our most Learned Men of the Colledge of Physicians in* London. *Natural Philosophy is therefore but young; but Civil Philosophy yet much younger, as being no older (I say it provoked, & that my Detractors may know how little they have wrought upon me) then my own Book de Cive. But what? Were there no Philosophers Natural nor Civil among the*

ancient Greeks? *There were men so called; Witness* Lucian; *by whom they are derided; Witness divers Cities, from which they have been often by publique Edicts banished. But it follows not that there was Philosophy. There walked in old* Greece *a certain Phantasme, for superficial gravity (though full within of fraud & filth) a little like Philosophy; which unwary men thinking to be it, adhered to the Professors of it, some to one, some to another (though they disagreed among themselves) and with great Salary put their children to them to be taught, instead of Wisdome, nothing but to dispute; & neglecting the Laws, to determine every Question according to their own fancies. The first Doctors of the Church next the Apostles, born in those times, whilest they endeavored to defend the Christian Faith against the* Gentiles *by Natural Reason, began also to make use of Philosophy, & with the Decrees of Holy Scripture to mingle the Sentences of Heathen Philosophers; & first some harmless ones of* Plato; *but afterwards also many foolish & false ones out of the Physicks & Metaphysicks of* Aristotle; *& bringing in the Enemies betrayed unto them the Cittadel of Christianity. From that time, instead of the Worship of God, there entred a thing called* School-Di-

vinity, *walking on one foot firmly, which is the Holy Scripture, but halted on the other rotten foot, which the Apostle* Paul *called* Vain, *& might have called* Pernicious Philosophy; *for it hath raised an infinite number of Controversies in the Christian World concerning Religion, & from those Controversies Wars. It is like that* Empusa *in the* Athenian *Comick Poet, which was taken in* Athens *for a Ghost that changed shapes, having one brazen leg, but the other was the leg of an Ass, & was sent (as was believed) by* Hecate, *as a signe of some approaching evil fortune. Against this* Empusa *I think there cannot be invented a better Exorcisme, then to distinguish between the Rules of Religion, that is, the Rules of Honoring God, which we have from the Laws, and the Rules of Philosophy, that is, the Opinions of private men; & to yeild what is due to Religion to the Holy Scripture, and what is due to Philosophy to Natural Reason And this I shall do, if I but handle the Elements of Philosophy truly & clearly as I endevour to do I therefore having in the 3d Section wch I have published & dedicated to your* Lordship *long since, reduced all Power Ecclesiastical and Civil by strong Arguments of Reason, without repugnance to Gods Word, to one and the same Soveraign Authority; I intend now,*

by putting into a clear Method the true Foundations of Natural Philosophy, to fright and drive away this Metaphysical Empusa; *not by skirmish, but by letting in the light upon her. For I am confident if any confidence of a Writing can proceed from the Writers fear, circumspection & diffidence) that in the three former parts of this Book, all that I have said is sufficiently demonstrated from Definitions; & all in the fourth part, from Suppositions not absurd. But if there appear to your* Lordship *any thing less fully demonstrated then to satisfie every Reader, the cause was this, that I professed to write not all to all, but some things to Geometricians onely. But that your* Lordship *will be satisfied I cannot doubt.*

There remains the second Section, which is concerning Man. *That part thereof where I handle the* Optiques, *containing six Chapters, together with the Tables of the Figures belonging to them, I have already written & engraven lying by me above these six years. The rest shall, as soon as I can, be added to it; though by the contumelies & petty injuries of some unskilful men, I know already by experience how much greater thanks will be due, then payed me, for telling Men the truth of what*

Men

Men are. But the burthen I have taken on me I mean to carry through; not ſtriving to appeaſe, but rather to revenge my ſelf of Envy, by encreaſing it. For it contents me that I *have your* Lordſhips *favour; which, (being all you require) I acknowledge; and for which, with my prayers to Almighty God for your* Lordſhips *ſafety, I ſhall (to my power) be always thankefull.*

London, *April* 23,
1655.

YOUR LORDSHIP[S]

most humble Servant

Thomas Hobbes.

The Authors Epiſtle

To the Reader.

THink not (courteous Reader) that the Philoſophy the Elements whereof I am going to ſet in order, is that which makes Philoſophers Stones, nor that which is found in the Metaphiſique Codes. But that it is the Natural Reaſon of Man buſily flying up and down among the Creatures, & bringing back a true report of their Order, Cauſes & Effects. Philoſophy therefore, the Childe of the World and your own Mind, is within your ſelf; perhaps not faſhioned yet, but like the World its Father, as it was in the beginning, a thing confuſed. Do therefore as the Statuaries do, who by hewing off that which is ſuperfluous, do not make but find the Image. Or imitate the Creation. If you will be a Philoſopher in good earneſt, let your Reaſon move upon the Deep of your own Cogitations and Experience. Thoſe things that lie in Confuſion muſt be ſet aſunder, diſtinguiſhed, and every one ſtampt with its own name ſet in order; that is to ſay, your Method muſt reſemble that of the Creation. The order of the Creation was, *Light*, *Diſtinction of Day and Night*, the *Firmament*, the *Luminaries*, *Senſible Creatures*, *Man*; and after the Creation, the *Commandement*. Therefore the order of Contemplation will be, *Reaſon*, *Definition*, *Space*, *the Starres*, *Senſible Quality*, *Man*; and after Man is grown up, *Subjection to Command*. In the firſt part of this Section which is entitled Logique, I ſet up the light of Reaſon. In the Second (which hath for title the Grounds of Philoſophy) I diſtinguiſh the moſt common Notions by accurate definition, for the avoiding of confuſion and obſcurity. The third part concerns the Expanſion of Space, that is, Geometry. The fourth contains the Motion of the Starres, together with the doctrine of Senſible Qualities.

To the Reader.

In the ſecond Section (if it pleaſe God) ſhall be handled Man. In the third Section the doctrine of Subjection is handled already. This is the Method I followed; and if it like you you may uſe the ſame; for I do but propound, not commend to you any thing of mine. But whatſoever ſhall be the Method you will like, I would very fain commend Philoſophy to you, that is to ſay, the ſtudy of Wiſdome, for want of which we have all ſuffered much dammage lately. For even they that ſtudy Wealth, do it out of love to Wiſdome; for their Treaſures ſerve them but for a Looking-glaſs, wherin to behold and contemplate their owne Wiſdome. Nor do they that love to be employed in publike buſineſs, aime at any thing but place wherein to ſhew their Wiſdome. Neither do Voluptuous men neglect Philoſophy, but onely becauſe they know not how great a pleaſure it is to the Mind of Man to be raviſhed in the vigorous and perpetual embraces of the moſt beauteous World. Laſtly, though for nothing elſe, yet (becauſe the Mind of Man is no leſs impatient of Empty Time, then Nature is of Empty Place) to the end you be not forced for want of what to do, to be troubleſome to men that have buſineſs, or take hurt by falling into idle Company, but have ſomewhat of your own wherewith to fill up your time, I recommend unto you the Study of Philoſophy. *Farewell.*

T. H.

The Titles of the CHAPTERS.

The first Part, or Logique.

CHAP.

1 *Of Philosophy.*
2 *Of Names.*
3 *Of Proportion.*
4 *Of Syllogisme.*
5 *Of Erring, Falsity and Captions.*
6 *Of Method.*

The Second Part, or The first Grounds of Philosophy.

7 *Of Place and Time.*
8 *Of Body and Accident.*
9 *Of Cause and Effect.*
10 *Of Power and Act.*
11 *Of Identity and Difference.*
12 *Of Quantity.*
13 *Of Analogisme, or the Same Proportion.*
14 *Of Straight and Crooked, Angle and Figure.*

The third Part, Of the Proportions of Motions and Magnitudes.

15 *Of the Nature, Properties, and divers considerations, of Motion and Endeavour.*
16 *Of Motion Accelerated and Uniform, and of Motion by Concourse.*
17 *Of Figures Deficient.*
18 *Of the Equation of Straight Lines, which the Crooked Lines of Parabolas, and other Figures made in imitation of Parabolas.*
19 *Of Angles of Incidence and Reflexion, equal by supposition.*
20 *Of the Dimension of a Circle, and the Division of Arches or Angles.*
21 *Of Circular Motion.*
22 *Of other Variety of Motions.*
23 *Of the Center of Equiponderation of Bodies pressing downwards in straight parallel lines.*
24 *Of Refraction and Reflexion.*

The fourth Part, of Physiques, or the *Phænomena* of Nature.

25 *Of Sense and Animall Motion.*
26 *Of the World and of the Starres.*
27 *Of Light, Heat, and of Colours.*
28 *Of Cold, Wind, Hard, Ice, Restitution of Bodies bent, Diaphanous, Lightning and Thunder, and of the Heads of Rivers.*
29 *Of Sound, Odour, Savour, and Touch.*
30 *Of Gravity.*

COMPUTATION OR LOGIQUE.

CHAP. 1.

Of Philosophy.

1 *The Introduction.* 2 *The Definition of Philosophy explained.* 3 *Ratiocination of the Mind.* 4 *Properties what they are.* 5 *How Properties are known by Generation, & contrarily.* 6 *The Scope of Philosophy.* 7 *The Utility of it.* 8 *The Subject.* 9 *The Parts of it.* 10 *The Epilogue.*

PHILOSOPHY seems to me to be amongst men now, in the same manner as Corn and Wine are said to have been in the world in ancient time. For from the beginning there were Vines and Ears of Corn growing here and there in the fields; but no care was taken for the planting and sowing of them. Men lived therefore upon Akorns; or if any were so bold as to venture upon the eating of those unknown and doubtfull fruits, they did it with danger of their health. In like manner, every man brought Philosophy, that is, Naturall Reason, into the world with him; for all men can reason to some degree, and concerning some things: but where there is need of a long series of Reasons, there most men wander out of the way, and fall into Error for want of Method, as it were for want of sowing and planting, that is, of improving their Reason. And from hence it comes to passe, that they who content themselves with daily experience, which may be likened to feeding upon Akorns, and either reject, or not much regard Philoso-

phy, are commonly esteemed, and are indeed, men of sounder judgement, then those, who from opinions, though not vulgar, yet full of uncertainty, and carelesly received, do nothing but dispute and wrangle, like men that are not well in their wits. I confesse indeed, that that part of Philophy by which Magnitudes and Figures are computed, is highly improved. But because I have not observed the like advancement in the other parts of it, my purpose is, as far forth as I am able, to lay open the few and first Elements of Philosophy in generall, as so many Seeds, from which pure and true Philosophy may hereafter spring up by little and little.

I am not ignorant how hard a thing it is to weed out of mens mindes such inveterate opinions as have taken root there, and been cõfirmed in them by the authority of most eloquent Writers; especially, seeing true (that is accurate) Philosophy, professedly rejects not only the paint and false colours of Language, but even the very ornaments and graces of the same; and the first Grounds of all Science, are not only not beautifull, but poore, aride, and in appearance deformed. Nevertheless, there being certainly some men, though but few, who are delighted with Truth and strength of Reason in all things, I thought I might do well to take this pains for the sake even of those few. I proceed therefore to the matter, and take my beginning from the very Definition of Philosophy, which is this.

2 PHILOSOPHY *is such knowledge of Effects or Appearances, as we acquire by true Ratiocination from the knowledge we have first of their Causes or Generation: And again, of such Causes or Generations as may be from knowing first their Effects.*

For the better understanding of which Definition, we must consider; first, that although Sense and Memory of things, which are common to Man and all living Creatures, be Knowledge, yet because they are given us immediately by Nature, and not gotten by Ratiocination, they are not Philosophy.

Secondly, Seeing Experience is nothing but Memory; and Prudence, or Prospect into the future time, nothing but Expectation of such things as we have already had experience of, Prudence also is not to be esteemed Philosophy.

By RATIOCINATION, I mean *Computation*. Now to compute

pute, is either to collect the ſum of many things that are added together, or to know what remains when one thing is taken out of another. *Ratiocination* therefore is the ſame with *Addition* and *Subſtraction*; and if any man adde *Multiplication* and *Diviſion*, I will not be againſt it, ſeeing Multiplication is nothing but Addition of equals one to another; and Diviſion nothing but a Subſtraction of equals one from another, as often as is poſſible. So that all Ratiocination is comprehended in theſe two operations of the minde, Addition and Subſtraction.

3 But how by the *Ratiocination* of our Minde, we Adde and Subſtract in our ſilent thoughts, without the uſe of words, it will be neceſſary for me to make intelligible by an example or two. If therefore a man ſee ſomething a far off and obſcurely, although no appellation had yet been given to any thing, he will notwithſtanding have the ſame Idea of that thing, for which now by impoſing a name on it, we call it *Body*. Again, when by comming neerer, he ſees the ſame thing thus and thus, now in one place and now in another, he will have a new Idea thereof, namely that, for which we now call ſuch a thing *Animated*. Thirdly, when ſtanding neerer he perceives the figure, hears the voice, and ſees other things, which are ſignes of a Rationall minde, he has a third Idea, though it have yet no appellation, namely, that for which we now call any thing *Rationall*. Laſtly, when by looking fully and diſtinctly upon it he conceaves all that he has ſeen as one thing, the Idea he has now, is compounded of his former Ideas, which are put together in the Minde, in the ſame order, in which theſe three ſingle names *Body*, *Animated*, *Rationall*, are in ſpeech compounded into this one name *Body-Animated-Rationall*, or *Man*. In like manner, of the ſeverall conceptions of *four ſides*, *equality of ſides*, *and right angles*, is compounded the conception of a *Square*. For the mind may conceive a figure of foure ſides without any conception of their equality; & of that equality without conceiving a right angle; and may joyne together all theſe ſingle conceptions into one conception or one Idea of a Square. And thus we ſee how the Conceptions of the mind are compounded. Again, whoſoever ſees a man ſtanding neer him, conceives the whole Idea of that man; and if as he goes away he follow him with his eyes onely, he will loſe the Idea of thoſe things which

were ſignes of his being Rationall, whileſt nevertheleſſe the Idea of a Body-Animated remaines ſtill before his eies; ſo that the Idea of Rationall is ſubſtracted from the whole Idea of Man, that is to ſay of Body-Animated-Rationall, and there remaines that of Body-Animated; & a while after at a greater diſtance the Idea of Animated will be loſt, & that of Body only will remain; ſo that at laſt, when nothing at all can be ſeen, the whole Idea will vaniſh out of ſight. By which examples, I thinke it is manifeſt enough, what is the internall Ratiocination of the Mind, without words.

We muſt not therefore thinke that Computation, that is, Ratiocination, has place onely in numbers; as if man were diſtinguiſhed from other living Creatures (which is ſaid to have been the opinion of *Pythagoras*) by nothing but the faculty of numbring; for *Magnitude*, *Body*, *Motion*, *Time*, *Degrees of Quality*, *Action*, *Conception*, *Proportion*, *Speech* and *Names* (in which all the kinds of Philoſophy conſiſt) are capable of Addition and Subſtraction. Now ſuch things as we adde or ſubſtract, that is, which we put into an account, we are ſaid to *conſider*, in Greek λογίζεσθαι; in which language alſo συλλογίσθαι ſignifies to *Compute*, *Reaſon* or *Reckon*.

4 But *Effects* and the *Appearances* of things to ſenſe, are faculties or Powers of Bodies, which make us diſtinguiſh them from one another; that is to ſay, conceive one Body to be equall or unequall, like or unlike to another Body; as in the example above, when by coming neer enough to any Body, we perceive the Motion and Going of the ſame, we diſtinguiſh it thereby from a Tree, a Column and other fixed Bodies; and ſo that motion, or going is the *Property* thereof, as being proper to living creatures, and a faculty by which they make us diſtinguiſh them from other Bodies.

5 How the knowledge of any Effect may be gotten from the knowledge of the Generation thereof, may eaſily be underſtood by the example of a Circle: For if there be ſet before us a plain figure having as neer as may be the figure of a Circle, we cannot poſſibly perceive by ſenſe whether it be a true Circle or no; then which nevertheleſſe nothing is more eaſie to be known, to him that knowes firſt the Generation of the propounded figure. For let it be known that the figure was made by the circumduction of

a Body whereof one end remained unmoved, and we may reaſon thus; a Body carried about, retaining alwayes the ſame length, applies it ſelfe firſt to one *Radius*, then to another, to a third, a fourth, and ſucceſſively to all; and therefore the ſame length, from the ſame point, toucheth the circumference in every part thereof; which is as much to ſay as all the *Radii* are equal. We know therefore that from ſuch generation proceeds a figure, from whoſe one middle point all the extreame points are reached unto by equal *Radii*. And in like manner, by knowing firſt what figure is ſet before us, we may come by Ratiocination to ſome Generation of the ſame, though perhaps not that by which it was made, yet that by wch it might have been made; for he that knows that a Circle has the property above declared, will eaſily know whether a Body carried about, as is ſaid, will generate a Circle or no.

6 The *End* or *Scope* of Philoſophy, is, that we may make uſe to our benefit of effects formerly ſeen; or that by applicatiō of Bodies to one another; we may produce the like effects of thoſe we conceive in our minde, as far forth as matter, ſtrength & induſtry will permit, for the commodity of humane life. For the inward glory and triumph of mind that a man may have, for the maſtering of ſome difficult and doutfull matter, or for the diſcovery of ſome hidden truth, is not worth ſo much paines as the ſtudy of Philoſophy requires; nor need any man care much to teach another what he knowes himſelfe, if he think that will be the onely benefit of his labour. The end of Knowledge is Power; and the uſe of Theoremes (which among Geometricians ſerve for the finding out of Properties) is for the conſtruction of Problemes; and laſtly, the ſcope of all ſpeculation is the performing of ſome action, or thing to be done.

7 But what the *Utility* of Philoſophy is, eſpecially of Natural Philoſophy and Geometry, will be beſt underſtood by reckoning up the chief commodities of which mankind is capable; and by comparing the manner of life of ſuch as enjoy them, with that of others which want the ſame. Now the greateſt commodities of mankind are the Arts, namely of meaſuring Matter and Motion; of moving ponderous Bodies; of Architecture; of Navigation; of making inſtruments for all uſes; of calculating the Cœleſtiall Motions, the Aſpects of the Stars, and the parts of Time; of Geography,

graphy &c. By which Sciences, how great benefits men receive, is more easily understood then expressed. These benefits are enjoyed by almost all the people of Europe, by most of those of Asia, and by some of Africa; but the Americans, and they that live neer the Poles do totally want them. But why? Have they sharper wits then these? Have not all men one kinde of soule, and the same faculties of mind? What then makes this difference, except Philosophy? Philosophy therefore is the cause of all these benefits. But the Utility of Morall and Civil Philosophy is to be estimated not so much by the commodities we have by knowing these Sciences, as by the calamities we receive from not knowing them. Now all such calamities as may be avoided by humane industry arise from warre, but chiefly from Civil warre; for from this proceed Slaughter, Solitude, and the want of all things. But the cause of warre is not that men are willing to have it; for the Will has nothing for Object but Good, at least that which seemeth good. Nor is it from this, that men know not that the effects of war are evil; for who is there that thinks not poverty and losse of life to be great evils? The cause therefore of Civill warre, is that men know not the causes neither of Warre nor Peace, there being but few in the world that have learned those duties which unite and keep men in peace, that is to say, that have learned the rules of civill life sufficiently. Now the knowledge of these rules is Morall Philosophy. But why have they not learned them, unlesse for this reason that none hitherto have taught them in a clear and exact method? For what shall we say? Could the ancient Masters of Greece, Egypt, Rome, and others perswade the unskillfull multitude to their innumerable opinions concerning the nature of their Gods, which they themselves knew not whether they were true or false, and which were indeed manifestly false & absurd; & could they not perswade the same multitude to civill duty, if they themselves had understood it? Or shall those few writings of Geometricians which are extant, be thought sufficient for the taking away of all controversy in the matters they treat of, and shall those innumerable and huge Volumes of *Ethicks* be thought unsufficient, if what they teach had been certain and well demonstrated? What then can be imagined to be the cause that the writings of

those

thoſe men have increaſed ſcience, and the writings of theſe have increaſed nothing but words, ſaving that the former were written by men that knew, and the later by ſuch as knew not the doctrine they taught onely for oſtentation of their wit and eloquence? Nevertheleſſe, I deny not but the reading of ſome ſuch books is very delightfull; for they are moſt eloquently written, and containe many cleer, wholſome and choice ſentences; which yet are not univerſally true, though by them univerſally pronounced. From whence it comes to paſſe, that the circumſtances of times, places and perſons being changed, they are no leſſe frequently made uſe of to confirme wicked men in their purpoſes, then to make them underſtand the precepts of Civill duties. Now that which is chiefly wanting in them, is a true and certaine rule of our actions, by which we might know whether that we undertake be juſt or unjuſt. For it is to no purpoſe to be bidden in every thing to do Right, before there be a certain Rule and meaſure of Right eſtabliſhed; which no man hitherto hath eſtabliſhed. Seeing therefore from the not knowing of Civill duties, that is, from the want of Morall ſcience proceed Civill warres, and the greateſt calamities of mankind, we may very well attribute to ſuch ſcience the production of the contrary commodities. And thus much is ſufficient, to ſay nothing of the prayſes and other contentment proceeding from Philoſophy, to let you ſee the Utility of the ſame in every kinde thereof.

8 The *Subject* of Philoſophy, or the matter it treats of, is every Body of which we can conceive any generation, and which we may by any conſideration thereof compare with other Bodies; or which is capable of compoſition and reſolution; that is to ſay, every Body, of whoſe Generation or Properties we can have any knowledge. And this may be deduced from the Definition of Philoſophy, whoſe profeſſion it is to ſearch out the Properties of Bodies from their Generation, or their Generation from their Properties; and therefore where there is no Generation nor Property, there is no Philoſophy. Therefore it excludes *Theology*, I meane the doctrine of God, Eternal, Ingenerable, Incomprehenſible, and in whom there is nothing neither to divide nor compound, nor any Generation to be conceived.

It excludes the doctrine of *Angels*, and all ſuch things as are

thought

thought to be neither Bodies, nor properties of Bodies; there being in them no place neither for compoſition, nor diviſion, nor any capacity of more and leſſe; that is to ſay, no place for Ratiocination.

It excludes *Hiſtory*, as well *Naturall* as *Politicall*, though moſt uſefull (nay neceſſary) to Philoſophy; becauſe ſuch Knowledge is but Experience, or Authority, and not Ratiocination.

It excludes all ſuch Knowledge as is acquired by Divine Inſpiration, or Revelation, as not derived to us by Reaſon, but by Divine grace in an inſtant, and as it were by ſome ſenſe ſupernaturall.

It excludes, not onely all Doctrines which are falſe, but ſuch alſo as are not well grounded; for whatſoever we know by right Ratiocination, can neither be falſe nor doubtfull; and therefore *Aſtrology*, as it is now held forth, and all ſuch Divinations rather then ſciences, are excluded.

Laſtly, the doctrine of *Gods Worſhip* is excluded from Philoſophy, as being not to be known by naturall reaſon, but by the authority of the Church; and as being the object of Faith, and not of Knowledge.

9 The principall parts of Philoſophy are two. For two chief kinds of Bodies, and very different from one another, offer themſelves to ſuch as ſearch after their Generation & Properties; One whereof being the worke of Nature, is called a *Naturall Body*; the other is called a *Commonwealth*, and is made by the wills and agreement of men. And from theſe ſpring the two parts of Philoſophy called *Naturall* and *Civill*. But ſeeing that for the knowledge of the Properties of a Common-wealth, it is neceſſary firſt to know the Diſpoſitions, Affections and Manners of men, Civill Philoſophy is againe commonly divided into two parts; whereof one which treats of mens Diſpoſitions and Manners is called *Ethicks*, and the other which takes cogniſance of their Civil Duties is called *Politicks* or ſimply *Civill Philoſophy*. In the firſt place therefore (after I have ſet downe ſuch Premiſſes as appertaine to the nature of Philoſophy in general) I will diſcourſe of *Bodies Naturall*; in the ſecond, of the *Diſpoſitions and Manners of men*; and in the third, of the *Civill Duties of Subjects*.

10 To conclude, ſeeing there may be many who will not like this

this my Definition of Philoſophy, and will ſay that from the liberty which a man may take of ſo defining as ſeemes beſt to himſelfe, he may conclude any thing from any thing (though I thinke it no hard matter to demonſtrate, that this Definition of mine agrees with the ſenſe of all men;) yet leſt in this point there ſhould be any cauſe of diſpute betwixt me and them, I here undertake no more then to deliver the Elements of that Science, by which the Effects of any thing may be found out from the known Generation of the ſame, or contrarily the Generation from the Effects; to the end that they who ſearch after other Philoſophy, may be admoniſhed to ſeeke it from other Principles.



C CHAP.

Chap. II.
Of Names.

1 *The necessity of sensible Moniments or Marks for the help of Memory, A Marke defined.* 2 *The necessity of Marks for the signification of the conceptions of the Mind.* 3 *Names supply both those necessities.* 4 *The Definition of a Name.* 5 *Names are Signes not of Things, but of our Cogitations.* 6 *What it is we give Names to.* 7 *Names Positive and Negative.* 8 *Contradictory Names.* 9 *A Common Name.* 10 *Names of the first and second Intention.* 11 *Universall, Particular, Individuall, and Indefinite Names.* 12 *Names Univocall and Equivocal.* 13 *Absolute and Relative Names.* 14 *Simple and Compounded Names.* 15 *A Predicament described.* 16 *Some things to be noted concerning Predicaments.*

1 

How unconstant and fading mens thoughts are, and how much the recovery of them depends upon chance, there is none but knows by infallible experience in himself. For no man is able to remember Quantities without sensible and present Measures, nor Colours without sensible and present Patterns, nor Number without the Names of Numbers disposed in order and learned by heart. So that whatsoever a man has put together in his mind by ratiocination without such helps, will presently slip from him, and not be revocable but by beginning his ratiocination anew. From which it followes, that for the acquiring of Philosophy some sensible Moniments are necessary, by which our past thoughts may be not onely reduced, but also registred every one in its own order. These Moniments I call Marks, namely, sensible things taken at pleasure, that by the sense of them such thoughts may be recalled to our mind, as are like those thoughts for which we took them.

2 Again,

2 Again, though ſome one man, of how excellent a wit ſoever, ſhould ſpend all his time, partly in reaſoning and partly in inventing Marks for the help of his memory, and advancing himſelf in learning; who ſees not, that the benefit he reapes to himſelfe will not be much, and to others none at all? For unleſſe he communicate his notes with others, his ſcience will periſh with him. But if the ſame notes be made common to many, and ſo one mans inventions be taught to others, ſciences will thereby be encreaſed to the generall good of mankind. It is therefore neceſſary for the acquiring of Philoſophy that there be certain Signes, by which what one man finds out may be manifeſted and made known to others. Now thoſe things we call SIGNES, are the *Antecedents of their Conſequents, and the Conſequents of their Antecedents, as often as we obſerve them to go before or follow after in the ſame manner*. For example, a thick Cloud is a Signe of Rain to follow; and Rain a Signe, that a Cloud has gone before, for this reaſon onely, that we ſeldom ſee Clouds without the Conſequence of Rain, nor Rain at any time but when a Cloud has gone before. And of Signs ſome are *Naturall*, whereof I have already given an example; others are *Arbitrary*, namely, thoſe we make choice of at our own pleaſure; as a buſh hung up, ſignifies that Wine is to be ſold there; a ſtone ſet in the ground, ſignifies the bound of a field; and words ſo and ſo connected, ſignifie the Cogitations and Motions of our Minde. The difference therefore betwixt Markes and Signes is this, that we make thoſe for our own uſe, but theſe for the uſe of others.

3 Words ſo connected, as that they become ſignes of our Thoughts, are called SPEECH, of which every part is a *Name*. But ſeeing (as is ſaid) both Markes and Signes are neceſſary for the acquiring of Philoſophy, (Marks by which we may remember our own thoughts, and Signes by which we may make our thoughts known to others,) Names do both theſe offices; but they ſerve for Marks before they be uſed as Signes. For though a man were alone in the world, they would be uſefull to him in helping him to remember; but to teach others, (unleſſe there were ſome others to be taught,) of no uſe at all. Again, Names, though ſtanding ſingly by themſelves, are Marks, becauſe they ſerve to recall our own thoughts to mind; but they cannot be Signes, otherwiſe then by being diſpoſed and ordered in Speech,

as parts of the ſame. For example, a man may begin with a word, whereby the hearer may frame an Idea of ſomething in his mind, which nevertheleſſe he cannot conceive to be the Idea which was in the mind of him that ſpake, but that he would ſay ſomething which began with that word, though perhaps not as by it ſelfe, but as part of another word. So that the nature of a name conſiſts principally in this, that it is a Mark taken for Memories ſake; but it ſerves alſo by accident, to ſignifie and make known to others what we remember our ſelves; and therefore I will define it thus:

4 *A* NAME *is a Word taken at pleaſure to ſerve for a Mark, which may raiſe in our Mind a thought like to ſome thought we had before, and which being pronounced to others, may be to them a Sign of what thought the ſpeaker had or had not before in his mind.* And it is for brevities ſake that I ſuppoſe the Originall of Names to be Arbitrary, judging it a thing that maybe aſſumed as unqueſtionable. For cõſideringthat new Names are daily made, and old ones laid aſide; that diverſe Nations uſe different Names, and how impoſſible it is either to obſerve ſimilitude, or make any compariſon betwixt a Name and a Thing, how can any man imagine that the Names of Things were impoſed from their natures? For though ſome Names of living creatures and other things, which our firſt Parents uſed, were taught by God himſelfe; yet they were by him arbitrarily impoſed, and afterwards both at the Tower of Babel, and ſince in proceſſe of time, growing every where out of uſe, are quite forgotten, and in their roomes have ſucceeded others, invented and received by men at pleaſure. Moreover, whatſoever the common uſe of words be, yet Philoſophers, who were to teach their knowledge to others, had alwayes the liberty, and ſometimes they both had and will have a neceſſity, of taking to themſelves ſuch Names as they pleaſe for the ſignifying of their meaning, if they would have it underſtood. Nor had Mathematicians need to aske leave of any but themſelves to name the figures they invented *Parabolas, Hyperboles, Ciſſoeides, Quadratrices, &c.* or to call one Magnitude A, another B.

5 But ſeeing Names ordered in ſpeech (as is defined) are ſignes of our Conceptions, it is manifeſt they are not ſignes of the Things themſelves; for that the ſound of this word *Stone* ſhould be the

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signe of a Stone, cannot be understood in any sense but this, that he that heares it, collects that he that pronounces it thinkes of a Stone. And therefore that disputation, whether Names signifie the Matter or Form, or something compounded of both, and other like subtleties of the *Metaphysicks*, is kept up by erring men, and such as understand not the words they dispute about.

6 Nor indeed is it at all necessary that every Name should be the Name of some Thing. For as these, a *Man*, a *Tree*, a *Stone*, are the Names of the Things themselves; so the Images of a Man, of a Tree and of a stone, which are represented to men sleeping, have their Names also, though they be not Things, but onely fictions and Phantasmes of things. For we can remember these; and therefore it is no lesse necessary that they have Names to mark and signifie them, then the Things themselves. Also this word *Future* is a Name, but no future thing has yet any being, nor do we know whether that which we call Future, shall ever have a being or no. Nevertheless, seeing we use in our mind to knit together things Past with those that are Present, the Name *Future* serves to signifie such knitting together. Moreover, that which neither is, nor has been, nor ever shall or ever can be, has a name, namely, *That which neither is, nor has been, &c.* Or more briefly this, *Impossible*. To conclude, this word *Nothing*, is a name, which yet cannot be the name of any thing. For when (for example) we substract 2 and 3 from 5, and so nothing remaining we would call that substraction to mind, this speech *Nothing remains*, and in it the word *Nothing* is not unusefull. And for the same reason we say truly *Lesse then Nothing* remaines, when we substract More from Lesse; for the minde feigns such remaines as these for Doctrines sake, and desires as often as is necessary, to call the same to memory. But seeing every name has some relation to that which is named, though that which we name be not alwaies a thing that has a being in Nature, yet it is lawfull for Doctrines sake to apply the word *Thing*, to whatsoever we name; as if it were all one whether that thing be truly existent or be onely feigned.

7 The first distinction of Names, is that some are *Positive*, or *Affirmative*; others *Negative*, which are also called *Privative* and *Indefinite*. *Positive* are such as we impose for the likenesse, Equality

lity or Identity of the things we conſider; *Negative* for the diverſity, Unlikneſſe, or Inequality of the ſame. Examples of the former are *a Man, a Philoſopher*; for *a Man* denotes any one of a multitude of men, and *a Philoſopher* any one of many Philoſophers by reaſon of their ſimilitude; Alſo *Socrates* is a Poſitive name, becauſe it ſignifies alwayes one and the ſame man. Examples of *Negatives* are ſuch Poſitives as have the Negative particle *Not* added to them, as *Not-Man, Not-Philoſopher*. But Poſitives were before Negatives; for otherwiſe there could have been no uſe at all of theſe. For when the name of *White* was impoſed upon certain things, and afterwards upon other things the names of *Black, Blew, Tranſparent, &c.* the infinite diſſimilitudes of theſe with *White* could not be comprehended in any one Name, ſave that which had in it the Negation of White, that is to ſay, the Name *Not-White*, or ſome other equivalent to it, in which the word *White* is repeated; ſuch as *Unlike to white, &c.* And by theſe *Negative names*, we take notice our ſelves, and ſignifie to others what we have not thought of.

8 Poſitive and Negative names are *Contradictory* to one another, ſo that they cannot both be the name of the ſame thing. Beſides, of Contradictory names, one is the name of any thing whatſoever; for whatſoever is, is either Man or Not-man, White or Not-white, and ſo of the reſt. And this is ſo manifeſt, that it needs no further proofe or explication; for they that ſay *the ſame thing cannot both be, and not be*, ſpeak obſcurely; but they that ſay, *Whatſoever is, either is, or is not*, ſpeake alſo abſurdly and ridiculouſly. The certainty of this Axiome, viz. *Of two Contradictory Names, one is the Name of any thing whatſoever, the other not*, is the originall and foundation of all Ratiocination, that is, of all Philoſophy; and therefore it ought to be ſo exactly propounded, that it may be of it ſelfe cleare and perſpicuous to all men; as indeed it is, ſaving to ſuch, as reading the long diſcourſes made upon this ſubject by the Writers of *Metaphyſicks* (which they beleeve to be ſome egregious learning) thinke they underſtand not, when they do.

9 Secondly, of Names, ſome are *Common* to many things, as a *Man, a Tree*; others *Proper* to one thing, as *he that writ the Iliad, Homer, this man, that man*. And a Common name, being the name of many things ſeverally taken, but not collectively of all together (as Man

is not the name of all mankind, but of every one, as of Peter, Iohn, and the reſt ſeverally) is therefore called an *Univerſall name*; and therefore this word *Univerſall* is never the name of any thing exiſtent in nature, nor of any Idea or Phantaſme formed in the mind, but alwayes the name of ſome word or name; ſo that when *a Living creature*, *a Stone*, *a Spirit*, or any other thing is ſaid to be *Univerſal*, it is not to be underſtood, that any Man, Stone, &c. ever was or can be Univerſall, but onely that theſe words, *Living creature*, *Stone*, *&c.* are *Univerſall names*, that is, Names common to many things; and the Conceptions anſwering them in our minde, are the Images and Phantaſmes of ſeverall Living Creatures, or other things. And therfore for the underſtanding of the extent of an Univerſal name, we need no other faculty but that of our imagination, by which we remember that ſuch names bring ſometimes one thing, ſometimes another into our minde. Alſo of Common Names ſome are more, ſome leſſe Common. *More Common*, is that which is the name of more things; *Leſſe Common*, the name of fewer things. As *Living-Creature* is more Common then *Man*, or *Horſe* or *Lion*, becauſe it comprehends them all; and therefore a more Common name, in reſpect of a leſſe Common, is called the *Genus* or a *Generall name*; and this in reſpect of that, the *Species*, or a *Speciall Name*.

10 And from hence proceeds the third diſtinction of Names, which is, that ſome are called names of the *Firſt*, others of the *Second Intention*. Of the *firſt Intention* are the names of Things, a *Man*, *Stone*, *&c.* of the *ſecond* are the names of names and ſpeeches, as *Univerſall*, *Particular*, *Genus*, *Species*, *Syllogiſme*, and the like. But it is hard to ſay why thoſe are called names of the *Firſt*, and theſe of the *Second Intention*, unleſſe perhaps it was firſt intended by us to give names to thoſe things which are of daily uſe in this life, and afterwards to ſuch things as appertaine to ſcience, that is, that our Second Intention was to give names to Names. But whatſoever the cauſe hereof may be, yet this is manifeſt, that *Genus*, *Species*, *Definition*, *&c.* are names of Words and Names onely; and therefore to put *Genus* and *Species* for Things, and *Definition* for the nature of any thing, as the Writers of *Metaphyſicks* have done, is not right, ſeeing they be only ſignifications of what we thinke of the nature of Things.

11 Fourthly, ſome Names are of *certaine* and *determined*, others

of

of *uncertaine* and *undetermined* signification. Of *determined* and *certain* signification is, first, that name which is giuen to any one thing by it selfe, and is called an *Individuall Name*; as *Homer*, *this tree*, *that liuing Creature*, &c. Secondly that which has any of these words *All*, *Every*, *Both*, *Either*, or the like added to it; and it is therefore called an Universall Name, because it signifies every one of those things to which it is Common; and of *certaine* signification for this reason, that he which heares, conceives in his minde the same thing that he which speakes would have him conceive. Of *Indefinite* signification is, first, that Name which has the word *some*, or the like added to it, and is called a *Particular name*; Secondly a Common Name set by it selfe without any note either of Universality or Particularity, as *Man*, *Stone*, and is called an *Indefinite Name*; but both *Particular* and *Indefinite* names are of uncertaine signification, because the Hearer knowes not what thing it is the Speaker would have him conceive; and therefore in Speech, Particular and Indefinite names are to be esteemed equivalent to one another. But these words, *All*, *Every*, *Some*, &c. which denote Universality and Particularity, are not Names, but parts onely of Names; So that *Every Man*, and *That Man which the Hearer conceives in his mind*, are all one; and *Some Man*, and *That Man which the Speaker thought of* signifie the same. From whence it is evident, that the use of signes of this kind, is not for a mans own sake, or for his getting of knowledge by his own private meditation (for every man has his own, Thoughts sufficiently determined without such helpes as these) but for the sake of others; that is, for the teaching and signifying of our Conceptions to others; nor were they invented onely to make us remember, but to make us able to discourse with others.

12 Fifthly, Names are usually distinguished into *Univocall*, and *Equivocall*. *Univocall* are those which in the same train of Discourse signifie alwayes the same thing; but *Equivocall* those which meane sometimes one thing, and sometimes another. Thus, the Name *Triangle* is said to be *Univocall*, because it is alwayes taken in the same sense; and *Parabola* to be *Equivocall*, for the signification it has sometimes of Allegory or Similitude, and sometimes of a certaine Geometricall figure. Also every *Metaphor* is by profession *Equivocall*. But this distinction belongs not so much to Names, as to those that use Names; for some use them properly and accurately

curately for the finding out of truth; others draw them from their proper ſenſe, for Ornament, or Deceipt.

13 Sixtly, of Names, ſome are *Abſolute*, others *Relative*. *Relative* are ſuch as are impoſed for ſome Compariſon, as *Father*, *Sonne*, *Cauſe*, *Effect*, *Like*, *Unlike*, *Equal*, *Unequal*, *Maſter*, *Servant*, *&c.* And thoſe that ſignifie no Compariſon at all are *Abſolute Names*. But as it was noted above, that Univerſality is to be attributed to Words & Names onely, and not to Things; ſo the ſame is to be ſaid of other diſtinctions of Names; for no Things are either *Univocall* or *Equivocall*, or *Relative* or *Abſolute*. There is alſo another diſtinction of Names into *Concrete* and *Abſtract*; but becauſe Abſtract Names proceed from Propoſition, and can have no place where there is no Affirmation, I ſhall ſpeake of them hereafter.

14 Laſtly, there are *Simple* and *Compounded Names*. But here it is to be noted, that a name is not taken in Philoſophy, as in Grammar, for one ſingle word, but for any number of words put together to ſignifie one Thing; for among Philoſophers *Sentient Animated Body*, paſſes but for one Name, being the Name of every living Creature; which yet, among Grāmarians is accounted three Names. Alſo a *Simple Name* is not here diſtinguiſhed from a *Compounded Name* by a Prepoſition, as in Grammar. But I call a *Simple Name*, that which in every kind is the moſt Common or moſt Univerſall; and that a *Compounded Name*, which by the joyning of another Name to it is made leſſe Univerſall, and ſignifies that more conceptions then one were in the mind, for which that later Name was added. For example, in the conception of Man (as is ſhewn in the former Chap.) Firſt, he is conceived to be ſomething that has Extenſion, which is marked by the word *Body*. *Body* therefore is a *Simple Name*, being put for that firſt ſingle Conception; Afterwards, upon the ſight of ſuch and ſuch motion, another Conception ariſes for which he is called an *Animated Body*; and this I here call a *Compounded Name*, as I doe alſo the name *Animal*, which is equivalent to an *Animated Body*. And in the ſame manner an *Animated Rational Body*, as alſo a *Man*, which is equivalent to it, is a more Compounded Name. And by this we ſee how the Compoſition of Conceptions in the mind is anſwerable to the Compoſition of Names; for as in the minde one Idea or Phantaſme ſucceeds to another, and to this a third; ſo to one Name is added another and another ſucceſ-

ſively, and of them all is made one *Compounded* Name. Neverthe-leſſe we muſt not thinke Bodies, which are without the Minde, are compounded in the ſame manner, namely that there is in Nature a Body, or any other imaginable Thing exiſtent, which at the firſt has no Magnitude, and then by the addition of Magnitude, comes to have Quantity, and by more or leſſe Quantity to have Denſity or Rarity, and again by the addition of Figure to be Figurate, and after this by the injection of Light or Colour, to become Lucid or Coloured; though ſuch has been the Philoſophy of many.

15 The Writers of *Logique* have endeavoured to digeſt the Names of all the kinds of Things into certaine *Scales* or Degrees, by the continual ſubordination of Names leſſe Common, to Names more Common. In the Scale of *Bodies* they put in the firſt and higheſt place *Body* ſimply, and in the next place under it leſſe Common Names, by which it may be more limited and determined, namely *Animated* and *Inanimated*, and ſo on till they come to *Individualls*. In like manner in the Scale of *Quantities* they aſſign the firſt place to *Quantity*, and the next to *Line*, *Superficies*, and *Solid*, which are Names of leſſe latitude; and theſe Orders or Scales of Names they uſually call *Prædicaments* and *Categories*; And of this Ordination not onely Poſitive but Negative Names alſo are capable; which may be exemplified by ſuch Formes of the *Prædicaments* as follow.

The Form of the Prædicament of *Body*.

Not-Body; or Accident

Body
- Not animated
- Animated
  - Not Living-Creature
  - Living-Creature
    - Not Man
    - Man
      - Not Peter
      - Peter

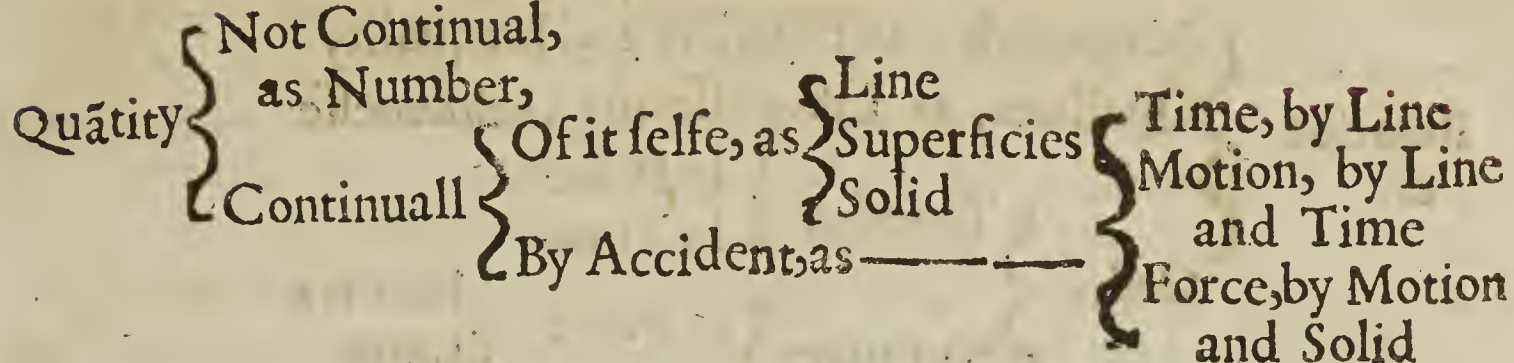
Both Accident and Body are conſidered
- Abſolutely as,
  - Quantity, or ſo much
  - Quality, or ſuch

or

- Comparatively, which is called their Relation

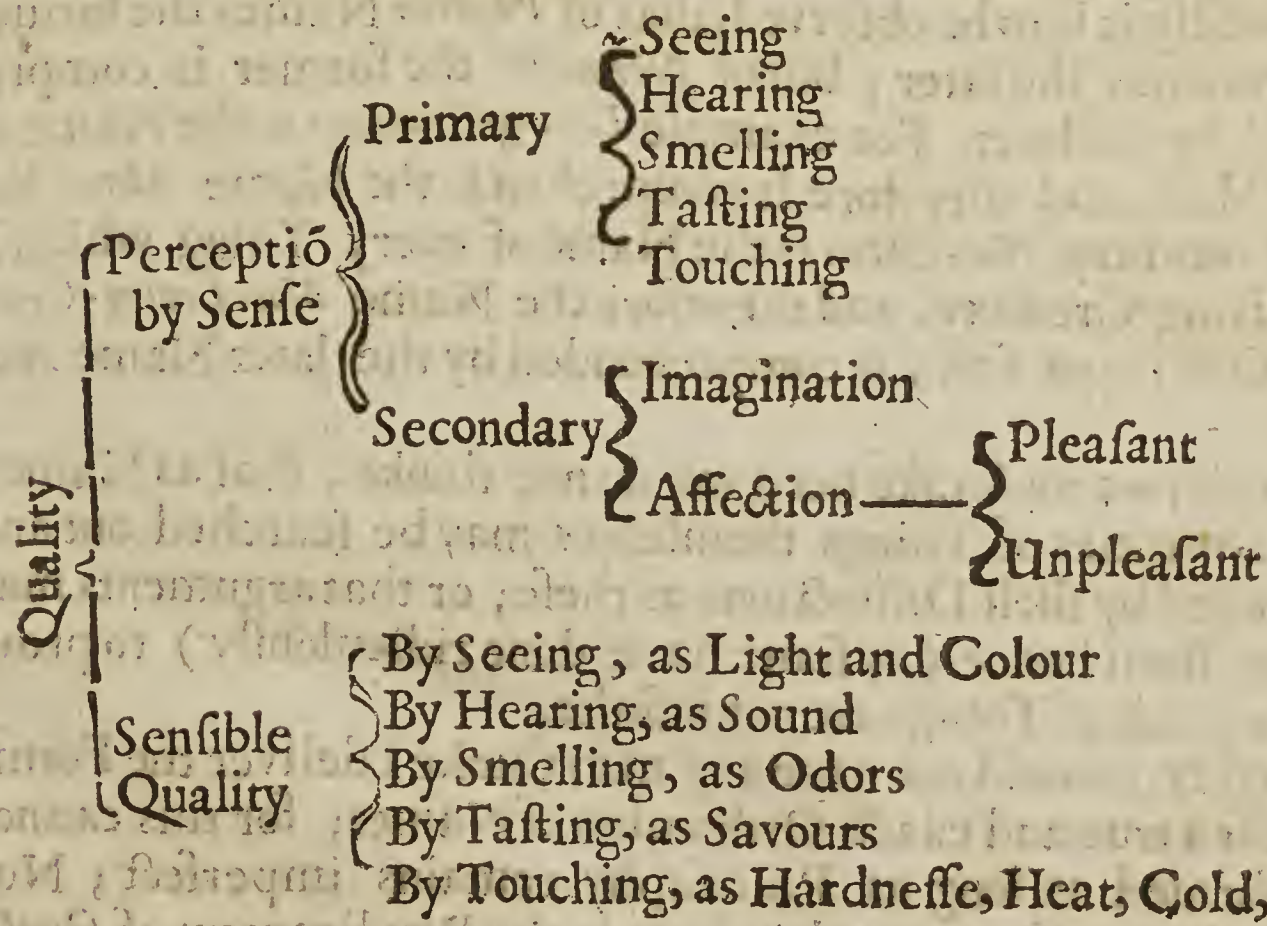
The

The Forme of the Prædicament of *Quantity*.



Where it is to be noted, that *Line*, *Superficies* and *Solid* may be said to be of such and such Quantity, that is, to be originally and of their own nature capable of Equality and Inequality; But we cannot say there is either Majority or Minority, or Equality, or indeed any Quantity at all, in *Time*, without the help of *Line* and *Motion*; nor in *Motion*, without *Line* and *Time*; nor in *Force*, otherwise then by *Motion* and *Solid*.

The Forme of the Prædicament of *Quality*.



The forme of the Prædicament of *Relation.*

- Relation of
  - Magnitudes, as Equality and Inequality
  - Qualities, as Likeneſſe and Unlikeneſſe
  - Order
    - Together
      - In Place
      - In Time
    - Not together
      - In Place
        - Former
        - Later
      - In Time
        - Former
        - Later

16 Concerning which Prædicaments it is to be noted in the firſt place, That as the diviſion is made in the firſt Prædicament into Contradictory Names, ſo it might have been done in the reſt. For as there, *Body* is divided into *Animated* and *Not-Animated*, ſo in the ſecond Prædicament *Continuall Quantity* may be divided into *Line* and *Not-line*, and again, *Not-line* into *Superficies* and *Not-Superficies*, and ſo in the reſt; but it was not neceſſary.

Secondly, it is to be obſerved, that of *Poſitive* Names the former comprehends the later; but of *Negatives* the former is comprehended by the later. For example *Living-Creature* is the Name of every Man, and therefore it comprehends the Name *Man*; but on the contrary *Not-Man* is the Name of every Thing which is Not-Living-Creature, and therefore the Name *Not-Living-Creature* which is put firſt, is comprehended by the later Name *Not-Man.*

Thirdly, we muſt take heed we do not thinke, that as Names, ſo the diverſities of Things themſelves may be ſearched out and determined by ſuch Diſtinctions as theſe; or that arguments may be taken from hence (as ſome have done ridiculouſly) to prove that the kinds of Things are not infinite.

Fourthly, I would not have any man thinke I deliver the Forms above for a true and exact Ordination of Names; for this cannot be performed as long as Philoſophy remains imperfect; Nor that by placing (for example) *Light* in the Prædicament of *Qualities*, while another places the ſame in the Prædicament of Bodies, I pretend that either of us ought for this to be drawn from

from his opinion; for this is to be done onely by Arguments and Ratiocination, and not by disposing of words into *Classes.*

Lastly, I confesse I have not yet seen any great use of the Prædicaments in Philosophy. I beleeve *Aristotle* when he saw he could not digest the Things themselves into such Orders, might nevertheless desire out of his owne Authority to reduce Words to such Formes, as I have done; but I doe it onely for this end, that it may be understood what this Ordination of Words is, and not to have it received or true, till it be demonstrated by good reason to be so.



CHAP.

Chap. III.
Of Propoſition.

1 *Divers Kinds of Speech.* 2 *Propoſition defined.* 3 *Subject, Prædicate and Copula what they are, and Abſtract and Concrete what. The Uſe and Abuſe of Names Abſtract.* 5 *Propoſition Univerſal and Particular.* 6 *Affirmative and Negative.* 7 *True and Falſe.* 8 *True and Falſe belongs to Speech, and not to Things.* 9 *Propoſition Primary, not Primary, Definion, Axiome, Petition.* 10 *Propoſition Neceſſary and Contingent.* 11 *Categoricall and Hypotheticall.* 12 *The ſame Propoſition diverſly pronounced.* 13 *Propoſitions that may be reduced to the ſame Categoricall Propoſition, are Equipollent.* 14 *Univerſal Propoſitions converted by Contradictory Names, are Equipollent.* 15 *Negative Propoſitions are the ſame, whether the Negation be before or after the Copula.* 16 *Particular Propoſitions ſimply converted, are Equipollent.* 17 *What are Subaltern, Contrary, Subcontrary and Contradictory Propoſitions.* 18 *Conſequence what it is.* 19 *Falſity cannot follow from Truth.* 20 *How one Propoſition is the Cauſe of another.*

1 From the Connection or Contexture of Names ariſe diverſe kinds of Speech, whereof ſome ſignifie the Deſires and Affections of Men; ſuch are firſt *Interrogations*, which denote the deſire of Knowing; as *Who is a good Man?* In which ſpeech there is one Name expreſſed, & another deſired and expected from him of whom we aske the ſame. Then *Prayers*, which ſignifie the deſire of having ſomething; *Promiſes*, *Threats*, *Wiſhes*, *Commands*, *Complaints*, and other ſignifications of other Affections. Speech may alſo be Abſurd and Inſignificant; as when there is a ſucceſſion of Words, to which there can be no ſucceſſion of Thoughts in the mind to anſwer them; and this happens often to ſuch, as underſtanding nothing in ſome ſubtil matter, doe nevertheleſſe, to make others beleeve they underſtand, ſpeake of the ſame in-

incoherently; For the connection of incoherent Words, though it want the end of Speech (which is Signification) yet it is Speech; and is used by the Writers of *Metaphysicks* almost as frequently as Speech significative. In Philosophy there is but one kinde of Speech usefull, which some call in Latine *Dictum*, others *Enuntiatum & Pronunciatum*; but most men call it *Proposition*, and is the speech of those that Affirm or Deny, and expresseth Truth or Falsity.

2 *A* PROPOSITION *is a Speech consisting of two Names copulated, by which he that speaketh signifies he conceives the later Name to be the Name of the same thing whereof the former is the Name; or* (which is all one) *that the former Name is comprehended by the later.* For example, this speech *Man is a Living Creature*, in which two Names are copulated by the verb *Is*, is a *Proposition*, for this reason, that he that speakes it conceives both *Living Creature* and *Man* to be Names of the same thing, or that the former Name *Man* is comprehended by the later Name *Living Creature*. Now the former Name is commouly called the *Subject*, or *Antecedent*, or the *Contained Name*, and the later the *Predicat*, *Consequent* or *Containing Name*. The signe of Connection amongst most Nations is either some word, as the word *is* in the Proposition *Man is a living Creature*, or some Case or Termination of a word, as in this Proposition, *Man walketh* (which is equivalent to this, *Man is walking*) the Termination by which it is said he *walketh*, rather then he *is walking*, signifieth that those two are understood to be copulated, or to be Names of the same Thing.

But there are, or certainly may be some Nations that have no word which answers to our Verbe *Is*, who nevertheless forme Propositions by the position onely of one Name after another, as if instead of *Man is a Living Creature*, it should be said *Man a Living Creature*; for the very order of the Names may sufficiently shew their connection; and they are as apt and usefull in Philosophy, as if they were copulated by the Verbe *Is*.

3 Wherefore in every Proposition three things are to be considered, *viz.* the two Names, which are the *Subject* and the *Predicate*, and their *Copulation*; both which Names raise in our Minde the Thought of one and the same Thing; but the Copulation makes us thinke of the Cause for which those Names were imposed on that Thing. As for example, when we say *a Body is moveable*, though we conceive the same thing to be designed by both

those

thoſe Names, yet our Minde reſts not there, but ſearches further what it is *to be a Body*, or *to be Moveable*, that is, wherein conſiſts the difference betwixt theſe and other Things, for which theſe are ſo called, others are not ſo called. They therefore that ſeeke what it is *to be* any thing, as *to be Moveable*, *to be Hot*, *&c.* ſeek in Things the cauſes of their Names.

And from hence ariſes that diſtinction of Names (touched in the laſt Chap.)into *Concrete* and *Abſtract*. For *Concrete* is the Name of any thing which we ſuppoſe to have a being, and is therefore called the *Subject*, in Latine *Suppoſitum*, and in Greek ὑποκείμενον; as *Body*, *Moveable*, *Moved*, *Figurate*, *a Cubit high*, *Hot*, *Cold*, *Like*, *Equal*, *Appius*, *Lentulus* and the like; and *Abſtract* is that which in any Subject denotes the Cauſe of the Concrete Name, as *to be a Body*, *to be Moveable*, *to be Moved*, *to be Figurate*, *to be of ſuch Quantity*, *to be Hot*, *to be Cold*, *to be Like*, *to be Equall*, *to be Appius*, *to be Lentulus*, *&c.* Or Names equivalent to theſe, which are moſt commonly called *Abſtract* Names, as *Corporeity*, *Mobility*, *Motion*, *Figure*, *Quantity*, *Heat*, *Cold*, *Likeneſſe*, *Equality*, and (as Cicero has it) *Appiety* and *Lentulity*. Of the ſame kind alſo are Infinitives; for *to Live* and *to Move* are the ſame with *Life* and *Motion*, or *to be Living*, and *to be Moved*. But *Abſtract Names* denote onely the Cauſes of *Concrete Names*, and not the Things themſelves. For example when we ſee any thing, or conceive in our Minde any Viſible thing, that Thing appears to us, or is conceived by us, not in one Point, but as having Parts diſtant from one another, that is, as being extended and filling ſome ſpace. Seeing therefore we call the Thing ſo conceived *Body*, the cauſe of that name is, that that Thing is *extended*, or the *Extenſion* or *Corporeity* of it. So when we ſee a Thing appeare ſometimes here, ſometimes there, and call it *Moved* or *Removed*, the Cauſe of that Name is that it *is Moved* or the *Motion* of the ſame.

And theſe Cauſes of Names; are the ſame with the Cauſes of our Conceptions, namely ſome Power or Action, or Affection of the Thing conceived, which ſome call the Manner by which any thing workes upon our ſenſes, but by moſt men they are called *Accidents*; I ſay Accidents, not in that ſenſe in which *Accident* is oppoſed to *Neceſſary*; but ſo, as being neither the Things themſelves, nor parts thereof, do nevertheleſſe accompany the

Things

Things in ſuch manner, that (ſaving *Extenſion*) they may all periſh and be deſtroyed, but can never be *abſtracted.*

4 There is alſo this difference betwixt *Concrete* and *Abſtract Names*, that thoſe were invented before Propoſitions, but theſe after; for theſe could have no being till there were Propoſitions, from whoſe *Copula* they proceed. Now in all matters that concerne this life, but chiefly in Philoſophy, there is both great Uſe and great Abuſe of *Abſtract Names*; and the Uſe conſiſts in this, that without them we cannot for the moſt part either Reaſon, or compute the Properties of Bodies; for when we would multiply, divide, adde or ſubſtract Heat, Light, or Motion, if we ſhould double or adde them together by *Concrete* Names, ſaying (for example) Hot is double to Hot, Light double to Light, or Moved double to Moved, we ſhould not double the Properties, but the Bodies themſelves that are Hot, Light, Moved, &c. which we would not doe. But the Abuſe proceeds from this, that ſome men ſeeing they can conſider, that is (as I ſaid before) bring into account the Increaſings and Decreaſings of Quantity, Heat and other Accidents, without conſidering their Bodies or Subjects (which they call *Abſtracting*, or making to exiſt apart by themſelves,) they ſpeake of Accidents, as if they might be ſeparated from all Bodies. And from hence proceed the groſſe errors of writers of *Metaphyſicks*; for, becauſe they can conſider Thought without the conſideration of Body, they inferre there is no need of a Thinking-Body; and becauſe Quantity may be conſidered without conſidering Body, they thinke alſo that Quantity may be without Body, and Body without Quantity; and that a Body has Quantity by the addition of Quantity to it. From the ſame fountaine ſpring thoſe inſignificant words, *Abſtract Subſtance*, *Separated Eſſence* and the like; as alſo that confuſion of words derived from the Latine Verb *Eſt*, as *Eſſence*, *Eſſentiality*, *Entity*, *Entitative*; beſides *Reality*, *Aliquiddity*, *Quiddity*, &c. which could never have been heard of among ſuch Nations as do not *Copulate* their Names by the Verbe *Is*, but by Adjective Verbs, as Runneth, Readeth, &c. or by the mere placing of one Name after another; and yet ſeeing ſuch Nations Compute and Reaſon, it is evident that Philoſophy has no need of thoſe words *Eſſence*, *Entity* and other the like barbarous Termes.

There are many Distinctions of Propositions, whereof the first is, that some are *Universall*, others *Particular*, others *Indefinite*, and others *Singular*; and this is commonly called the distinction of *Quantity*. An *Universall Proposition* is that whose subject is affected with the sign of an Universall Name, as *Every man is a Living Creature*. *Particular*, that whose subject is affected with the sign of a Particular Name, as *Some Man is learned*. An *Indefinite Proposition* has for its Subject a Common Name, and put without any sign; as *Man is a Living Creature*, *Man is Learned*. And a *Singular Proposition* is that whose Subject is a singular Name, as *Socrates is a Philosopher*, *This man is black*.

6 The Second Distinction is into *Affirmative* and *Negative*, and is called the Distinction of *Quality*. An *Affirmative Proposition* is that whose Prædicate is a Positive Name, as *Man is a Living Creature*. *Negative*, that whose Prædicate is a Negative Name, as *Man is Not-a-stone*.

7 The third Distinction is, that one is *True*, another *False*. A *True Proposition* is that, whose Prædicate containes, or comprehends its Subject, or whose Prædicate is the Name of every thing, of which the Subject is the Name; as *Man is a Living Creature* is therefore a True Proposition, because whatsoever is called *Man*, the same is also called *Living Creature*; and *Some Man is sick*, is True, because *sick* is the Name of *Some Man*. That which is not True, or that whose Prædicate does not containe its Subject, is called a *False Proposition*, as *Man is a Stone*.

Now these words *True*, *Truth*, and *True Proposition* are equivalent to one another; For Truth consists in Speech, and not in the Things spoken of; and though *True* be sometimes opposed to *Apparent* or *Feigned*, yet it is alwayes to be referred to the Truth of Proposition; for the Image of a Man in a Glasse, or a Ghost, is therefore denyed to be a very Man, because this Proposition, *A Ghost is a man*, is not True; for it cannot be denied but that a Ghost is a very Ghost. And therefore Truth or Verity is not any Affection of the Thing, but of the Proposition concerning it. As for that which the Writers of *Metaphysicks* say, that *A thing*, *One thing*, and *a Very thing*, are equivalent to one another, it is but trifling and childish; for who does not know, that *A Man*, *One Man*, and a *Very Man*, signifie the same.

8 And

8 And from hence it is evident, that Truth and Falſity have no place but amongſt ſuch Living Creatures as uſe Speech. For though ſome brute Creatures, looking upon the Image of a Man in a Glaſſe, may be affected with it as if it were the Man himſelfe, and for this reaſon feare it or fawne upon it in vain; Yet they doe not apprehend it as True or Falſe, but onely as Like; and in this they are not deceived. Wherefore, as men owe all their True Ratiocination to the right underſtanding of Speech; So alſo they owe their Errors to the miſunderſtanding of the ſame; and as all the Ornaments of Philoſophy proceed onely from Man, ſo from Man alſo is derived the ugly abſurdity of Falſe opinions. For Speech has ſomething in it like to a Spiders Web (as it was ſaid of old of *Solons* laws;) for by contexture of words tender and delicate Wits are inſnared and ſtopt; but ſtrong Wits breake eaſily through them.

From hence alſo this may be deduced, that the firſt Truths were arbitrarily made by thoſe that firſt of all impoſed Names upon Things, or received them from the impoſition of others. For it is true (for example) that *Man is a Living Creature*; but it is for this reaſon, that it pleaſed men to impoſe both thoſe Names on the ſame thing.

9 Fourthly, Propoſitions are diſtinguiſhed into *Primary* and *not Primary*. *Primary* is that wherein the Subject is explicated by a Prædicate of many Names, as *Man, is a Body Animated Rationall*; for that which is comprehended in the Name *Man* is more largely expreſſed in the Names *Body*, *Animated*, and *Rationall* joyned together; and it is called *Primary*, becauſe it is firſt in Ratiocination; for nothing can be proved, without underſtanding firſt the name of the Thing in queſtion. Now *Primary Propoſitions* are nothing but Definitions, or parts of Definitions; and theſe onely are the principles of Demonſtration, being Truths conſtituted arbitrarily by the Inventors of Speech, and therefore not to be demonſtrated. To theſe Propoſitions, ſome have added others, which they call *Primary* and *Principles*; namely *Axiomes* and *Common Notions*; which (though they be ſo evident that they need no proofe, yet) becauſe they may be proved, are not truly Principles; & the leſſe to be received for ſuch, in regard Propoſitions not intelligible, and ſometimes manifeſtly falſe, are thruſt on us under the Name of *Principles*

by the clamour of Men, who obtrude for evident to others all that they themſelves thinke True. Alſo certaine *Petitions* are commonly received into the number of Principles; as for example, *That a ſtreight line may be drawne between two points*, and other *Petitions* of the Writers of Geometry; and theſe are indeed the Principles of *Art* or *Conſtruction*, but not of *Science* and *Demonſtration*.

10 Fiftly, Propoſitions are diſtinguiſhed into *Neceſſary*, that is, neceſſarily True; and True, but not neceſſarily, which they call *Contingent*. A *Neceſſary Propoſition* is when nothing can at any time be conceived or feigned, whereof the Subject is the Name, but the Prædicate alſo is the Name of the ſame thing; as *Man is a Living-Creature* is a neceſſary Propoſition, becauſe at what time ſoever we ſuppoſe the name *Man* agrees with any thing, at that time the name *Living-Creature* alſo agrees with the ſame. But a *Contingent Propoſition* is that which at one time may be true, at another time falſe; as *Every Crow is Black*; which may perhaps be true now, but falſe hereafter. Again, in every *Neceſſary Propoſition*, the Prædicate is either equivalent to the Subject, as in this, *Man is a Rational Living-Creature*, or part of an equivalent Name, as in this, *Man is a Living-Creature*, for the Name *Rational Living-Creature*, or *Man*, is compounded of theſe two, *Rational* and *Living-Creature*. But in a *Contingent Propoſition* this cannot be; for though this were true, *Every Man is a Lyar*, yet becauſe the word *Lyar* is no part of a Compounded name equivalent to the Name *Man*, that Propoſition is not to be called *Neceſſary*, but *Contingent*, though it ſhould happen to be true alwayes. And therefore thoſe Propoſitions only are *Neceſſary*, which are of Sempiternal Truth, that is, true at all times. From hence alſo it is manifeſt, that Truth adheres not to Things, but to Speech onely; for ſome Truths are eternal; for it will be eternally true, *if Man, then Living-Creature*; but that any *Man*, or *Living-Creature* ſhould exiſt eternally, is not neceſſary.

11 A ſixth Diſtinction of Propoſitions is into *Categorical* and *Hypotheticall*. A *Categoricall Propoſition* is that which is ſimply or abſolutely pronounced, as *Every Man is a Living Creature*, *No Man is a Tree*; and *Hypotheticall* is that which is pronounced conditionally, as, *If any thing be a Man the ſame is alſo a Living Creature*, *If any thing be a Man the ſame is alſo Not-a-ſtone*.

A Categoricall Propoſition, and an *Hypothetical* anſwering it, doe both

both ſignifie the ſame, if the Propoſitions be Neceſſary; but not if they be Contingent. For example, if this, *Every Man is a Living-Creature*, be true, this alſo will be true, *If any thing be a Man, the ſame is alſo a Living-Creature*; but in Contingent Propoſitions, though this be true, *Every Crow is Black*; yet this, *If any thing be a Crow the ſame is Black*, is falſe. But an *Hypotheticall Propoſition* is then rightly ſaid to be true, when the Conſequence is true; as *Every Man is a Living-Creature* is rightly ſaid to be a true Propoſition, becauſe of whatſoever it is truly ſaid *That is a Man*, it cannot but be truly ſaid alſo *The ſame is a Living-Creature*. And therefore whenſoever an *Hypotheticall Propoſition* is true, the *Categoricall* anſwering it is not only true, but alſo neceſſary; which I thought worth the noting, as an argument, that Philoſophers may in moſt things reaſon more ſolidly by *Hypotheticall* then *Categoricall Propoſitions*.

12 But ſeeing every Propoſition may be & uſes to be pronounced and written in many formes, and we are obliged to ſpeake in the ſame manner as moſt men ſpeake; yet they that learne Philoſophy from Maſters, had need to take heed they be not deceived by the Variety of expreſſions. And therefore whenſoever they meet with any obſcure Propoſition, they ought to reduce it to its moſt Simple and Categoricall forme; in which the Copulative word *Is* muſt be expreſſed by it ſelfe, and not mingled in any manner either with the Subject or Prædicate, both which muſt be ſeparated and cleerly diſtinguiſhed one from another. For example, if this Propoſition *Man can not ſinne*, be compared with this, *Man cannot ſinne*, their difference will eaſily appeare if they be reduced to theſe, *Man is able not to ſinne*, and *Man is not able to ſinne*, where the Prædicates are manifeſtly different. But they ought to doe this ſilently by themſelves, or betwixt them and their Maſters onely; for it will be thought both ridiculous and abſurd, for a man to uſe ſuch language publiquely. Being therefore to ſpeake of *Equipollent Propoſitions*, I put in the firſt place all thoſe for *Equipollent*, that may be reduced purely to one and the ſame Categoricall Propoſition.

13 Secondly, that which is Categoricall and Neceſſary, is *Equipollent* to its Hypotheticall Propoſition; as this Categorical, *A Right-lined Triangle has its three Angles equal to two Right angles*, to this Hypotheticall, *If any Figure be a Right-lined Triangle, the three Angles of it are equal to two Right Angles.*

14 A

14 Alſo, any two Univerſall Propoſitions, of which the Termes of the one (that is, the Subject and Prædicate) are Contradictory to the Termes of the other, and their Order inverted, as theſe, *Every man is a Living Creature*, and *Every thing that is not a Living Creature, is not a Man*, are *Equipollent*. For ſeeing *Every Man is a Living Creature* is a true Propoſition, the Name *Living Creature* containes the Name *Man*; but they are both Poſitive Names; and therefore (by the laſt Article of the præcedent Chapter) the Negative Name *Not Living Creature* containes the Negative Name *Not Man*; Wherefore *Every thing that is not a Living Creature is not a Man* is a true Propoſition. Likewiſe theſe, *No Man is a Tree*, *No Tree is a Man*, are *Equipollent*. For if it be true that *Tree* is not the Name of any *Man*, then no one thing can be ſignified by the two Names *Man* and *Tree*, wherefore *No Tree is a Man* is a true Propoſition. Alſo to this, *Whatſoever is not a Living Creature is not a Man*, where both the Termes are Negative, this other Propoſition is *Equipollent*, *Onely a Living Creature is a Man*.

15 Fourthly, Negative Propoſitions, whether the particle of Negation be ſet after the *Copula* as ſome Nations doe, or before it, as it is in Latine and Greeke, if the Termes be the ſame, are *equipollent*; as for example, *Man is not-a-Tree* and *Man is not-a-Tree*, are equipollent, though *Ariſtotle* deny it. Alſo theſe, *Every Man is not a Tree*, and *No Man is a Tree* are equipollent, and that ſo manifeſtly, as it needs not be demonſtrated.

16 Laſtly, all Particular Propoſitions that have their Termes inverted, as theſe, *Some Man is Blind*, *Some Blind thing is a Man*, are *equipollent*; for either of the two Names, is the Name of ſome one and the ſame Man; and therefore in which ſoever of the two Orders they be connected, they ſignifie the ſame Truth.

17 Of Propoſitions that have the ſame Termes, and are placed in the ſame Order, but varied either by Quantity or Quality, ſome are called *Subaltern*, others *Contrary*, others *Subcontrary*, and others *Contradictory*.

Subaltern, are Univerſal and Particular Propoſitions of the ſame Quality; as, *Every Man is a Living Creature*, *Some Man is a Living Creature*; or, *No Man is Wiſe*, *Some Man is not Wiſe*. Of theſe if the Univerſal be true, the Particular will be true alſo.

Contrary, are Univerſal Propoſitions of different Quality; *as*

Every

Every Man is happy, *No Man is happy*. And of these if one be true the other is false; also they may both be false, as in the example given.

Subcontrary, are Particular Propositions of different Quality; as *Some Man is learned*, *Some Man is not learned*; which cannot be both false, but they may be both true.

Contradictory are those that differ both in Quantity and Quality; as *Every Man is a Living Creature*, *Some Man is not a Living Creature*; which can neither be both true, nor both false.

18 A Proposition is said to *follow* from two other Propositions, when these being granted to be true, it cannot be denyed but the other is true also. For example, let these two Propositions, *Every Man is a Living Creature*, and *Every Living Creature is a Body*, be supposed true, that is, that *Body* is the Name of *Every Living Creature*, *and Living Creature*, the Name of *Every Man*. Seeing therefore, if these be understood to be true, it cannot be understood that *Body* is not the name of *Every man*, that is, that *Every Man is a Body*, is false, this Proposition will be said to *follow* from those two, or to be necessarily *inferred* from them.

19 That a true Proposition may follow from false Propositions, may happen sometimes; but false from true, never. For if these, *Every Man is a Stone*, and *Every Stone is a Living Creature*, (which are both false) be granted to be true, it is granted also that *Living Creature* is the name of *Every Stone*, and *Stone* of *Every Man*, that is, that *Living Creature* is the Name of *Every Man*; that is to say, this Proposition *Every Man is a Living Creature*, is true, as it is indeed true. Wherefore a true Proposition may sometimes follow from false. But if any two Propositions be true, a false one can never follow from them. For if true follow from false, for this reason onely, that the false are granted to be true, then truth from two truths granted will follow in the same manner.

20 Now seeing none but a true Proposition will follow from true, and that the understanding of two Propositions to be true, is the cause of understanding that also to be true which is deduced from them; the two Antecedent Propositions are commonly called the Causes of the inferred Proposition, or Conclusion. And from hence it is that Logicians say, the *Premisses* are Causes of the *Conclusion*; which may passe, though it be not properly spoken; for

hough Understanding be the cause of Understanding, yet Speech is not the cause of Speech. But when they say, the Cause of the Properties of any thing, is the Thing it self, they speake absurdly. For example, if a Figure be propounded which is Triangular; Seeing every Triangle has all its angles together equal to two right angles, from whence it follows that all the angles of that Figure are equal to two right angles, they say for this reason, that that Figure is the Cause of that Equality. But seeing the Figure does not it self make its angles, and therefore cannot be said to be the *Efficient-Cause*, they call it the *Formall-Cause*; whereas indeed it is no Cause at all; nor does the Property of any Figure follow the Figure, but has its Being at the same time with it; only the Knowledge of the Figure goes before the Knowledge of the Properties; and one Knowledge is truly the Cause of another Knowledge, namely the *Efficient-Cause*.

And thus much concerning *Proposition*; which in the Progress of Philosophy is the first Step, like the moving towards of one Foot. By the due addition of another Step I shall proceed to *Syllogisme*, and make a compleat Pace. Of which in the next Chapter.

CHAP.

CHAP. IV.
Of Syllogiſme.

1 *The Definition of Syllogiſme.* 2 *In a Syllogiſme there are but three Termes.* 3 *Major, Minor and Middle Term; alſo Major and Minor Propoſition, what they are.* 4 *The Middle Terme in every Syllogiſme ought to be determined in both the Propoſitions to one and the ſame thing.* 5 *From two Particular Propoſitions nothing can be concluded.* 6 *A Syllogiſme is the Collection of two Propoſitions into one Summe.* 7 *The Figure of a Syllogiſme what it is.* 8 *What is in the mind anſwering to a Syllogiſme.* 9 *The firſt Indirect Figure how it is made.* 10 *The ſecond Indirect Figure how made.* 11 *How the third Indirect Figure is made* 12 *There are many Moods in every Figure, but moſt of them Uſeleſſe in Philoſophy.* 13 *An Hypotheticall Syllogiſme when equipollent to a Categoricall.*

1 A Speech conſiſting of three Propoſitions, from two of which the third followes, is called a SYLLOGISME; and that which followes is called the *Concluſion*; the other two *Premiſſes*. For example this Speech, *Every man is a Living Creature, Every Living Creature is a Body*, therefore, *Every Man is a Body*, is a *Syllogiſme*, becauſe the third Propoſition follows from the two firſt; that is, if thoſe be granted to be true, this muſt alſo be granted to be true.

2 From two Propoſitions which have not one Terme common, no Concluſion can follow; and therefore no *Syllogiſme* can be made of them. For let any two Premiſſes, *A man is a Living Creature*, *A Tree is a Plant*, be both of them true; yet becauſe it cannot be collected from them that *Plant* is the Name of a *Man*, or *Man* the Name of a *Plant*, it is not neceſſary that this Concluſion, *A Man is a Plant* ſhould be true. Corollary, Therefore in the *Premiſſes* of a *Syllogiſme* there can be but three *Termes*.

Beſides there can be no Terme in the *Concluſion*, which was not in the *Premiſſes*. For let any two Premiſſes be, *A Man is a Living Creature*, *A Living Creature is a Body*, yet if any other Terme be put

in the Concluſion, as *Man is two footed*; though it be true, it cannot follow from the Premiſſes, becauſe from them it cannot be collected, that the Name *Two footed* belongs to a *Man*; and therefore againe, In every *Syllogiſme* there can be but three *Termes*.

3 Of theſe Termes, that which is the *Predicate* in the Concluſion, is commonly called the *Major*; that which is the *Subject* in the Concluſion, the *Minor*, and the other is the *Middle Term*; as in this Syllogiſme, *A Man is a Living Creature*, *A Living Creature is a Body*, therefore, *A Man is a Body*, *Body* is the *Major*, *Man* the *Minor* and *Living Creature* the *Middle Term*. Alſo of the Premiſſes, that in which the *Major Terme* is found, is called the *Major Propoſition*, and that which has the *Minor Term* the *Minor Propoſition*.

4 If the Middle Terme be not in both the Premiſſes determined to one and the ſame ſingular thing, no Concluſion will follow, nor Syllogiſme be made. For let the *Minor Terme* be *Man*, the *Middle Terme Living Creature*, and the *Major Term Lyon*; and let the Premiſſes be *Man is a Living Creature*, *Some Living Creature is a Lyon*, yet it will not follow that, *Every* or *Any Man is a Lyon*. By which it is manifeſt, that in every Syllogiſme, that Propoſition which has the *Middle Terme* for its *Subject*, ought to be either *Univerſal* or *Singular*, but not *Particular* nor *Indefinite*. For example, this Syllogiſm, *Every Man is a Living Creature*, *ſome Living Creature is four-footed*, therefore *ſome Man is four-footed*, is therefore faulty, becauſe the *Middle Term*, *Living Creature*, is in the firſt of the Premiſſes determined onely to *Man*, for there the Name of *Living Creature* is given to *Man onely*, but in the later Premiſſe it may be underſtood of ſome other Living Creature beſides Man. But if the later Premiſſe had been *Univerſall*, as here, *Every Man is a Living Creature*, *Every Living Creature is a Body*, therefore *Every Man is a Body*, the Syllogiſme had been true; for it would have followed that *Body* had been the Name of *Every Living Creature*, that is of *Man*, that is to ſay, the Concluſion *Every Man is a Body* had been true. Likewiſe when the *Middle Term* is a *Singular* Name, a Syllogiſme may be made, I ſay a true Syllogiſme, though uſeleſſe in Philoſophy, as this, *Some Man is Socrates*, *Socrates is a Philoſopher*, therefore *Some Man is a Philoſopher*; for the Premiſſes being granted, the Concluſion cannot be denyed.

5 And therefore of two Premiſſes, in both which the *Middle Terme*

Terme is *Particular*, a Syllogiſme cannot be made; for whether the *Middle Terme* be the *Subject* in both the Premiſſes, or the *Predicate* in both, or the *Subject* in one and the *Predicate* in the other, it will not be neceſſarily determined to the ſame thing. For let the Premiſſes be,

Some Man is blind
Some Man is learned
} In both which the *Middle Term* is the *Subject*,

It will not follow, that *Blind* is the Name of any *learned Man*, or *Learned* the Name of any *Blind Man*, ſeeing the Name *Learned* does not containe the Name *Blind*, nor this that; and therefore it is not neceſſary that both ſhould be Names of the ſame Man. So from theſe Premiſſes.

Every Man is a Living Creature
Every Horſe is a Living Creature
} In both which the *Middle Terme* is the *Predicate*,

Nothing will follow. For ſeeing *Living Creature* is in both of them *Indefinite*, which is equivalent to *Particular*, and that *Man* may be one kind of *Living Creature*, and *Horſe* another kind, it is not neceſſary that *Man* ſhould the be Name of *Horſe*, or *Horſe* of *Man*. Or if the Premiſſes be,

Every Man is a Living Creature
Some Living Creature is four-footed
} In one of which the *Middle Terme* is the *Subject*, and in the other the *Predicate*,

The Concluſion will follow, becauſe the Name *Living Creature* being not determined, it may in one of them be underſtood of *Man*, in the other of *Not-Man*.

6 Now it is manifeſt from what has been ſaid, that a Syllogiſme is nothing but a Collection of the ſumme of two Propoſitions, joyned together by a common Term, which is called the *Middle Terme*. And as Propoſition is the Addition of two Names, ſo Syllogiſme is the adding together of three.

7 Syllogiſmes are uſually diſtinguiſhed according to their diverſity of *Figures*, that is, by the diverſe poſition of the Middle Term. And againe in Figure there is a diſtinction of certain

Moods, which consist of the differences of Propositions in *Quantity* & *Quality*. The first Figure is that, in which the Terms are placed one after another according to their latitude of Signification; in which order the *Minor Term* is first, the *Middle Term* next, and the *Major* last; as if the *Minor* Term be Man, the *Middle Term Living Creature* and the *Major Term Body*, then, *Man is a Living Creature, is a Body*, will be a Syllogisme in the first *Figure*; in which, *Man is a Living Creature*, is the *Minor* Proposition; the *Major*, *Living Creature is a Body*, and the *Conclusion* or sum of both *Man is a Body*. Now this *Figure* is called *Direct*, because the Termes stand in direct Order; and it is varied by *Quantity* and *Quality* into four *Moods*; of which the first is that wherein all the Terms are *Positive*, and the Minor Term *Universal*, as *Every Man is a Living Creature, Every Living Creature is a Body*; in which all the Propositions are *Affirmative* and *Universall*. But if the Major Term be a Negative Name, and the Minor an Universall Name, the *Figure* will be in the second *Mood*, as, *Every Man is a Living Creature, Every Living Creature is not a Tree*, in which the Major Proposition and Conclusion are both *Universall* and *Negative*. To these two are commonly addded two more, by making the Minor Term *Particular*. Also it may happen that both the Major and Middle Termes are *Negative* Terms, and then there arises another *Mood*, in which all the Propositions are Negative, and yet the Syllogisme will be good; as, if the Minor Term be *Man*, the Middle Term *Not a Stone*, and the Major Terme, *Not a Flint*, this Syllogisme, *No Man is a Stone, Whatsoever is not a Stone is not a Flint*, therefore *No Man is a Flint*, is true, though it consist of three Negatives. But in Philosophy, the Profession whereof is to establish Universall Rules concerning the Properties of Things, seeing the difference betwixt *Negatives* and *Affirmatives* is onely this, that in the former the Subject is affirmed by a Negative Name, and by a Positive in the later, it is superfluous to consider any other *Mood* in direct *Figure*, besides that, in which all the Propositions are both Universal and Affirmative.

8 The Thoughts in the mind answering to a Direct Syllogism, proceed in this manner; First, there is conceived a Phantasme of the thing named, with that Accident or Quality thereof for which it is in the Minor Proposition called by that name which

is the Subject; next, the Mind has a Phantasme of the same thing with that Accident or Quality for which it hath the name that in the same Proposition is the Predicate; Thirdly, the Thought returns of the same thing as having that Accident in it, for which it is called by the Name that is in the Predicate of the Major Proposition; and lastly, remembring that all those are the Accidents of one and the same thing, it concludes that those three Names are also Names of one and the same thing; that is to say, the Conclusion is true. For example, when this Syllogisme is made, *Man is a Living Creature*, *A Living Creature is a Body*, therefore *Man is a Body*, the Mind conceives first an image of a Man *speaking* or discoursing, and remembers that that which so appears, is called *Man*; then it has the image of the same Man *moving*, and remembers that that which appeares so is called *Living Creature*; thirdly, it conceives an image of the same Man as *filling some place* or space, and remembers that what appeares so is called *Body*; and lastly, when it remembers, that that thing which was extended, and moved and spake, was one and the same thing, it concludes that the three Names *Man*, *Living Creature*, and *Body*, are Names of the same thing, and that therefore *Man is a Living Creature* is a true Proposition. From whence it is manifest, that Living Creatures that have not the use of Speech, have no Conception or Thought in the Mind, answering to a Syllogisme made of Universall Propositions; seeing it is necessiary to Thinke not only of the Thing, but also by turnes to remember the diverse Names, which for diverse considerations thereof are applied to the same.

9 The rest of the Figures arise either from the Inflexion, or Inversion of the first or direct Figure; which is done by changing the Major, or Minor, or both the Propositions into converted Propositions æquipollent to them.

From whence follow three other Figures, of which, two are *Inflected*, and the third *Inverted*. The first of these three is made by the Conversion of the Major Proposition. For let the Minor, Middle and Major Terms stand in direct order, thus, *Man is a Living Creature*, *Is not a Stone*, which is the first or direct Figure; the Inflexion will be by converting the Major Proposition in this manner,

Man

Man is a Living Creature,
A ſtone is not a Living Creature; And this is the ſecond

Figure, or the firſt of the Indirect Figures; in which the Concluſion will be, *Man is not a ſtone*. For (having ſhewn in the laſt Chap. 14 Article, that Univerſall Propoſitions converted by contradiction of the Termes are æquipollent,) both thoſe Syllogiſmes conclude alike; ſo that if the Major be read (like Hebrew) backwards, thus, *A Living Creature is not a Stone*, it will be direct again, as it was before. In like manner this Direct Syllogiſme, *Man is not a Tree, is not a Pear-tree*, will be made Indirect by converting the Major Propoſition (by contradiction of the Termes) into another æquipollent to it, thus,

Man is not a Tree,
A Pear-tree is a Tree; for the ſame Concluſion will follow, *Man is not a Pear-tree.*

But for the Converſion of the Direct Figure into the firſt Indirect Figure, the Major Terme in the Direct Figure ought to be *Negative*. For though this Direct, *Man is a Living Creature, is a Body* be made Indirect, by converting the Major Propoſition, thus,

Man is a Living Creature.
Not a Body is not a Living Creature, Therefore
Every Man is a Body; yet this Converſion appeares ſo obſcure, that this Mood is of no uſe at all. By the Converſion of the Major Propoſition, it is manifeſt, that in this Figure, the Midle Terme is alwayes the Predicate in both the Premiſſes.

10 The ſecond Indirect Figure is made by converting the Minor Propoſition, ſo as that the Middle Term is the Subject in both. But this never concludes Uniuerſally, and therefore is of no uſe in Philoſophy. Nevertheleſſe I will ſet down an example of it; by which this Direct *Every Man is a Living Creature,*
Every Living Creature is a Body,
by Converſion of the Minor Propoſition will ſtand thus,

Some Living Creature is a Man,
Every Living Creature is a Body, Therefore
Some Man is a Body.

For *Every Man is a Living Creature*, cannot be converted into this, *Every Living Creature is a Man*; and therefore if this Syllogiſme

giſme be reſtored to its Direct forme, the Minor Propoſition will be *Some Man is a Living Creature*, and conſequently the Concluſion will be *Some Man is a Body*, ſeeing the Minor Terme *Man*, which is the Subject in the Concluſion, is a Particular Name.

11 The third Indirect or Inverted Figure, is made by the Converſion of both rhe Premiſſes. For Example, this Direct Syllogiſme,

Every Man is a Living Creature,
Every Living Creature is not a Stone, Therefore
Every Man is not a Stone; being Inverted will ſtand thus,
Every Stone is not a Living Creature
Whatſoever is not a Living Creature, is not a Man, Therefore
Every Stone is not a Man. Which Concluſion is the Converſe of the Direct Concluſion, and æquipollent to the ſame.

The Figures therefore of Syllogiſms, if they be numbred by the diverſe ſcituation of the Middle Terme onely, are but three; in the firſt whereof the Middle Term has the Middle place; in the ſecond, the laſt; and in the third, the firſt place. But if they be numbred according to the ſcituation of the Termes ſimply, they are four; for the firſt may be diſtinguiſhed againe into two, namely into Direct and Inverted. From whence it is evident, that the controverſie among Logicians concerning the fourth Figure, is a meer λογόμαχια, or contention about the Name thereof; for as for the thing it ſelfe, it is plain, that the ſcituation of the Termes (not conſidering the Quantity or Quality by which the Moods are diſtinguiſhed) makes four differences of Syllogiſmes, which may be called Figures, or have any other Name at pleaſure.

12 In every one of theſe Figures there are many Moods, which are made by varying the Premiſſes according to all the differences they are capable of, by Quantity and Quality; as namely, in the Direct Figure there are ſix Moods; in the firſt Indirect Figure, four; in the ſecond, fourteen; and in the third, eighteen. But becauſe from the Direct Figure I rejected as ſuperfluous all Moods beſides that which conſiſts of Univerſal Propoſitions, and whoſe Minor Propoſition is Affirmative, I doe together with it reject the Moods of the reſt of the Figures which are made by Converſion of the Premiſſes in the Direct Figure.

13 As

13 As it was ſhewed before, that in Neceſſary Propoſitions a Categoricall and Hypotheticall Propoſition are æquipollent; ſo likewiſe it is manifeſt that a Categoricall and Hypotheticall Syllogiſme are æquivalent. For every Categoricall Syllogiſme, as this,

Every Man is a Living Creature,
Every Living Creature is a Body, Therefore
Every Man is a Body, is of equall force with this Hypotheticall Syllogiſme.

If any thing be a Man, the ſame is alſo a Living Creature,
If any thing be a Living Creature, the ſame is a Body, Therefore
If any thing be a Man, the ſame is a Body.

In like manner, this Categorical Syllogiſme in an Indirect Figure

No Stone is a Living Creature,
Every Man is a Living Creature, Therefore
No Man is a Stone, or
No Stone is a Man, is æquivalent to this Hypotheticall Syllogiſme.

If any thing be a Man, the ſame is a Living Creature,
If any thing be a a Stone, the ſame is not a Living Creature, Therefore
If any thing be a Stone, the ſame is not a Man, or
If any thing be a Man, the ſame is not a Stone.

And thus much ſeemes ſufficient for the nature of Syllogiſmes; (for the doctrine of Moods and Figures is cleerely delivered by others that have written largely and profitably of the ſame.) Nor are Precepts ſo neceſſary as Practice for the attaining of true Ratiocination; and they that ſtudy the Demonſtrations of Mathematicians, will ſooner learn true Logick, then they that ſpend time in reading the Rules of Syllogizing which Logicians have made; no otherwiſe then little Children learn to goe, not by Precepts, but by exerciſing their feet. This therefore may ſerve for the firſt Pace in the way to Philoſophy.

In the next place I ſhall ſpeak of the Faults and Errors, into which men that reaſon unwarily, are apt to fall; and of their Kinds and Cauſes.

CHAP.

CHAP. V.
Of Erring, Falsity and Captions.

1. *Erring and Falsity how they differ. Error of the Mind by it selfe without the use of Words, how it happens.* 2 *A sevenfold Incoherency of Names, every one of which makes allwayes a false Proposition.* 3. *Examples of the first manner of Incoherency.* 4 *Of the second.* 5 *Of the third.* 6 *Of the fourth.* 7 *Of the fifth.* 8 *Of the sixth.* 9 *Of the seventh.* 10 *Falsity of Propositions detected by resolving the Terms with Definitions continued till they come to Simple Names, or Names that are the most Generall of their kind.* 11 *Of the fault of a Syllogisme consisting in the Implication of the Termes which the Copula* 12 *Of the fault which consists in Equivocation.* 13 *Sophisticall Captions are oftner faulty in the matter then in the forme of Syllogismes.*

1 MEn are subject to *Erre* not onely in Affirming and Denying;but also in Perception, and in silent Cogitation. In Affirming and Denying, when they call any thing by a Name, which is not the Name thereof; as if from seeing the Sun first by reflection in Water, and afterwards again directly in the Firmament, we should to both those appearances give the Name of Sunne, and say there are two Sunnes; which none but men can doe; for no other Living Creatures have the use of Names. This kind of Error onely deserves the name of *Falsity*,as arising, not from sense, nor from the Things themselves but from pronouncing rashly; for Names have their constitution, not from the Species of Things, but from the Will and Consent of Men. And hence it comes to passe, that men pronounce *Falsely* by their own negligence, in departing from such appellations of things as are agreed upon, and are not deceived neither by the Things, nor by the Sense; for they do not perceive that the thing they see is called Sunne, bnt they give it that Name from their owne will and agreement. Tacite Errors, or the Errors of

Senſe and Cogitation, are made, by paſſing from one Imagination to the Imagination of another different thing; or by feigning that to be Paſt, or Future, which never was, nor ever ſhall be; as when by ſeeing the Image of the Sunne in Water, we imagine the Sunne it ſelfe to be there; or by ſeeing ſwords, that there has been or ſhall be fighting, becauſe it uſes to be ſo for the moſt part; or when from Promiſes we feigne the mind of the Promiſer to be ſuch and ſuch; or laſtly, when from any Signe we vainly imagine ſomething to be ſignified, which is not. And Errors of this ſort are common to all things that have ſenſe; and yet the Deception proceeds neither from our ſenſes, nor from the Things we perceive; but from our ſelves, while we feigne ſuch things as are but meer Images, to be ſomething more then Images. But neither Things, nor Imaginations of Things can be ſaid to be Falſe, ſeeing they are truly what they are; nor doe they as Signes promiſe any thing which they do not performe; for they indeed do not promiſe at all, but we from them; nor doe the Clouds, but we from ſeeing the Clouds, ſay it ſhall rain. The beſt way therefore to free our ſelves from ſuch Errors as ariſe from naturall Signes, is firſt of all, before we begin to reaſon concerning ſuch conjecturall things, to ſuppoſe our ſelves ignorant, and then to make uſe of our Ratiocination; for theſe Errors proceed from the want of Ratiocination; whereas Errors which conſiſt in Affirmation and Negation, (that is, the Falſity of Propoſitions) proceed only from Reaſoning amiſſe. Of theſe therefore, as repugnant to Philoſophy, I will ſpeake principally.

2 Errors which happen in Reaſoning, that is, in Syllogizing, conſiſt either in the Falſity of the Premiſſes, or of the Inference. In the firſt of theſe caſes, a Syllogiſme is ſaid to be faulty in the *Matter* of it; and in the ſecond caſe, in the *Forme*. I will firſt conſider the Matter, namely how many wayes a Propoſition may be falſe; and next the Forme, and how it comes to paſs, that when the Premiſſes are True, the Inference is notwithſtanding Falſe.

Seeing therefore that Propoſition onely is True, (Chap. 3. Art. 7.) in which are copulated two Names of one and the ſame thing; and that alwayes Falſe, in which Names of different things are copulated; look how many wayes Names of different things

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may be copulated, and ſo many wayes a Falſe Propoſition may be made.

Now all things to which we give Names, may be reduced to theſe four kinds, namely, *Bodies*, *Accidents*, *Phantaſmes*, and *Names* themſelves; and therefore in every true Propoſition it is neceſſary that the Names copulated, be both of them Names of *Bodies*, or both Names of *Accidents*, or both Names of *Phantaſmes*, or both Names of *Names*. For Names otherwiſe copulated are incoherent, and conſtitute a Falſe Propoſition. It may happen alſo that the Name of a *Body*, of an *Accident*, or of a *Phantaſme* may be copulated with the Name of a *Speech*. So that copulated Names may be Incoherent ſeven manner of wayes.

1	If the Name of a *Body*	be copulated with	the Name of an *Accident*.
2	If the Name of a *Body*		the Name of a *Phantaſme*.
3	If the Name of a *Body*		the Name of a *Name*.
4	If the Name of an *Accident*		the Name of a *Phantaſme*.
5	If the Name of an *Accident*		the Name of a *Name*.
6	If the Name of a *Phantaſme*		the Name of a *Name*.
7	If the Name of a *Body*, of an *Accident* or of a *Phantaſme*		the Name of a *Speech*.

Of all which I will give ſome examples.

3 After the firſt of theſe wayes Propoſitions are falſe, when Abſtract Names are copulated with Concrete Names; as (in Latine and Greek) *Eſſe eſt Ens*, *Eſſentia eſt Ens*, τὸ τι ἦν εἶναι (*i.*) *Quidditas eſt Ens*, and many the like which are found in Ariſtotles *Metaphyſicks*. Alſo, the *Underſtanding worketh*, the *Underſtanding underſtandeth*, the *Sight ſeeth*, A *Body* is *Magnitude*, A *Body* is *Quantity*, A *Body* is *Extenſion*, To *Be a Man* is *a Man*, *Whiteneſſe* is a *White thing*, *&c.* which is as if one ſhould ſay The *Runner* is the *Running*, or the *Walke Walketh*. Moreover, *Eſſence* is *ſeparated*, *Subſtance* is *Abſtracted*; and others like theſe, or derived from theſe (with which common Philoſophy abounds). For ſeeing no *Subject* of an *Accident*, (that is, No *Body*) is an *Accident*; no Name of an *Accident* ought to be given to a *Body*, nor of a *Body* to an *Accident*.

4 Falſe in the ſecond Manner are ſuch Propoſitions as theſe,

A *Ghoſt is a Body*, or a *Spirit*, that is, a thinne Body; *Senſible Species fly up and down in the Air*, or *are moved hither and thither*, which is proper to Bodies; alſo, A *Shadow is Moved*, or *is a Body*; *Light is Moved*, or *is a Body*; *Colour is the Object of Sight*, *Sound* of *Hearing*; *Space* or *Place is Extended*; and innumerable others of this kind. For ſeeing *Ghoſts*, *ſenſible Species*, a *Shadow*, *Light*, *Colour*, *Sound*, *Space*, *&c.* appeare to us no leſſe ſleeping then waking, they cannot be things without us, but onely *Phantaſmes* of the mind that imagins them; and therefore the Names of theſe, copulated with the Names of *Bodies*, cannot conſtitute a true Propoſition.

5 Falſe Propoſitions of the third kind, are ſuch as theſe; *Genus eſt Ens*, *Univerſale eſt Ens*, *Ens de Ente Prædicatur*. For *Genus*, and *Univerſale*, and *Prædicare* are Names of *Names*, and not of *Things*. Alſo *Number is Infinite*, is a falſe Propoſition; for no number can be Infinite, but onely the word *Number* is then called an Indefinite Name when there is no determined Number anſwering to it in the Mind.

6 To the fourth kind belong ſuch falſe Propoſitions as theſe, *An Object is of ſuch Magnitude or Figure as appeares to the Beholders*; *Colour*, *Light*, *Sound*, *are in the Object*; and the like. For the ſame Object appeares ſometimes *greater*, ſometimes *leſſer*, ſometimes *ſquare*, ſometimes *round* according to the diverſity of the Diſtance and Medium; but the true Magnitude and Figure of the thing ſeen is allwayes one and the ſame; ſo that the magnitude and figure which appeares, is not the true magnitude and figure of the Object, nor any thing but Phantaſme; and therefore in ſuch Propoſitions as theſe, the Names of *Accidents* are copulated with the Names of *Phantaſmes*.

7 Propoſitions are falſe in the fifth manner, when it is ſaid that *The Definition is the Eſſence of a thing*; *Whiteneſſe*, or ſome other Accident *is the Genus*, *or Univerſal*. For *Definition* is not the *Eſſence* of any thing, but a ſpeech ſignifying what we conceive of the Eſſence thereof; and ſo alſo not Whiteneſſe it ſelfe, but the word *Whiteneſſe*, is a *Genus*, or an *Univerſall Name*.

8 In the ſixth manner they Erre, that ſay the *Idea of any thing is Univerſal*; as if there could be in the Mind an Image of *a Man* which were not the Image of *ſome one Man*, but of *Man ſimply*, which is impoſſible; for every *Idea* is one, and of one thing; but they are de-

deceived in this, that they put the *Name* of the thing for the *Idea* thereof.

9 They erre in the ſeventh manner, that make this diſtinction between things that have being, that ſome of them *exiſt by themſelves*, others *by Accident*; Namely, becauſe *Socrates is a Man* is a Neceſſary Propoſition, and *Socrates is a Muſician* a Contingent Propoſition, therefore they ſay ſome things *exiſt neceſſarily* or *by themſelves*, others *contingently* or *by Accident*; whereby, ſeeing *Neceſſary*, *Contingent*, *By it ſelfe*, *By Accident*, are not Names of *Things*, but of *Propoſitions*, they that ſay *any thing that has being*, *exiſts by Accident*, copulate the Name of a *Propoſition* with the Name of a *Thing*. In the ſame manner alſo they Erre, which place ſome Ideas in the Underſtanding, others in the Fancy; as if from the Underſtanding of this Propoſition *Man is a Living Creature*, we had one Idea or Image of a Man derived from ſenſe to the Memory, and another to the Underſtanding; wherein that which deceives them is this, that they think one *Idea* ſhould be anſwerable to a *Name*, another to a *Propoſition*; which is falſe; for *Prepoſition* ſignifies onely the order of thoſe things one after another, which we obſerve in the ſame *Idea* of Man; ſo that this Propoſition *Man is a Living Creature*, raiſes but one *Idea* in us, though in that *Idea* we conſider that firſt, for which he is called *Man*, and next that for which he is called *Living Creature*. The Falſities of Propoſitions in all theſe ſeveral manners, is to be diſcovered by the Definitions of the Copulated Names.

10 But when Names of *Bodies* are copulated with Names of *Bodies*, Names of *Accidents* with Names of *Accidents*, Names of *Names* with Names of *Names*, and Names of *Phantaſmes* with Names of *Phantaſmes*, if we nevertheleſſe remaine ſtill doubtfull whether ſuch Propoſitions are true; we ought then in the firſt place to find out the Definition of both thoſe Names, and againe the Definitions of ſuch Names as are in the former Definition, and ſo proceed by a continuall Reſolution till we come to a ſimple Name, that is, to the moſt Generall or moſt Univerſall Name of that kind; and if after all this the Truth or Falſity thereof be not evident, we muſt ſearch it out by Philoſophy, and Ratiocination, beginning from Definitions. For every Propoſition Univerſally true, is either a Definition, or part of a Definition, or the evidence of it depends upon Definitions.

11. That

11 That fault of a Syllogiſme which lyes hid in the Forme thereof, will allwayes be found either in the implication of the Copula with one of the Termes, or in the Æquivocation of ſome word; and in either of theſe wayes there will be *four Terms*, which (as I have ſhewne) cannot ſtand in a true Syllogiſme. Now the implication of the Copula with either Terme, is eaſily detected by reducing the Propoſitions to plain and cleere Prædications, as (for example) if any man ſhould argue thus,

The Hand toucheth the Pen,
The Pen toucheth the Paper, Therefore
The Hand toucheth the Paper; the Fallacy will eaſily appear by reducing it, thus,

The Hand, is, touching the Pen,
The Pen, is, touching the Paper, Therefore
The Hand, is, touching the Paper; where there are manifeſtly theſe four Termes, *The Hand, Touching the Pen, The Pen,* and *Touching the Paper.* But the danger of being deceived by Sophiſmes of this kind, does not ſeem to be ſo great, as that I need inſiſt longer upon them.

12 And though there may be Fallacy in Æquivocal Terms, yet in thoſe that be manifeſtly ſuch there is none at all; nor in *Metaphors*; for they profeſſe the transferring of Names from one thing to another. Nevertheleſſe ſometimes Æquivocalls (and thoſe not very obſcure) may deceive; as in this argumentation, *It belongs to Metaphyſicks, to treat of Principles; But the firſt Principles of all, is, that the ſame thing cannot both exiſt and not exiſt at the ſame time; and therefore it belongs to Metaphyſicks to treat whether the ſame thing may both exiſt and not exiſt at the ſame time*; where the Fallacy lies in the Æquivocation of the word *Principle*; for whereas Ariſtotle in the beginning of his Metaphyſicks ſayes, that *the treating of Principles belongs to primary ſcience*, he underſtands by *Principles*, *Cauſes* of things, and certaine *Exiſtences* which he calls *Primary*; but where he ſayes *a Primary Propoſition is a Principle*, by *Principle* there he means the *beginning and cauſe of Knowledge*, that is the underſtanding of words, which if any man want, he is incapable of learning.

13 But the *Captions* of Sophiſts and Scepticks, by which they were wont of old to deride and oppoſe Truth, were faulty for the

the moſt part, not in the Forme, but in the Matter of Syllogiſme; and they deceived not others oftner then they were themſelves deceived. For the force of that famous argument of Zeno againſt Motion, conſiſted in this Propoſition, *Whatſoever may be divided into parts infinite in number, the ſame is infinite*; which he without doubt thought to be true, yet nevertheleſſe is falſe. For to be divided into *infinite parts*, is nothing elſe but to be divided into *as many parts as any man will*. But it is not neceſſary that a Line ſhould have parts infinite in number, or be infinite, becauſe I can divide and ſubdivide it as often as I pleaſe; for how many parts ſoever I make, yet their number is finite; but becauſe he that ſayes *Parts*, ſimply, without adding *how many*, does not limit any number, but leaves it to the determination of the Hearer, therefore we ſay commonly a line may be divided infinitely; which cannot be true in any other ſenſe.

And thus much may ſuffice concerniug *Syllogiſme*, which is as it were the firſt Pace towards Philoſophy; in which I have ſaid as much as is neceſsary, to teach any man from whence all true argumentation has its force. And to enlarge this Treatiſe with all that may be heaped together, would be as ſuperfluous, as if one ſhould (as I ſaid before) give a young child Precepts for the teaching of him to goe; for the Art of Reaſoning is not ſo well learned by Precepts as by Practice, and by the reading of thoſe books in which the Concluſions are all made by ſevere Demonſtration. And ſo I paſs on to the way of Philoſophy, that is, to the Method of Study.

CHAP.

CHAP. VI.
Of Method.

1 *Method and Science defined.* 2 *It is more easily known concerning Singular then Universall things, That they are; and contrarily, it it more easily knowne concerning Universall then Singular things, Why they are, or what are their Causes.* 3 *What it is Philosophers seek to know.* 4 *The first Part, by which Principles are found out is purely Analyticall.* 5 *The highest Causes, and most Universall in every kind, are knowne by themselves.* 6 *Method from Principles found out, tending to Science simply, what it is.* 7 *That Method of Civill and Naturall Science which proceeds from Sense to Principles, is Analytical; and againe that which [illegible] Principles, is Syntheticall.* 8 *The Method of searching out, whether any thing propounded, be Matter or Accident.* 9 *The Method of seeking whether any Accident be in this, or in that Subject.* 10 *The Method of searching after the Cause of any Effect propounded.* 11 *Words serve to Invention, as Markes; to Demonstration, as Signes.* 12 *The Method of Demonstration is Synthetical.* 13 *Definitions onely are Primary and Universal Propositions.* 14 *The Nature and Definition of a Definition.* 15 *The Properties of a Definition.* 16 *The Nature of a Demonstration.* 17 *The Properties of a Demonstration, and Order of things to be demonstrated.* 18 *The Faults of a Demonstration.* 19 *Why the Analyticall Method of Geometricians cannot be treated of in this place.*

FOr the understanding of *Method*, it will be necessary for me to repeat the definition of Philosophy, delivered above (Chap. 1. Art. 2.) in this manner, *Philosophy is the knowledge we acquire by true Ratiocination, of Appearances, or apparent Effects, from the knowledge we have of some possible Production or Generation of the same; and of such Production as has been or may be, from the knowledge we have of the Effects.* METHOD therefore in the Study of Philosophy, *is the shortest way of finding out Effects by their known*

known Causes, or of Causes by their known Effects. But we are then said to *know* any Effect, when we know, *that there be Causes of the same,* and *in what Subiect those Causes are*, and *in what Subiect they produce that Effect*, and *in what Manner they work the same.* And this is the Science of Causes, or as they call it of the Διότι. All other Science, which is called the Ὅτι, is either Perception by Sense, or the Imagination, or Memory remaining after such Perception.

The first Beginnings therefore of Knowledge, are the Phantasmes of Sense and Imagination; and that there be such Phantasmes we know well enough by Nature; but to know why they be, or from what Causes they proceed, is the work of Ratiocination; which consists (as is said above, in the 1. Chap. 2. Art.) in *Composition*, and Division or *Resolution.* There is therefore no Method, by which we find out the Causes of things, but is either *Compositive*, or *Resolutive*, or *partly Compositive*, and *partly Resolutive.* And the Resolutive is commonly called *Analyticall* Method, as the Compositive is called *Syntheticall.*

2 It is common to all sorts of Method, to proceed from known things to unknown; and this is manifest from the cited Definition of Philosophy. But in Knowledge by Sense, the whole object is more known, then any part thereof; as when we see a Man, the Conception or *whole Idea* of that *Man* is first or more known, then the *particular Ideas* of his being *figurate*, *animate*, *and rationall*; that is, we first see the whole Man, and take notice of his *Being*, before we observe in him those other Particulars. And therefore in any knowledge of the Ὅτι, or that any thing *is*, the beginning of our search is from the *whole Idea*; and contrarily, in our knowledge of the Διότι, or of the Causes of any thing, that is, in the *Sciences*, we have more knowledge of the Causes of the Parts, then of the Whole. For the Cause of the Whole is compounded of the Causes of the Parts; but it is necessary that we know the things that are to be compounded, before we can know the whole Compound. Now by Parts, I do not here mean Parts of the thing it self, but Parts of its Nature; as, by the Parts of Man I do not understand his Head, his Shoulders, his Arms, &c. but his Figure, Quantity, Motion, Sense, Reason, and the like; which Accidents being compounded or put together, constitute the whole Nature of Man, but not the man himselfe. And this is the meaning of that

common saying, namely, that some things are more knowne to *Us*, others more known to *Nature*; for I do not thinke that they which so distinguish, mean, that something is known to Nature, which is known to no man; and therefore, by those things that are more known to Us, we are to understand, things we take notice of by our Senses, and by more known to Nature, those we acquire the knowledge of by Reason; for in this sense it is, that the *Whole*, that is, those things that have Universal Names, (which for brevities sake I call *Universall*) are more knowne to us then the *Parts*, that is, such things as have Names lesse Uniuersal, (which I therefore call *Singular*;) and the Causes of the Parts, are more known to Nature then the Cause of the Whole; that is, Universalls then Singulars.

3 In the Study of Philosophy men search after Science either Simply, or Indefinitely; that is, to know as much as they can, without propounding to themselves any limited question; or they enquire into the Cause of some determined Appearance, or endeavour to find out the certainty of something in question; as what is the cause of *Light*, of *Heat*, of *Gravity*, of a *Figure* propounded, and the like; or in what *Subiect* any propounded *Accident* is inhærent; or what may conduce most to the *generation* of some propounded *Effect* from many *Accidents*; or in what manner particular Causes ought to be compounded for the production of some certaine Effect. Now according to this variety of things in question, sometimes the *Analyticall Method* is to be used, and sometimes the *Syntheticall*.

4 But to those that search after Science indefinitely, which consists in the knowledge of the Causes of all things, as far forth as it may be attained, and the Causes of Singular things are compounded of the Causes of Universall or Simple things, it is necessary that they know the Causes of Universall things, or of such Accidents as are common to all Bodies, that is, to all Matter, before they can know the Causes of Singular things, that is, of those Accidents by which one thing is distinguished from another. And againe they must know what those Universall things are, before they can know their Causes. Moreover, seeing Universall things are contained in the Nature of Singular things, the knowledge of them is to be acquired by Reason, that is, by Resolution

ſolution. For example, if there be propounded a Conception or *Idea* of ſome Singular thing, as of a *Square*, this Square is to be reſolved into a *Plain, terminated with a certaine number of equall and ſtraight lines and right angles.* For by this Reſolution we have theſe things Univerſall or agreeable to all Matter, namely, *Line, Plain,* (which containes *Superficies,*) *Terminated, Angle, Straightneſs, Rectitude and Equality*; and if we can find out the Cauſes of theſe, we may compound them all together into the Cauſe of a Square. Againe, if any man propound to himſelfe the Conception of *Gold*, he may by Reſolving come to the Ideas of *Solid, Viſible, Heavy,* (that is, tending to the Center of the Earth, or downwards,) and many other more Univerſall then Gold it ſelfe; and theſe he may Reſolve againe, till he come to ſuch things as are moſt Univerſall. And in this manner by Reſolving continually, we may come to know what thoſe things are, whoſe Cauſes being firſt known ſeverally, and afterwards compounded, bring us to the Knowledge of Singular things. I conclude therefore, that the Method of attaining to the Univerſall Knowledge of Things, is purely *Analyticall.*

5 But the Cauſes of Univerſall things (of thoſe at leaſt that have any Cauſe) are manifeſt of themſelues, or (as they ſay commonly) known to Nature; ſo that they need no Method at all; for they have all but one Univerſall Cauſe, which is Motion. For the variety of all Figures ariſes out of the variety of thoſe Motions by which they are made; and Motion cannot be underſtood to have any other Cauſe beſides Motion; nor has the Variety of thoſe things we perceive by Senſe, as of *Colours, Sounds, Savours, &c.* any other Cauſe then Motion, reſiding partly in the Objects that work upon our Senſes, and partly in our ſelves, in ſuch manner, as that it is manifeſtly ſome kind of Motion, though we cannot without Ratiocination come to know what kind. For though many cannot underſtand till it be in ſome ſort demonſtrated to them, that all Mutation conſiſts in Motion; yet this happens not from any obſcurity in the thing it ſelfe, (for it is not intelligible that any thing can depart either from Reſt, or from the Motion it has, except by Motion;) but either by having their Naturall Diſcourſe corrupted with former Opinions received from their Maſters, or elſe for this, that they do not at all bend their mind to the enquiring out of Truth,

6 By the Knowledge therefore of Universalls, and of their Causes (which are the first Principles by which we know the Διότι of things,) we have in the first place their Definitions, (which are nothing but the explication of our Simple Conceptions.) For example, he that has a true Conception of *Place*, cannot be ignorant of this Definition, *Place is that space which is possessed or filled adæquately by some Body*; and so, he that conceives *Motion* aright, cannot but know, that *Motion is the privation of one Place, andthe acquisition of another*. In the next place, we have their Generations or Descriptions; as, (for example,) that *a Line is made by the Motion of a Point, Superficies by the Motion of a Line*, and *one Motion by another Motion, &c.* It remains, that we enquire, what Motion begets such and such Effects; as, what Motion makes a Straight line, and whata Circular; what Motion thrusts, what drawes, and by what way; what makes a thing which is seen or heard, to be seen or heard sometimes in one manner, sometimes in another. Now the Method of this kind of Enquiry, is *Compositive*. For first we are to observe what Effect a Body moved produceth, when we consider nothing in it besides its Motion; and we see presently that this makes a Line, or length; next, what the Motion of a long Body produces; which we find to be Superficies; and so forwards, till we see what the Effects of Simple Motion are; and then in like manner, we are to observe what proceeds from the Addition, Multiplication, Substraction and Division of these Motions, and what Effects, what Figures, and what Properties they produce; from which kind of Contemplation sprung that part of Philosophy which is called *Geometry*.

From this consideration of what is produced by Simple Motion, we are to passe to the consideration of what Effects one Body moved worketh upon another; and because there may be Motion in all the severall parts of a Body, yet so as that the whole Body remain still in the same place, we must enquire, first, what Motion causeth such and such Motion in the whole, that is, when one Body invades another Body which is either at Rest, or in Motion, what way, and with what swiftnesse the invaded Body shall move; and again, what Motion this second Body will generate in a third, and so forwards. From which Contemplation shall be drawn that part of Philosophy which treats of Motion.

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In the Third place we muſt proceed to the Enquiry of ſuch Effects, as are made by the Motion of the Parts of any Body, as, how it comes to paſſe, that things when they are the ſame, yet ſeeme not to be the ſame, but changed. And here the things we ſearch after are ſenſible Qualities, ſuch as *Light*, *Colour*, *Tranſparency*, *Opacity*, *Sound*, *Odour*, *Savour*, *Heat*, *Cold* and the like; which becauſe they cannot be known till we know the Cauſes of Senſe it ſelfe, therefore the conſideration of the Cauſes of *Seeing*, *Hearing*, *Smelling*, *Taſting* and *Touching* belongs to this third place; and all thoſe qualities and Changes above mentioned are to be referred to the fourth place; which two conſiderations comprehend that part of Philoſophy which is called *Phyſiques*. And in theſe four parts is contained whatſoever in Naturall Philoſophy may be explicated by Demonſtration properly ſo called. For if a Cauſe were to be rendred of Natural Appearances in ſpecial, as, what are the Motions and Influences of the heavenly Bodies, and of their parts, the reaſon hereof muſt either be drawn from the parts of the Sciences above mentioned, or no reaſon at all will be given, but all left to uncertaine conjecture.

After *Phyſiques* we muſt come to *Morall Philoſophy*; in which we are to conſider the Motions of the Mind, namely *Appetite*, *Averſion*, *Love*, *Benevolence*, *Hope*, *Fear*, *Anger*, *Emulation*, *Envy*, *&c.* what Cauſes they have, and of what they be Cauſes. And the reaſon why theſe are to be conſidered after *Phyſiques*, is, that they have their Cauſes in Senſe and Imagination, which are the Subject of *Phyſicall* Contemplation. Alſo the reaſon why all theſe Things are to be ſearched after in the order aboveſaid, is, that Phyſiques cannot be underſtood, except we know firſt what Motions are in the ſmalleſt parts of Bodies; nor ſuch Motion of Parts, till we know what it is that makes another Body move; nor this, till we know what Simple Motion will effect. And becauſe all Appearance of things to ſenſe is determined, and made to be of ſuch and ſuch Quality and Quantity by Compounded Motions, every one of which has a certaine degree of Velocity, and a certaine and determined way; therefore in the firſt place we are to ſearch out the wayes of Motion ſimply, (in which Geometry conſiſts;) next the wayes of ſuch generated Motions as are manifeſt; and laſtly the wayes of internal and inviſible Motions, (which is the

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Enquiry of Naturall Philoſophers.) And therefore they that ſtudy Naturall Philoſophy, ſtudy in vaine, except they begin at Geometry; and ſuch Writers or Diſputers thereof, as are ignorant of Geometry, do but make their Readers and Hearers loſe their time.

7 *Civill* and *Morall Philoſophy* doe not ſo adhere to one another, but that they may be ſevered. For the Cauſes of the Motions of the Mind are known not onely by Ratiocination, but alſo by the Experience of every man that takes the paines to obſerve thoſe Motions within himſelfe. And therefore not only they that have attained the knowledge of the Paſſions and Perturbations of the Mind, by the *Syntheticall Method* and from the very firſt Principles of Philoſophy, may by proceeding in the ſame way come to the Cauſes and Neceſſity of conſtituting Common-wealths, and to get the Knowledge of what is Naturall Right, and what are Civill Duties; and in every kind of Goverment, what are the Rights of the Commonwealth, and all other Knowledge apperTaining to Civill Philoſophy, for this reaſon, that the Principles of the Politiques conſiſt in the Knowledge of the Motions of the Mind, and the Knowledge of theſe Motions from the knowledge of Senſe and Imagination; but even they alſo that have not learned the firſt part of Philoſophy, namely *Geometry* and *Phyſiques*, may notwithſtanding attain the Principles of Civill Philoſophy, by the *Analyticall Method*. For if a Queſtion be propounded, as *Whether ſuch an Action be Juſt or Uniuſt*; if that *Uniuſt* be reſolved into *Fact againſt Law*, and that notion of *Law* into the *Command* of him or them that have *Coercive Power*; and that *Power* be derived from the *Wills* of Men that conſtitute ſuch Power to the end they may live in Peace, they may at laſt come to this, that the Appetites of Men and the Paſſions of their Minds are ſuch, that unleſſe they be reſtrained by ſome Power, they will alwayes be making warre upon one another; which may be known to be ſo by any mans experience, that will but examine his owne Mind. And therefore from hence he may proceed by Compounding, to the determination of the Juſtice or Injuſtice of any propounded Action. So that it is manifeſt by what has been ſaid, that the Method of Philoſophy to ſuch as ſeek Science ſimply, without propounding to themſelves the Solution of any Particular queſtion, is partly *Analyticall*,

nalyticall, and partly *Syntheticall*; namely, that which proceeds from Senſe to the invention of Principles, *Analyticall*; and the reſt *Syntheticall*.

8 To thoſe that ſeek the Cauſe of ſome certaine and propounded Appearance or Effect, it happens ſometimes, that they know not whether the thing whoſe Cauſe is ſought after, be Matter or Body, or ſome Accident of a Body. For though in *Geometry*, when the Cauſe is ſought of Magnitude, or Proportion, or Figure, it be certainly known that theſe things, namely Magnitude, Proportion and Figure are Accidents; yet in Naturall Philoſophy, where all queſtions are concerning the Cauſes of the Phantaſmes of ſenſible things, it is not ſo eaſie to diſcern between the things themſelves from which thoſe Phantaſmes proceed, and the Appearances of thoſe things to the ſenſe; which have deceived many, eſpecially when the Phantaſmes have been made by Light. For Example, a Man that looks upon the Sunne, has a certaine ſhining *Idea* of the Magnitude of about a foot over; and this he calls the Sunne, though he know the Sunne to be truly a great deale bigger; and in like manner, the Phantaſme of the ſame thing appears ſometimes round, by being ſeen a farre off, and ſometimes ſquare, by being neerer. Whereupon it may well be doubted whether that Phantaſme be Matter, or ſome Body Naturall, or onely ſome Accident of a Body; in the examination of which doubt we may uſe this Method. The Properties of Matter and Accidents already found out by Us by the *Syntheticall Method* from their Definitions, are to be compared with the *Idea* we have before us; and if it agree with the Properties of Matter or Body, then it is a Body; otherwiſe it is an Accident. Seeing therefore Matter cannot by any endeavour of ours be either Made or Deſtroyed, or Encreaſed, or Diminiſhed, or Moved out of its place; whereas that *Idea* Appeares, Vaniſhes, is Encreaſed, and Diminiſhed, and Moved hither and thither at pleaſure; we may certainly conclude that it is not a Body, but an Accident onely. And this Method is *Syntheticall*.

9 But if there be a doubt made concerning the Subject of any known Accident, (for this may be doubted ſometimes, as in the præcedent example doubt may be made in what Subject that Splendor and apparent Magnitude of the Sunne is) then our enquiry

quiry muſt proceed in this manner. Firſt, Matter in Generall muſt be divided into parts, as into Object, Medium, and the Sentient it ſelfe, or ſuch other parts as ſeem moſt conformable to the thing propounded. Next, theſe parts are ſeverally to be examined how they agree with the Definition of the Subject; and ſuch of them as are not capable of that Accident are to be rejected. For example, If by any true Ratiocination the Sunne be found to be greater then its apparent Magnitude, then that Magnitude is not in the Sunne; If the Sunne be in one determined ſtraight line, and one determined diſtance, and the Magnitude and Splendor be ſeen in more lines and diſtances then one, as it is in Reflection or Refraction, then neither that Splendor nor apparent Magnitude are in the Sun it ſelf, and therefore the Body of the Sun cannot be the Subject of that Splendor and Magnitude. And for the ſame reaſons the Aire and other parts will be rejected, till at laſt nothing remain which can be the Subject of that Splendor and Magnitude but the Sentient it ſelfe. And this Method, in regard the Subject is divided into parts is *Analitycall*; and in regard the Properties both of the Subject and Accident are compared with the Accident concerning whoſe Subject the enquiry is made, it is *Syntheticall*.

10 But when we ſeek after the Cauſe of any propounded Effect, we muſt in the firſt place get into our Mind an exact Notion or *Idea* of that which we call Cauſe, namely, that *A Cauſe is the Summe or Aggregate of all ſuch Accidents both in the Agents and the Patient, as concurre to the producing of the Effect propounded; all which exiſting together, it cannot be underſtood but that the Effect exiſteth with them; or that it cannot poſſibly exiſt if any one of them be abſent.* This being known, in the next place we muſt examine ſingly every Accident that accompanies or præcedes the Effect, as farre forth as it ſeemes to conduce in any manner to the production of the ſame, and ſee whether the propounded Effect may be conceived to exiſt, without the exiſtence of any of thoſe Accidents; and by this meanes ſeparate ſuch Accidents as do not concurre, from ſuch as concurre to produce the ſaid Effect; which being done, we are to put together the concurring Accidents, and conſider whether we can poſſibly conceive that when theſe are all preſent, the Effect propounded will not follow; and if it be evident that the

the Effect will follow, then that Aggregate of Accidents is the entire Cauſe, otherwiſe not; but we muſt ſtill ſearch out and put together other Accidents. For example, if the Cauſe of Light be propounded to be ſought out; firſt, we examine things without us, and find that whenſoever Light appeares, there is ſome principall Object, as it were the fountaine of Light, without which we cannot have any perception of Light; and therefore the concurrence of that Object is neceſſary to the generation of Light. Next we conſider the Medium, and find that unleſſe it be diſpoſed in a certaine manner, namely, that it be *tranſparent*, though the Object remain the ſame, yet the Effect will not follow; and therefore the concurrence of Tranſparency is alſo neceſſary to the generation of Light. Thirdly, we obſerve our own Body, and find that by the indiſpoſition of the Eyes, the Brain, the Nerves, and the Heart, that is, by Obſtructions, Stupidity and Debility we are deprived of Light, ſo that a fitting diſpoſition of the Organs to receive impreſſions from without is likewiſe a neceſſary part of the Cauſe of Light. Again, of all the Accidents inhærent in the Object, there is none that can conduce to the effecting of Light, but onely Action, (or a certain Motion,) which cannot be conceived to be wanting, whenſoever the Effect is preſent; for, that any thing may ſhine, it is not requiſite that it be of ſuch or ſuch Magnitude or Figure, or that the whole Body of it be moved out of the place it is in, (unleſſe it may perhaps be ſaid, that in the Sun or other Body, that which cauſeth Light is the light it hath in it ſelfe; which yet is but a trifling exception, ſeeing nothing is meant thereby but the Cauſe of Light; as if any man ſhould ſay that the Cauſe of Light is that in the Sunne which produceth it;) it remaines therefore that the Action by which Light is generated, is Motion only in the parts of the Object. Which being underſtood, we may eaſily conceive what it is the Medium contributes, namely, the continuation of that Motion to the Eye; and laſtly what the Eye and the reſt of the Organs of the Sentient contribute, namely, the continuation of the ſame Motion to the laſt Organ of Senſe, the Heart. And in this manner the Cauſe of Light may be made up of Motion continued from the Original of the ſame Motion, to the Original of Vitall Motion, Light being nothing but the alteration of Vitall Motion, made by the impreſſion upon it of Motion continued from the Ob-

ject. But I give this onely for an example, for I ſhall ſpeak more at large of Light, and the generation of it in its proper place. In the mean time it is manifeſt, that in the ſearching out of Cauſes, there is need partly of the *Analyticall*, and partly of the *Syntheticall Method*; of the *Analyticall*, to conceive how circumſtances conduce ſeverally to the production of Effects; and of the *Syntheticall*, for the adding together and compounding of what they can effect ſingly by themſelves. And thus much may ſerve for the Method of Invention. It remaines that I ſpeake of the Method of Teaching, that is, of Demonſtration, and of the Meanes by which we demonſtrate.

11 In the Method of Invention the uſe of words conſiſts in this, that they may ſerve for *Marks*, by which, whatſoever we have found out may be recalled to memory; for without this all our Inventions periſh, nor will it be poſſible for us to go on from Principles beyond a Syllogiſme or two, by reaſon of the weakneſſe of Memory. For example, if any man by conſidering a Triangle ſet before him, ſhould find that all its angles together taken are equall to two right angles, and that by thinking of the ſame tacitely, without any uſe of words either underſtood or expreſſed; and it ſhould happen afterwards that another Triangle unlike the former, or the ſame in different ſcituation ſhould be offered to his conſideration, he would not know readily whether the ſame property were in this laſt or no; but would be forced as often as a different Triangle were brought before him (and the difference of Triangles is infinite) to begin his contemplation anew; which he would have no need to do if he had the uſe of Names; for every Univerſal Name denotes the conceptions we have of infinite Singular things. Nevertheleſſe as I ſaid above, they ſerve as *Markes* for the helpe of our Memory, whereby we regiſter to our ſelves our own Inventions; but not as *Signes* by which we declare the ſame to others; ſo that a man may be a Philoſopher alone by himſelfe without any Maſter; Adam had this capacity; But to Teach, that is to Demonſtrate, ſuppoſes two at the leaſt, and Syllogiſticall Speech.

12 And ſeeing Teaching is nothing but leading the Mind of him we teach, to the knowledge of our Inventions, in that Track by which we attained the ſame with our own Mind; therefore the

the ſame Method that ſerved for our Invention, will ſerve alſo for Demonſtration to others, ſaving that we omit the firſt part of Method which proceeded from the Senſe of Things to Univerſal Principles; which becauſe they are Principles, cannot be demonſtrated; and ſeeing they are known by Nature (as was ſaid above in the 5th. Article) they need no Demonſtration, though they need Explication. The whole Method therefore of Demonſtration is *Syntheticall*, conſiſting of that order of Speech, which begins from Primary or moſt Univerſall Propoſitions, which are manifeſt of themſelves, and proceeds by a perpetuall compoſition of Propoſitions into Syllogiſmes, till at laſt the Learner underſtand the truth of the Concluſion ſought after.

13 Now ſuch Principles are nothing but Definitions; whereof there are two ſorts; one, of Names that ſignifie ſuch things as have ſome conceiveable Cauſe, and another of ſuch Names as ſignifie things of which we can conceive no Cauſe at all. Names of the former kind are, *Body* or *Matter*, *Quantity* or *Extenſion*, *Motion*, and whatſoever is common to all Matter. Of the ſecond kind are, *ſuch a Body*, *ſuch and ſo great Motion*, *ſo great Magnitude*, *ſuch Figure*, and whatſoever we can diſtinguiſh one Body from another by. And Names of the former kind are well enough defined, when by Speech as ſhort as may be, we raiſe in the Mind of the Hearer perfect and cleer Ideas or Conceptions of the Things named, as when we Define Motion to be *the leaving of one place, and the acquiring of another continually*; for though no Thing Moved, nor any Cauſe of Motion be in that Definition, yet at the hearing of that Speech, there will come into the Mind of the Hearer an *Idea* of Motion cleer enough. But Definitions of things which may be underſtood to have ſome Cauſe, muſt conſiſt of ſuch Names as expreſſe the Cauſe or Manner of their Generation, as when we Define a Circle to be a Figure made by the circumduction of a ſtraight line in a plaine, &c. Beſides Definitions, there is no other Propoſition that ought to be called Primary, or (according to ſevere truth) be received into the number of Principles. For thoſe *Axiomes* of *Euclide*, ſeeing they may be demonſtrated are no Principles of Demonſtration, though they have by the conſent of all Men gotten the authority of Principles, becauſe they need not be Demonſtrated. Alſo thoſe *Petitions* or *Poſtulata* (as they call them) though they be Principles

yet they are not Principles of Demonſtration, but of Conſtruction onely; that is, not of Science, but of Power; or (which is all one) not of *Theoremes*, which are Speculations, but of *Problemes*, which belong to Practice, or the doing of ſomething. But as for thoſe common received Opinions, *Nature abhorres Vanity*, *Nature doth nothing in Vaine*, and the like, which are neither evident in themſelves, nor at all to be demonſtrated, and which are oftner falſe then true, they are much leſſe to be ackowledged for Principles.

To returne therefore to Definitions, The reaſon why I ſay that the Cauſe and Generation of ſuch things as have any Cauſe or Generation, ought to enter into their Definitions, is this. The End of Science, is the Demonſtration of the Cauſes and Generations of Things; which if they be not in the Definitions, they cannot be found in the Concluſion of the firſt Syllogiſme that is made from thoſe Definitions; and if they be not in the firſt Concluſion, they will not be found in any further Concluſion deduced from that; and therefore by proceeding in this manner we ſhall never come to Science; which is againſt the ſcope and intention of Demonſtration.

14 Now ſeeing Definitions (as I have ſaid) are Principles or Primary Propoſitions, they are therefore Speeches; and ſeeing they are uſed for the raiſing of an *Idea* of ſome Thing in the mind of the Learner, whenſoever that Thing has a Name, the Definition of it can be nothing but the Explication of that Name by Speech; and if that Name be given it for ſome compounded Conception, the Definition is nothing but a Reſolution of that Name into its moſt Univerſall parts. As when we define Man, ſaying, *Man is a Body Animated*, *Sentient*, *Rationall*, thoſe Names *Body Animated*, *&c.* are parts of that whole Name *Man*; ſo that Definitions of this kind alwayes conſiſt of *Genus* and *Difference*; the former Names being all till the laſt, *Generall*; and the laſt of all, *Difference*. But if any Name be the moſt Univerſall in its kind, then the Definition of it cannot conſiſt of *Genus* and *Difference*, but is to be made by ſuch circumlocution as beſt explicateth the force of that Name. Again, it is poſſible, and happens often that the *Genus* and *Difference* are put together, and yet make no Definition; as theſe Words *a Straight Line* containe

both

both the *Genus* and *Difference*; but are not a Definition, unleſſe we ſhould thinke a Straight Line may be thus defined, *A Straight Line is a Straight Line*; and yet if there were added another Name conſiſting of different Words, but ſignifying the ſame thing which theſe ſignifie, then theſe might be the Definition of that Name. From what has been ſaid it may be underſtood how a Definition ought to be defined, namely, *That it is a Propoſition, whoſe Prædicate Reſolves the Subiect, when it may, and when it may not, it exemplifies the ſame.*

15 The Properties of a Definition are,

Firſt, that it takes away Æquivocation, as alſo all that multitude of Diſtinctions, which are uſed by ſuch as think they may learn Philoſophy by Diſputation. For the Nature of a Definition is to define, that is, to determine the ſignification of the defined Name, and to pare from it all other Signification beſides what is contained in the Definition it ſelfe; and therefore one Definition does as much, as all the Diſtinctions (how many ſoever) that can be uſed about the Name defined.

Secondly, That it gives an Univerſall Notion of the thing defined, repreſenting a certaine Univerſall Picture thereof, not to the Eye, but to the Mind. For as when one paints a Man, he paints the image of ſome Man; ſo he that defines the Name *Man*, makes a Repreſentation of ſome Man to the mind.

Thirdly, That it is not neceſſary to diſpute whether Definitions are to be admitted or no. For when a Maſter is inſtructing his Scholar, if the Scholar underſtand all the parts of the thing defined, which are Reſolved in the Definition, and yet will not admit of the Definition, there needs no further Controverſie betwixt them, it being all one as if he refuſed to be taught. But if he underſtand nothing, then certainely the Definition is faulty; for the nature of a Definition conſiſts in this, that it exhibit a cleare Idea of the thing defined; and Principles are either known by themſelves, or elſe they are not Principles.

Fourthly, That in Philoſophy, Definitions are before defined Names. For in teaching Philoſophy, the firſt beginning is from Definitions, and all progreſſion in the ſame till we come to the Knowledge of the thing compounded, is Compoſitive. Seeing therefore Definition is the explication of a Compounded Name

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by Reſolution, and the progreſſion is from the parts to the compound, Definitions muſt be underſtood before Compounded Names; nay when the Names of the parts of any Speech be explicated, is it not neceſſary that the Definition ſhould be a Name Compounded of them. For example, when theſe Names; *Æquilaterall*, *Quadrilaterall*, *Right-angled*, are ſufficiently underſtood, it is not neceſſary in Geometry that there ſhould be at all ſuch a Name as *Square*; for defined Names are received in Philoſophy for brevities ſake onely.

Fiftly, That Compounded Names which are defined one way in ſome one part of Philoſophy, may in another part of the ſame be otherwiſe defined; as a *Parabola* and an *Hyperbole* have one Definition in Geometry, and another in Rhetorique; for Definitions are inſtituted and ſerve for the underſtanding of the Doctrine which is treated of. And therefore as in one part of Philoſophy, a Definition may have in it ſome one fit Name for the more briefe explanation of ſome propoſition in Geometry; ſo it may have the ſame liberty in other parts of Philoſophy; for the uſe of Names is particular (even where many agree to the ſetling of them) and arbitrary.

Sixtly, That no Name can be defined by any one Word; becauſe no one Word is ſufficient for the Reſolving of one or more words.

Seventhly, That a defined Name ought not to be repeated in the Definition. For a defined Name, is the whole Compound, and a Definition is the Reſolution of that Compound into parts; but no Totall can be part of it ſelfe.

16 Any two Definitions that may be compounded into a Syllogiſme, produce a Concluſion; which becauſe it is derived from Principles, that is, from Definitions, is ſaid to be *Demonſtrated*; and the Derivation or Compoſition it ſelfe is called a *Demonſtration*. In like manner, if a Syllogiſme be made of two Propoſitions, whereof one is a Definition, the other a Demonſtrated Concluſion, or neither of them is a Definition, but both formerly demonſtrated, that Syllogiſme is alſo called a Demonſtration, and ſo ſucceſſively. The Definition therefore of a Demonſtration is this, *A* DEMONSTRATION *is a Syllogiſm or Series of Syllogiſms derived and continued from the Definitions of Names, to the laſt Concluſion.*

And

And from hence it may be underſtood, that all true Ratiocination, which taketh its beginning from true Principles, produceth Science, and is true Demonſtration. For as for the Originall of the Name; although that which the Greeks called ἀποδείξις and the Latines *Demonſtratio* was underſtood by them for that ſort onely of Ratiocination, in which by the deſcribing of certaine Lines and Figures, they placed the thing they were to prove, as it were before mens Eyes, which is properly ἀποδεικνύειν, or to *ſhew* by the Figure; yet they ſeem to have done it for this reaſon, that unleſſe it were in Geometry (in which only there is place for ſuch Figures) there was no Ratiocination certaine, and ending in Science, their Doctrines concerning all other things being nothing but Controverſie and Clamour; which nevertheleſſe hapned not becauſe the Truth to which they pretended could not be made evident without Figures, but becauſe they wanted true Principles, from which they might derive their Ratiocination; and therefore there is no reaſon but that if true Definitions were præmiſed in all ſorts of Doctrines, the Demonſtrations alſo would be true.

17 It is proper to Methodical Demonſtration,

Firſt, That there be a true Succeſſion of one Reaſon to another, according to the Rules of Syllogizing delivered above.

Secondly, That the Præmiſſes of all Syllogiſmes be demonſtrated from the firſt Definitions.

Thirdly, That after Definitions, he that Teaches or Demonſtrates any thing, proceed in the ſame Method by which he found it out; namely, that in the firſt place thoſe things be demonſtrated which immediately ſucceed to Univerſal Definitions (in which is contained that part of Philoſophy which is called *Philoſophia Prima.*) Next, thoſe things which may be demonſtrated by Simple Motion (in which Geometry conſiſts.) After Geometry, ſuch things as may be taught or ſhewed by manifeſt Action, that is, by Thruſting from, or Pulling towards. And after theſe, the Motion or Mutation of the inviſible parts of Things, and the Doctrine of Senſe & Imagination & of the internal Paſſions, eſpecially thoſe of Men, in which are comprehended the Grounds of Civil Duties, or Civil Philoſophy; which takes up the laſt place. And that this Method ought to be kept in all ſorts of Philoſophy, is evident from hence, that ſuch things as I have ſaid are to be taught

laſt,

laſt cannot be demonſtrated, till ſuch as are propounded to be firſt treated of, be fully uuderſtood. Of which Method no other Example can be given, but that Treatiſe of the Elements of Philoſophy, which I ſhall begin in the next Chapter, and continue to the end of the worke.

18 Beſides thoſe *Paralogiſmes*, whoſe fault lies either in the Falſity of the Præmiſſes, or the want of true Compoſition, of which I have ſpoken in the præcedent Chapter, there are two more which are frequent in Demonſtration; one whereof is commonly called *Petitio Principii*; the other is the ſuppoſing of a *Falſe Cauſe*; and theſe do not onely deceive Unskilfull Learners but ſometimes Maſters themſelves, by making them take that for well demonſtrated which is not demonſtrated at all. *Petitio Principii*, is when the Concluſion to be proved, is diſguiſed in other Words, and put for the Definition or Principle from whence it is to be demonſtrated; and thus by putting for the Cauſe of the Thing ſought, either the Thing it ſelfe or ſome Effect of it, they make a Circle in their Demonſtration. As for example, He that would Demonſtrate that the Earth ſtands ſtill in the Center of the World, and ſhould ſuppoſe the Earths Gravity to be the Cauſe thereof, and define Gravity to be a quality by which every heavy Body tends towards the Center of the World, would loſe his labour; for the queſtion is, What is the Cauſe of that quality in the Earth; and therefore he that ſuppoſes Gravity to be the Cauſe, puts the Thing it ſelfe for its own Cauſe.

Of a *Falſe Cauſe* I find this example in a certaine Treatiſe where the thing to be demonſtrated is the Motion of the Earth. He begins therefore with this, that ſeeing the Earth and the Sun are not alwayes in the ſame ſcituation, it muſt needs be that one of them be locally moved; which is true; next he affirms that the Vapours which the Sun raiſes from the Earth and Sea are by reaſon of this Motion neceſſarily moved; which alſo is true; from whence he infers the Winds are made, and this may paſſe for granted; and by theſe Winds he ſayes the Waters of the Sea are moved, and by their Motion the bottome of the Sea, as if it were beaten forwards, moves round; and let this alſo be granted; wherefore he concludes, the Earth is moved; which is nevertheleſſe is a Paralogiſme. For if that wind were

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the Cauſe why the Earth was from the beginning moved round, and the Motion either of the Sunne or the Earth were the Cauſe of that Wind, then the Motion of the Sunne or the Earth was before the Wind it ſelf; and if the Earth were Moved before the Wind was made, then the Wind could not be the cauſe of the Earths revolution; but if the Sunne were Moved, and the Earth ſtand ſtill, then it is manifeſt the Earth might remain Unmoved notwithſtanding that Wind; and therefore that motion was not made by the Cauſe which he alledgeth. But Paralogiſmes of this kind are very frequent among the Writers of *Phyſiques*, though none can be more elaborate then this in the Example given.

19 It may to ſome men ſeem pertinent to treat in this place of that Art of the Geometricians, which they call *Logiſtica*, that is, the Art, by which, from ſuppoſing the thing in queſtion to be true, they proceed by Ratiocination, till either they come to ſomething knowne, by which they may demonſtrate the truth of the thing ſought for; or to ſomething which is impoſſible, from whence they collect that to be falſe which they ſuppoſed true. But this Art cannot be explicated here, for this reaſon, that the Method of it can neither be practiſed, nor underſtood unleſſe by ſuch as are well verſed in Geometry; and among Geometricians themſelves, they that have moſt Theoremes in readineſs, are the moſt ready in the uſe of this *Logiſtica*; ſo that indeed it is not a diſtinct thing from Geometry it ſelfe; for there are in the Method of it three parts; the firſt whereof conſiſts in the finding out of Equality betwixt known and unknown things, which they call Equation; and this Equation cannot be found out but by ſuch as know perfectly the Nature, Properties and Tranſpoſitions of Proportion, as alſo the Addition, Subſtraction, Multiplication, and Diviſion of Lines and Superficies, and the Extraction of Roots; which are the parts of no meane Geometrician. The Second is, when an Equation is found, to be able to judge whether the Truth or Falſity of the Queſtion may be deduced from it or no; which yet requires greater Knowledge. And the third is, when ſuch an Equation is found, as is fit for the ſolution of the Queſtion, to know how to Reſolve the ſame in ſuch manner, that the Truth or Falſity may there-

by manifestly appeare; which in hard questions cannot be done without the Knowledge of the Nature of Crooked-lined Figures; but he that understands readily the Nature and Properties of these, is a Compleat Geometrician. It happens besides, that for the finding out of Equations there is no certaine Method, but he is best able to do it, that has the best Naturall Wit.

THE

THE FIRST GROVNDS OF PHILOSOPHY.

CHAP. VII.
Of Place and Time.

1 *Things that have no existence, may nevertheleſſe be underſtood and computed.* 2 *What is ſpace.* 3 *Time.* 4 *Part.* 5 *Diviſion.* 6 *One.* 7 *Number.* 8 *Compoſition.* 9 *The Whole.* 10 *Spaces and Times Contiguous, and Continuall.* 11 *Beginning, End, Way, Finite, Infinite.* 12 *What is Infinite in Power. Nothing Infinite can be truly ſaid to be either Whole, or One; Nor Infinite Spaces or Times, Many.* 13 *Diviſion proceeds not to the Leaſt.*

1 IN the Teaching of Naturall Philoſophy, I cannot begin better (as I have already ſhewn) then from *Privation*; that is, from feigning the World to be annihilated. But if ſuch annihilation of all things be ſuppoſed, it may perhaps be aſked, what would remain for any Man (whom onely I except from this Univerſal annihilation of things) to conſider as the Subject of Philoſophy, or at all to reaſon upon; or what to give Names unto for Ratiocinations ſake.

I ſay therefore there would remain to that Man Ideas of the World, and of all ſuch Bodies as he had, before their annihilation, ſeen with his eies, or perceived by any other Senſe; that is to ſay, the Memory and Imagination of Magnitudes, Motions, Sounds, Colours, &c. as alſo of their order & parts. All wch things, though they be nothing but Ideas & Phantaſms, hapning internally to him that imagineth; yet they will appear as if they were externall, and not at all depending upon any power of the Mind. And theſe are

the things to which he would give Names, and ſubſtract them from, and compound them with one another. For ſeeing that after the deſtruction of all other things, I ſuppoſe Man ſtill remaining, and namely that he thinkes, imagines, and remembers, there can be nothing for him to thinke of but what is Paſt; Nay, if we do but obſerve diligently what it is we doe when we conſider and reaſon, we ſhall find, that though all things be ſtill remaining in the world, yet we compute nothing but our own Phantaſmes. For when we calculate the magnitude and motions of Heaven or Earth, we doe not aſcend into Heaven that we may divide it into parts, or meaſure the motions thereof, but we doe it ſitting ſtill in our Cloſets or in the Darke. Now things may be conſidered, that is, be brought into Account, either as internal Accidents of our Mind, in which manner we conſider them when the queſtion is about ſome Faculty of the Mind; or as Species of external things, not as really exiſting, but appearing onely to exiſt, or to have a Being without Us. And in this manner we are now to conſider them.

2 If therefore we remember, or have a Phantaſme of any thing that was in the world before the ſuppoſed annihilation of the ſame; and conſider, not that the thing was ſuch or ſuch, but onely that it had a Being without the Mind, we have preſently a Conception of that we call *Space*: an Imaginary Space indeed, becauſe a meere Phantaſme, yet that very thing which all men call ſo. For no man calls it Space for being already filled, but becauſe it may be filled; nor does any man think Bodies carry their Places away with them, but that the ſame Space contains ſometimes one, ſometimes another Body; which could not be if Space ſhould alwayes accompany the Body which is once in it. And this is of it ſelfe ſo manifeſt, that I ſhould not thinke it needed any explaining at all, but that I finde Space to be falſely defined by certaine Philoſophers, who inferre from thence, One, that the world is Infinite; for taking *Space* to be the Extenſion of Bodies, and thinking Extenſion may encreaſe continually, he inferres that Bodies may be infinitely Extended; and Another from the ſame Definition concludes raſhly, that it is impoſſible even to God himſelfe to create more Worlds then one; for if another World were to be created, he ſayes, that ſeeing there is nothing without this world, and therefore (according to his

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Definition) no Space, that new world muſt be placed in nothing; but in nothing nothing can be placed; which he affirms onely, without ſhewing any reaſon for the ſame; whereas the contrary is the truth: for more cannot be put into a Place allready filled, ſo much is Empty Space fitter then that which is Full for the receiving of new Bodies. Having therefore ſpoken thus much for theſe mens ſakes, and for theirs that aſſent to them, I return to my purpoſe, and define *Space* thus, SPACE *is the Phantaſme of a Thing exiſting without the Mind ſimply*; that is to ſay, that Phantaſme, in which we conſider no other Accident, but onely that it appears without us.

3 As a Body leaves a Phantaſme of its Magnitude in the mind, ſo alſo a Moved Body leaves a Phantaſme of its Motion, namely an Idea of that Body paſſing out of one Space into another by continuall ſucceſſion. And this Idea or Phantaſme, is that which (without receding much from the common opinion, or from *Ariſtotles* Definition) I call *Time*. For ſeeing all men confeſſe a Yeare to be Time, and yet do not think a Year to be the Accident or Affection of any Body, they muſt needs confeſſe it to be, not in the things without Us, but only in the Thought of the Mind. So when they ſpeake of the Times of their Predeceſſors, they do not think after their Predeceſſors are gone, that their Times can be any where elſe then in the Memory of thoſe that remember them. And as for thoſe that ſay, Dayes, Years and Moneths are the Motions of the Sunne and Moon, ſeeing it is all one to ſay, Motion *Paſt* and Motion *Deſtroyed*, and that *Future* Motion is the ſame with Motion which *Is not yet begun*, they ſay, that which they do not meane, that there neither is, nor has been, nor ſhall be any Time: for of whatſoever it may be ſaid, *It has been* or *It ſhall be*, of the ſame alſo it might have been ſaid heretofore, or may be ſaid hereafter, *It is*. What then can Dayes, Moneths and Yeares be, but the Names of ſuch Computations made in our Mind? *Time* therefore is a Phantaſme, but a Phantaſme of Motion, for if we would know by what Moments Time paſſes away, we make uſe of ſome Motion or other, as of the Sun, of a Clock, of the ſand in an Hour-glaſſe, or we mark ſome Line upon which we imagine ſomething to be Moved, there being no other means by which we can take notice of any Time at all. And yet when I ſay *Time* is a Phantaſme

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of Motion, I doe not say this is sufficient to define it by ; for this word *Time* comprehends the notion of *Former* and *Later*, or of *Succession* in the motion of a Body, in as much as it is first *Here* then *There*. Wherefore a compleat Definition of *Time* is such as this, TIME *is the Phantasme of Before and After in Motion* ; which agrees with this Definition of *Aristotle*, *Time is the Number of Motion according to Former and Later* ; for that Numbring is an act of the Mind ; and therefore it is all one to say, *Time is the Number of Motion according to Former and Later* ; and *Time is a Phantasme of Motion Numbred*. But that other Definition, *Time is the Measure of Motion*, is not so exact ; for we measure Time by Motion and not Motion by Time.

4 One Space is called *Part* of another Space, and one Time *Part* of another Time, when this containes that and something besides. From whence it may be collected, that nothing can rightly be called a PART, but that which is compared with something that contains it.

5 And therefore to *make parts*, or to *Part* or DIVIDE Space or *Time*, is nothing else but to consider One and Another within the same; so that if any Man *Divide* Space or Time, the diverse Conceptions he has are more by one, then the Parts he makes ; for his first Conception is of that which is to be divided, then of some Part of it, and again of some other Part of it, and so forwards as long as he goes on in Dividing.

But it is to be noted, that here by *Division*, I doe not mean the severing or pulling asunder of one Space or Time from another (for does any man think that one Hemisphere may be separated from the other Hemisphere, or the first Hour from the second ?) but Diversity of Consideration ; so that Division is not made by the operation of the Hands but of the Mind.

6 When Space or Time is considered among other Spaces or Times, it is said to be ONE, namely *One of them* ; for except One Space might be added to another, and substracted from another Space, and so of Time, it would be sufficient to say Space or Time simply, and superfluous to say One Space or One Time, if it could not be conceived that there were another. The common Definition of *One*, namely, that *One is that which is Undivided*, is obnoxious to an absurd Consequence; for it may thence be inferred, that whatsoever is Divided, is many things, that is, that every Divided

vided thing, is Divided Things, which is Insignificant.

7 NUMBER is One and One, or One One and One, and so forwards; namely One and One make the Number *Two*, and One One and One, the Number *Three*; and so are all other Numbers made; which is all one as if we should say, *Number is Unities*.

8 To COMPOUND Space of Spaces, or Time of Times, is first to Consider them one after another, and then altogether as One; as if one should reckon first the Head, the Feet, the Armes and the Body severally, and then for the account of them all together put *Man*. And that which is so put for all the severalls of which it consists, is called the WHOLE; and those severalls, when by the Division of the Whole, they come again to be considered singly, are parts thereof; and therefore the *Whole*, and *all the Parts taken together*, are the same thing. And as I noted above, that in *Division* it is not necessary to pull the Parts asunder; so in *Composition* it is to be understood, that for the making up of a whole there is no need of putting the Parts together, so as to make them touch one another, but onely of collecting them into one summe in the Mind. For thus all Men being considered together, make up the Whole of Mankind, though never so much dispersed by Time and Place; and twelve Hours, though the hours of severall dayes, may be Compounded into one Number of Twelve.

9 This being well understood, it is manifest, that nothing can rightly be called a Whole, that is not conceived to be compounded of Parts, and that it may be divided into parts; so that if we deny that a thing has parts, we deny the same to be a Whole. For example, if we say the soul can have no Parts, we affirme that no soul can be a Whole soul. Also it is manifest, that Nothing has Parts till it be Divided; and when a Thing is Divided, the Parts are onely so many as the Division makes them. Againe, that a Part of a Part is a Part of the Whole; & thus any Part of the Number *Four*, as *Two*, is a Part of the Number *Eight*; for *Four* is made of *Two* and *Two*; but *Eight* is compounded of *Two*, *Two* and *Four*; and therefore *Two* which is a Part of the Part *Four*, is also a Part of the whole *Eight*.

10 Two Spaces are said to be CONTIGUOUS, when there is no other Space betwixt them. But two Times, betwixt which there

there is no other Time, are called IMMEDIATE, A B C as AB, BC. And any two Spaces as well as Times are said to be CONTINUALL, when they have one common part, A B C D as AC, BD, where the part BC is common; and more Spaces and Times are Continual, when every two which are next one another are Continual.

11 That Part which is between two other Parts, is called a MEAN; & that which is not between two other parts, an EXTREME. And of Extremes, that which is first reckoned is the BEGINNING, and that which last, the END; and all the Means together taken, are the WAY. Also *Extreme Parts* and *Limits* are the same thing. And from hence it is manifest, that *Beginning* and *End* depend upon the order in which we number them; and that to *Terminate* or *Limit* Space and Time, is the same thing with *imagining their Beginning and End*; as also that every thing is FINITE or INFINITE, acording as we imagine or not imagine it *Limited* or *Terminated* every way; and that the *Limits* of any Number are *Unities*, and of these, that which is the first in our Numbering is the *Beginning*, and that which we number last, is the *End*. When we say Number is *Infinite*, we mean only that no Number is expressed; for when we speak of the Numbers *Two*, *Three*, *a Thousand*, *&c.* they are always *Finite*. But when no more is said but this, *Number is Infinite*, it is to be understood as if it were said, this Name *Number* is an *Indefinite* Name.

12 Space or Time is said to be *Finite in Power*, or *Terminable*, when there may be assigned a Number of finite Spaces or Times, as of Paces or Hours, than which there can be no greater Number of the same measure, in that Space or Time; and *Infinite in Power* is that Space or Time, in which a greater Number of the said Paces or Hours may be assigned, than any Number that can be given. But we must note, that although in that Space or Time which is Infinite in Power, there may be numbered more Paces or Hours then any number that can be assigned, yet their number will alwayes be Finite; for every Number is Finite. And therefore his Ratiocination was not good, that undertaking to prove the World to be Finite, reasoned thus; *If the world be Infinite, then there may be taken in it some Part which is distant from us an Infinite number of Paces: But no such Part can be taken; wherefore the*

World

world is not infinite; becauſe that Conſequence of the Major Propoſition is falſe; for in an Infinite ſpace, whatſoever we take, or deſign in our Mind, the diſtance of the ſame from us is a Finite ſpace; for in the very deſigning of the place thereof, we put an End to that ſpace, of whch we our ſelves are the Beginning; and whatſoever any man with his Mind cuts off both wayes from Infinite, he determines the ſame, that is, he makes it Finite.

Of Infinite Space or Time, it cannot be ſaid that it is a *Whole*, or *One*; not a *Whole*, becauſe not compounded of Parts; for ſeeing Parts, how many ſoever they be, are ſeverally Finite, they will alſo when they are all put together make a whole Finite; Nor *One*, becauſe nothing can be ſaid to be One, except there be Another to compare it with; but it cannot be conceived that there are two Spaces, or two Times Infinite. Laſtly, when we make queſtion whether the World be Finite or Infinite, we have nothing in our Minde anſwering to the Name *World*; for whatſoever we Imagine, is therefore Finite, though our Computation reach the fixed Stars, or the ninth or tenth, nay, the thouſanth Sphere. The meaning of the Queſtion is this onely, whether God has actually made ſo great an Addition of Body to Body, as we are able to make of Space to Space.

13 And therefore that which is commonly ſaid, that Space and Time may be divided Infinitely, is not to be ſo underſtood, as if there might be any Infinite or Eternal Diviſion; but rather to be taken in this ſenſe, *Whatſoever is Divided, is divided into ſuch Parts as may again be Divided*; or thus, *The Leaſt Diviſible thing is not to be given*; or as Geometricians have it, *No Quantity is ſo ſmall, but a Leſs may be taken*; which may eaſily be demonſtrated in this manner. Let any Space or Time (that which was thought to be the Leaſt Diviſible) be divided into two equal Parts A and B. I ſay either of them, as A, may be divided again. For ſuppoſe the Part A to be contiguous to the Part B of one ſide, and of the other ſide to ſome other Space equal to B. This whole Space therefore (being greater then the Space given) is diviſible. Wherefore if it be divided into two equal Parts, the Part in the middle, which is A, will be alſo divided into two equal Parts; and therefore A was Diviſible.

Chap. VIII.
Of Body and Accident.

1 *Body defined.* 2 *Accident defined.* 3 *How an Accident may be understood to be in its ſubject.* 4 *Magnitude, what it is.* 5 *Place what it is, and that it is Immoveable.* 6 *What is Full and Empty.* 7 *Here, There, Somewhere, what they ſignifie.* 8 *Many Bodies cannot be in One place, nor One Body in Many places.* 9 *Contiguous and Continual what they are.* 10 *The definition of Motion. No Motion intelligible but with Time.* 11 *What it is to be at Reſt, to have been Moved, and to be Moved. No Motion to be conceived, without the conception of Paſt and Future.* 12 *A Point, a Line, Superficies and Solid, what they are.* 13 *Equal, Greater and Leſſe in Bodies and Magnitudes, what they are.* 14 *One and the ſame Body has alwayes one and the the ſame Magnitude.* 15 *Velocity what it is.* 16 *Equal, Greater and Leſſe in Times what they are.* 17 *Equal, Greater and Leſſe in Velocity, what.* 18 *Equal, Greater and Leſſe in Motion, what.* 19 *That which is at Reſt will alwayes be at Reſt except it be Moved by ſome external thing; and that which is Moved will alwayes be Moved, unleſs it be hindered by ſome external thing.* 20 *Accidents are Generated and Deſtroyed, but Bodies not ſo.* 21 *An Accident cannot depart from its Subject.* 22 *Nor be Moved.* 23 *Eſſence, Form, and Matter, what they are.* 24 *Firſt Matter, what.* 25 *That the whole is greater then any Part thereof, why demonſtrated.*

1 HAving underſtood what Imaginary Space is, in which we ſuppoſed nothing remaining without us, but all thoſe things to be deſtroyed that by exiſting heretofore left Images of themſelves in our Minds; let us now ſuppoſe ſome one of thoſe things to be placed again in the World, or created anew. It is neceſſary therefore that this new created or replaced thing do not onely fill ſome part of the Space above-mentioned, or be coincident and coextended with it, but alſo, that it have no dependance upon our thought. And this is that which for the Ex-

Extenſion of it we commonly call *Body*; and becauſe it depends not upon our Thought, we ſay is *a thing ſubſiſting of it ſelf*; as alſo *exiſting*, becauſe without Us; and laſtly, it is called the *Subject*, becauſe it is ſo placed in and *ſubjected* to Imaginary Space, that it may be underſtood by Reaſon, as well as perceived by Senſe. The Definition therefore of *Body* may be this, A BODY *is that which having no dependance upon our Thought is coincident or coextended with ſome part of Space.*

2 But what an *Accident* is, cannot ſo eaſily be explained by any Definition, as by Examples. Let us imagine therefore that a Body fills any Space, or is coextended with it, that Coextention is not the coextended Body; And in like manner, let us imagine that the ſame Body is removed out of its place, that Removing is not the removed Body; Or let us think the ſame not removed, that not-removing or Reſt, is not the reſting Body. What then are theſe things? They are *Accidents* of that Body. But the thing in queſtion is *What is an Accident*; which is an Enquiry after that which we know already, and not that which we ſhould enquire after. For who does not alwayes and in the ſame manner underſtand him that ſayes any thing is Extended, or Moved, or not Moved? But moſt men will have it be ſaid that *an Accident is ſomething*, namely ſome part of a natural thing, when indeed it is no part of the ſame. To ſatisfie theſe men, as well as may be, they anſwer beſt that define an *Accident* to be *the Manner by which any Body is conceived*, which is all one as if they ſhould ſay, *An Accident is that faculty of any Body by which it works in us a Conception of it ſelf.* Which Definition though it be not an Anſwer to the Queſtion propounded, yet it is an Anſwer to that Queſtion which ſhould have been propounded, namely, *Whence does it happen that one part of any Body appears here, another part there?* For this is well anſwered thus, *It happens from the Extenſion of that Body.* Or, *How comes it to paſs that the whole Body by ſucceſſion is ſeen now here now there?* and the anſwer will be, *By reaſon of its Motion.* Or laſtly, *Whence is it that any Body poſſeſſeth the ſame ſpace for ſometime?* And the anſwer will be, *becauſe it is not moved.* For if concerning the Name of a Body, that is, concerning a Concrete Name, it be asked, *what is it?* the anſwer muſt be made by Definition; for the Queſtion is concerning the ſignification of the Name. But if it be asked concerning an Abſtract Name, *what is it?* the Cauſe

is demanded why a thing appears ſo or ſo. As if it be asked, *what is Hard?* The Anſwer will be, Hard is that, whereof no Part gives place, but when the Whole gives place. But if it be demanded, *what is Hardneſs?* A Cauſe muſt be ſhewn why a Part does not give place except the Whole give place. Wherefore I define an ACCIDENT to be *the Manner of our conception of Body.*

3 When an *Accident* is ſaid *to be in a Body*, it is not ſo to be underſtood, as if any thing were conteined in that Body; as if, for example, Redneſs were in Blood, in the ſame manner, as Blood is in a bloody cloth, that is, as a Part in the Whole; for ſo an Accident would be a Body alſo. But as Magnitude, or Reſt, or Motion, is in that which is Great, or which Reſteth, or which is Moved (which how it is to be underſtood every man underſtands) ſo alſo it is to be underſtood, that every other Accident *is in* its Subject. And this alſo is explicated by *Ariſtotle* no otherwiſe then negatively, namely, that *An Accident is in its Subject, not as any part thereof, but ſo as that it may be away, the Subject ſtill remaining*; which is right, ſaving that there are certain Accidents which can never periſh except the Body periſh alſo; for no Body can be conceived to be without Extenſion, or without Figure. All other Accidents, which are not common to all Bodies, but peculiar to ſome onely, as *To be at Reſt, to be Moved, Colour, Hardneſs*, and the like, do periſh continually, and are ſucceeded by others; yet ſo, as that the Body never periſheth. And as for the opinion that ſome may have, that all other Accidents are not in their Bodies in the ſame manner that Extenſion, Motion, Reſt, or Figure are in the ſame; for example, that Colour, Heat, Odour, Vertue, Vice and the like, are otherwiſe in them, and (as they ſay) *inherent*; I deſire they would ſuſpend their judgement for the preſent, and expect a little, till it be found out by Ratiocination, whether theſe very Accidents are not alſo certain Motions, either of the Mind of the perceiver; or of the Bodies themſelves which are perceived; for in the ſearch of this, a great part of Naturall Philoſophy conſiſts.

4 The Extenſion of a Body, is the ſame thing with the MAGNITUDE of it, or that which ſome call *Real Space*. But this *Magnitude* does not depend upon our Cogitation, as Imaginary Space doth; for this is an Effect of our Imagination, but *Magnitude* is the Cauſe of it; this is an Accident of the Mind, that of a Body exiſting out of the Mind.

5 That

5 That Space (by which word I here understand Imaginary Space) which is coincident with the Magnitude of any Body, is called the PLACE of that Body; and the Body it self is that which we call the *Thing Placed*. Now *Place*, and the *Magnitude* of the Thing Placed differ: First in this, that a Body keeps alwayes the same *Magnitude* both when it is at Rest, and when it is Moved; but when it is Moved, it does not keep the same *Place*. Secondly, in this, that *Place* is a Phantasme of any Body of such and such Quantity and Figure; but *Magnitude* is the peculiar Accident of every Body; for one Body may at several times have several Places, but has always one and the same Magnitude. Thirdly, in this, that *Place* is nothing out of the Mind, nor *Magnitude* any thing within it. And lastly, *Place* is feigned Extension but *Magnitude* true Extension, and a Placed Body is not Extension, but a Thing Extended. Besides, *Place is Immoveable*; for seeing that which is Moved, is understood to be carried from Place to Place, if Place were Moved, it would also be carried from Place to Place, so that one Place must have another Place, and that Place another Place, and so on infinitely, which is ridiculous. And as for those, that by making *Place* to be of the same Nature with *Real Space*, would from thence maintain it to be Immoveable, they also make Place (though they do not perceive they make it so) to be a meer Phantasme. For whilest One affirms that Place is therefore said to be Immoveable, because Space in general is considered there; if he had remembred that nothing is General or Universal besides Names or Signes, he would easily have seen that that Space which he sayes is considered in general, is nothing but a Phantasme in the Mind or the Memory, of a Body of such Magnitude and such Figure. And whilest another sayes, Real Space is made Immoveable by the Understanding; as when under the Superficies of running water, we imagine other and other water to come by continual succession, that Superficies fixed there by the Understanding is the *Immoveable Place* of the River, what else does he make it to be but a Phantasm, though he doe it obscurely, and in perplexed words? Lastly, the nature of *Place* does not consist in the *Superficies of the Ambient*, but in *Solid Space*; for the whole Placed Body is coextended with its whole Place, and every part of it with every answering part of the same Place; but seeing every

Placed Body is a Solid thing, it cannot be underſtood to be coextended with Superficies. Beſides, how can any whole Body be Moved, unleſs all its parts be moved together with it? Or how can the internall Parts of it be Moved, but by leaving their Place? But the internal Parts of a body cannot leave the Superficies of an external part contiguous to it; and therefore it followes, that if Place be the Superficies of the Ambient, then the parts of a Body Moved, that is Bodies moved, are not Moved.

6 Space (or Place) that is poſſeſſed by a Body, is called FULL, and that which is not ſo poſſeſſed is called EMPTY.

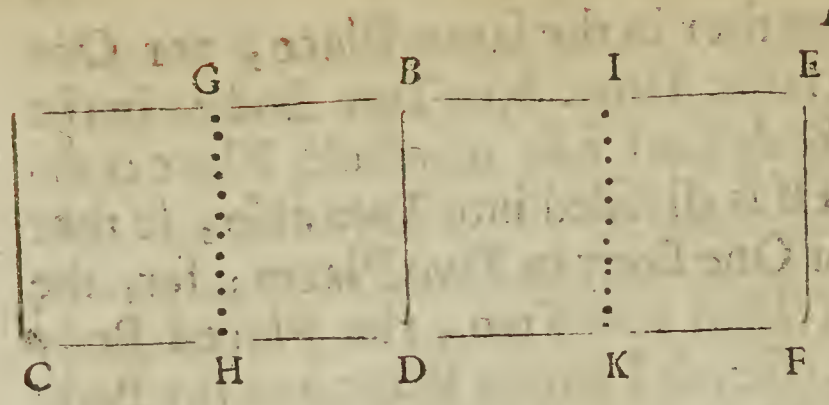
7 *Here*, *There*, *In the Country*, *In the City*, and other the like Names by which anſwer is made to the queſtion *Where is it*, are not properly Names of Place, nor doe they of themſelves bring into the mind the Place that is ſought; for *Here* and *There* ſignifie nothing, unleſſe the thing be ſhewn at the ſame time with the finger or ſomething elſe; but when the Eye of him that ſeeks, is by pointing, or ſome other ſigne directed to the thing ſought, the Place of it is not hereby defined by him that anſwers, but found out by him that askes the queſtion. Now ſuch Shewings as are made by words onely, as when we ſay, *In the Countrey*, or *In the City*, are ſome of greater latitude then others, as when we ſay *In the Countrey*, *In the City*, *In ſuch a Street*, *In a Houſe*, *In the Chamber*, *In Bed*, &c. For theſe do by little and little direct the Seeker neerer to the proper Place; & yet they do not determine the ſame, but onely reſtrain it to a leſſer Space, & ſignifie no more then that the Place of the Thing is within a certain Space deſigned by thoſe Words, as a Part is in the Whole. And all ſuch Names (by which anſwer is made to the queſtion *Where*) have for their higheſt *Genus* the Name *Somewhere*. From whence it may be underſtood, that whatſoever is Somewhere, is in ſome Place properly ſo called, which Place is part of that greater Space that is ſignified by ſome of theſe Names, *In the Countrey*, *In the City*, or the like.

8 A Body, and the Magnitude, and the Place thereof, are divided by one and the ſame act of the Mind; for, to divide an Extended Body, and the Extenſion thereof, and the Idea of that Extenſion, which is Place, is the ſame with dividing any one of them; becauſe they are coincident, and it cannot be done but by the Mind, that is by the Diviſion of Space. From whence it is manifeſt, that nei-

neither Two Bodies can be together in the ſame Place, nor One Body be in Two Places at the ſame Time. Not Two Bodies in the ſame Place; becauſe when a Body that fills its whole Place is divided into Two, the Place it ſelf is divided into Two alſo, ſo that there will be Two Places. Nor One Body in Two Places; for, the Place that a Body fills being divided into Two, the Placed Body will be alſo divided into Two, (for, as I ſaid, a Place and the Body that fills that Place are divided both together) and so there will be two Bodies.

9 Two Bodies are ſaid to be *Contiguous* to one another, and *Continual*, in the ſame manner as Spaces are; namely, *thoſe are Contiguous, between which there is no Space*. Now by Space I underſtand here as formerly an Idea or Phantaſme of a Body. Wherefore, though between two Bodies there be put no other Body, and conſequently no Magnitude, or (as they call it) Real Space, yet if another Body may be put between them, that is, if there intercede any imagined Space which may receive another Body, then thoſe Bodies are not Contiguous. And this is ſo eaſie to be underſtood, that I ſhould wonder at ſome men, who being otherwiſe skilful enough in Philoſophy, are of a different opinion, but that I finde that moſt of thoſe that affect Metaphyſical ſubtilties, wander from Truth, as if they were led out of their way by an *Ignis Fatuus*. For can any man that has his natural Senſes, think that two Bodies muſt therefore neceſſarily Touch one another, becauſe no other Body is between them? Or that there can be no *Vacuum*, becauſe *Vacuum* is nothing, or as they call it, *Non Ens*? Which is as childiſh, as if one ſhould reaſon thus; No Man can Faſt, becauſe to Faſt is to eat Nothing; but Nothing cannot be eaten. *Continual, are any two Bodies that have a common part; and more then two are Continual, when every two that are next to one another, are continual.*

10 MOTION, *is a continual relinquiſhing of one Place, and acquiring of another*; and that Place which is relinquiſhed is commonly called the *Terminus à quo*, as that which is acquired is called the *Terminus ad quem*; I ſay a continual Relinquiſhing, becauſe no Body, how little ſoever, can totally and at once go out of its former Place into another, ſo, but that ſome part of it will be in a part of Place which is common to both, namely, to the relinquiſhed and the acquired Places. For example, let any Body be in the Place



AC B D; the ſame Body cãnot come into the Place B D E F, but it muſt firſt be in G H I K, whoſe part G H B D is common to both the Places A C B D, and G H I K; and whoſe part B D I K is common to both the Places G H I K, and B D E F. Now it cannot be conceived that any thing can be Moved without Time; for Time, is by the Definition of it, a Phantaſme, that is a Conception of Motion; and therefore to conceive that any thing may be Moved without Time, were to conceive Motion without Motion, which is impoſſible.

11 *That is ſaid to be at Reſt, which during any time is in one place; and that to be Moved, or to have been Moved, which whether it be now at Reſt, or Moved, was formerly in another Place then that which it is now in.* From which Definitions it may be inferred, Firſt, that *Whatſoever is Moved, has been Moved*; for if it be ſtill in the ſame Place in which it was formerly, it is at Reſt, that is, it is not Moved, by the Definition of *Reſt*; but if it be in another Place, it has been Moved, by the Definition of *Moved*. Secondly, that *what is Moved, will yet be Moved*; for that which is Moved, leaveth the Place where it is, and therefore will be in another Place, and conſequently will be moved ſtill. Thirdly, that *whatſoever is Moved, is not in One place during any time, how little ſoever that time be*; for by the Definition of Reſt, that which is in one Place during any time, is at Reſt.

There is a certain Sophiſme againſt Motion, which ſeems to ſpring from the not underſtanding of this laſt Propoſition. For they ſay, that, *If any Body be Moved, it is Moved either in the Place where it is, or in the Place where it is not; both which are falſe, and therefore nothing is Moved.* But the falſity lies in the Major Propoſition; for that which is Moved, is neither Moved in the Place where it is, nor in the Place where it is not; but from the Place where it is, to the Place where it is not. Indeed it cannot be denied but that whatſoever is Moved, is Moved ſomewhere, that is, within ſome Space; but then the Place of that Body is not that whole Space, but a part of it, as is ſaid above in the ſeventh Article. From what is above demonſtrated, namely, that whatſoever is Moved, has alſo

been

been Moved, and will be Moved, this alſo may be collected, that there can be no conception of Motion, without conceiving Paſt and Future time.

12 Though there be no Body which has not ſome Magnitude, yet if when any Body is moved, the Magnitude of it be not at all conſidered, the way it makes it called a LINE, or one ſingle Dimenſion; & the Space through which it paſſeth, is called LENGTH; and the Body it ſelf, a POINT; in which ſenſe the Earth is called a *Point*, and the Way of its yearly Revolution, the *Ecliptick Line*. But if a Body which is Moved, be conſidered as *long*, and be ſuppoſed to be ſo Moved, as that all the ſeveral parts of it be underſtood to make ſeveral Lines, then the Way of every part of that Body is called BREADTH, and the Space which is made is called SUPERFICIES, conſiſting of two Dimenſions, one whereof to every ſeveral part of the other is applyed whole. Again, if a Body be conſidered as having *Superficies*, and be underſtood to be ſo Moved, that all the ſeveral parts of it deſcribe ſeveral Lines, then the Way of every part of that Body is called THICKNESS, or DEPTH, and the Space which is Made is called SOLID, conſiſting of three Dimenſions, any two whereof are applyed whole to every ſeveral part of the third.

But if a Body be conſidered as *Solid*, then it is not poſſible that all the ſeveral parts of it ſhould deſcribe ſeveral lines; for what way ſoever it be Moved, the way of the following part will fall into the way of the part before it, ſo that the ſame Solid will ſtill be made which the formoſt Superficies would have made by it ſelf. And therefore there can be no other Dimenſion in any Body, as it is a Body, then the three which I have now deſcribed; though as it ſhall be ſhewed hereafter, *Velocity*, which is Motion according to *Length*, may by being applyed to all the parts of a *Solid*, make a Magnitude of Motion conſiſting of four Dimenſions; as the goodneſs of Gold computed in all the parts of it makes the Price and Value thereof.

13 *Bodies* (how many ſoever they be) that can fill every one the place of every one, are ſaid to be EQUAL every one to every other. Now one Body may fill the ſame Place which another Body filleth, though it be not of the ſame Figure with that other Body, if ſo be that it may be underſtood to be reducible to the ſame

Figure, either by Flexion or Transpoſition of the parts. And *One Body is* GREATER *then another Body, when a part of that is equal to all this*; *and* LESSE, *when all that is equal to a part of this.* Alſo *Magnitudes* are *Equal*, or *Greater*, or *Leſſer* then one another for the ſame conſideration, namely, when the Bodies of which they are the Magnitudes, are either *Equal* or *Greater* or *Leſſe*, &c.

14 One and the ſame Body, is alwayes of one and the ſame Magnitude. For ſeeing a Body and the Magnitude and Place thereof cannot be comprehended in the Minde, otherwiſe then as they are Coincident, if any Body be underſtood to be at Reſt, that is, to remain in the ſame Place during ſome time, and the Magnitude thereof be in one part of that time Greater, and in another part Leſſe, that Bodies Place, which is one and the ſame, will be coincident ſometimes with Greater, ſometimes with Leſſe Magnitude, that is, the ſame Place will be greater and leſſe then it ſelf, which is impoſſible. But there would be no need at all of Demonſtrating a thing that is in it ſelf ſo maniteſt, if there were not ſome, whoſe opinion concerning Bodies and their Magnitudes is, that a Body may exiſt ſeparated from its Magnitude, and have greater or leſſe Magnitude beſtowed upon it, making uſe of this Principle for the explication of the nature of *Rarum* and *Denſum*.

15 Motion, in as much as a certain Length may in a certain Time be tranſmitted by it, is called VELOCITY or *Swiftneſs*: &c. For though *Swift* be very often underſtood with relation to *Slower* or *leſs Swift*, as Great is in reſpect of Leſs, yet nevertheleſs, as Magnitude is by Philoſophers taken abſolutely for Extenſion, ſo alſo *Velocity* or *Swiftneſs* may be put abſolutely for Motion according to Length.

16 Many Motions are ſaid to be made in Equal Times, when every one of them begins and ends together with ſome other Motion, or if it had begun together, would alſo have ended together with the ſame. For Time which is a Phantaſme of Motion, cannot be reckoned but by ſome expoſed Motion; as in Dials by the Motion of the Sun or of the Hand; and if two or more Motions begin and end with this Motion, they are ſaid to be made in equal times; from whence alſo it is eaſie to underſtand what it is to be moved in Greater or Longer time, & in leſſe time or not ſo long; namely, that that is longer Moved, which beginning with another

other, ends later; or ending together, began ſooner.

17 Motions are ſaid to be Equally Swift, when Equal lengths are tranſmitted in Equal times; and Greater Swiftneſs is that, wherein Greater length is paſſed in Equal time, or Equal length in leſs time. Alſo that Swiftneſs by which Equal lengths are paſſed in Equal parts of time, is called *Uniform* Swiftneſs or Motion; and of Motions *not Uniform*, ſuch as become Swifter or Slower by equal Increaſings or Decreaſings in equal parts of time, are ſaid to be *Accelerated* or *Retarded Uniformly*.

18 But Motion is ſaid to be Greater, Leſſe, and Equal, not onely in regard of the Length which is tranſmitted in a certain time, that is, in regard of Swiftneſs onely, but of Swiftneſs applyed to every ſmalleſt particle of Magnitude; For when any Body is Moved, every part of it is alſo Moved; and ſuppoſing the parts to be halves, the Motions of thoſe halves have their Swiftneſs equal to one another, and ſeverally equal to that of the Whole; but the Motion of the Whole is equal to thoſe two Motions, either of which is of equal Swiftneſs with it; and therefore it is one thing for two Motions to be Equal to one another, & another thing for them to be Equally Swift. And this is manifeſt in two Horſes that draw abreaſt, where the Motion of both the Horſes together is of Equal Swiftneſs with the Motion of either of them ſingly; but the Motion of both is Greater then the Motion of one of them, namely Double. Wherefore *Motions are ſaid to be ſimply Equal to one another when the Swiftneſs of one computed in every part of its Magnitude, is Equal to the Swiftnes of the other cõputed alſo in every part of its Magnitude: & Greater then one another, when the Swiftneſs of one computed as above, is Greater then the Swiftneſs of the other ſo computed; and Leſſe, when Leſſe.* Beſides, the Magnitude of Motion computed in this manner is that which is commonly called FORCE.

19 *Whatſoever is at Reſt, will alwayes be at Reſt, unleſs there be ſome other Body beſides it, which by endeavouring to get into its Place by motion, ſuffers it no longer to remain at Reſt.* For ſuppoſe that ſome Finite Body exiſt, and be at Reſt, and that all Space beſides be Empty; if now this Body begin to be Moved, it will certainly be Moved ſome way; Seeing therefore there was nothing in that Body which did not diſpoſe it to Reſt, the reaſon why it is Moved this way is in ſomething out of it; and in like manner, if it had been Moved any

other way, the reaſon of Motion that way had alſo been in ſomething out of it; but ſeeing it was ſuppoſed that Nothing is out of it, the reaſon of its Motion one way would be the ſame with the reaſon of its Motion every other way; wherefore it would be Moved alike all wayes at once; which is impoſſible.

In like manner, *Whatſoever is Moved, will alwayes be Moved, except there be ſome other Body beſides it, which cauſeth it to Reſt.* For if we ſuppoſe Nothing to be without it, there will be no reaſon why it ſhould Reſt now, rather then at another time; wherefore its Motion would ceaſe in every particle of time alike; which is not intelligible.

When we ſay a Living Creature, a Tree, or any other ſpecified Body is *Generated*, or *Deſtroyed*, it is not to be ſo underſtood as if there were made a Body of that which is not-Body, or not a Body of a Body, but of a Living Creature not a Living Creature, of a Tree not a Tree, &c. that is, that thoſe Accidents for which we call one thing a Living Creature, another thing a Tree, and another by ſome other Name, are Generated and Deſtroyed; and that therefore the ſame Names are not to be given to them now, which were given them before. But that Magnitude for which we give to any thing the Name of Body is neither Generated nor Deſtroyed. For though we may feign in our Mind that a Point may ſwell to a huge bulk, and that this may again contract it ſelfe to a Point; that is, though we may imagine ſomething to ariſe where before was Nothing, and Nothing to be there where before was ſomething, yet we cannot comprehend in our Minde how this may poſſibly be done in Nature. And therefore Philoſophers, who tye themſelves to Naturall Reaſon, Suppoſe that a Body can neither be Generated nor Deſtroyed, but onely that it may appear otherwiſe then it did to Us, that is under different *Species*, and conſequently be called by other and other Names; ſo that that which is now called Man, may at another time have the Name of Not-Man; but that which is once called Body, can never be called Not-Body. But it is manifeſt, that all other Accidents beſides Magnitude or Extenſion may be Generated and Deſtroyed; as when a White thing is made Black, the Whiteneſs that was in it Periſheth, and the Blackneſs that was not in it is now Generated; and therefore Bodies, and the Accidents under which they appear diverſly, have

this

this difference, that Bodies are Things, and not Generated; Accidents are Generated, and not Things.

21 And therefore when any thing appears otherwise then it did, by reason of other and other Accidents, it is not to be thought that an Accident goes out of one Subject into another (for they are not, as I said above, in their Subjects as a Part in the Whole, or as a Conteined thing in that which Contains it, or as a Master of a Family in his House,) but that one Accident Perisheth, and another is Generated. For example, when the Hand being Moved, Moves the Pen, Motion does not go out of the Hand into the Pen, for so the Writing might be continued though the Hand stood still, but a new Motion is Generated in the Pen, and is the Pens Motion.

22 And therefore also it is improper to say an Accident is Moved; as when in stead of saying, *Figure is an Accident of a Body carried away*, we say, *A Body carries away its Figure*.

23 Now that Accident for which we give a certain Name to any Body, or the Accident which denominates its Subject, is commonly called the ESSENCE thereof; as Rationality is the Essence of a Man, Whiteness; of any White Thing and Extension the Essence of a Body. And the same Essence in as much as it is Generated, is called the FORM. Again, a Body, in respect of any accident is called the SUBJECT, and in respect of the Form it is called the MATTER.

Also, the Production or Perishing of any Accident, makes its Subject be said *to be Changed*; onely the Production or Perishing of Form, make it be said it is *Generated* or *Destroyed*; but in all Generation and Mutation, the name of *Matter* still remains. For a Table made of Wood, is not onely Wooden, but Wood; and a Statue of Brass is Brass as well as Brazen; though *Aristotle* in his *Metaphysiques* say, that whatsoever is made of any thing ought not to be called ἐκεινὸ, but ἐκείνινον, as that which is made of Wood not ξύλον, but ξύλινον, that is, not Wood, but Wooden.

24 And as for that Matter which is common to all things, and which Philosophers following *Aristotle*, usually call *Materia Prima*, that is, *First Matter*, it is not any Body distinct from all other Bodies, nor is it one of them. What then is it? A mere Name; yet a Name which is not of vain Use; for it signifies a conception of

Body

Body without the conſideration of any Form or other Accident except onely Magnitude or Extenſion, & aptneſs to receive Form & other Accidents;ſo that whenſoever we have uſe of the Name *Body in general*, if we uſe that of *Materia Prima*,we do well.For as, when a Man not knowing which was firſt, Water or Ice,would finde out which of the two were the Matter of both,he would be fain to ſuppoſe ſome third Matter which were neither of theſe two; ſo he that would finde out what is the Matter of all things,ought to ſuppoſe ſuch as is not the Matter of any thing that exiſts. Wherefore *Materia Prima* is no Thing; and therefore they do not attribute to it either Form or any other Accident beſides Quantity; whereas all ſingular things have their Forms and Accidents certain.

Materia Prima therefore is Body in general, that is Body conſidered Univerſally, not as having neither Form nor any Accident, but in which no Form nor any other Accident but Quantity are at all conſidered, that is, they are not drawn into Argumentation.

25 From what has been ſaid, thoſe Axiomes may be demonſtrated which are aſſumed by *Euclide* in the beginning of his firſt Element about the Equality and Inequality of Magnitudes; of (which omitting the reſt) I will here demonſtrate onely this one, *The Whole is greater then any Part thereof*; to the end that the Reader may know that thoſe Axioms are not indemonſtrable,& therefore not Principles of Demonſtration; and from hence learn to be wary how he admits any thing for a Principle, which is not at leaſt as evident as theſe are. *Greater* is defined to be that, whoſe Part is Equal to the Whole of another. Now if we ſuppoſe any Whole to be A, and a Part of it to be B; ſeeing the Whole B is Equal to it ſelf, and the ſame B is a Part of A; therefore a Part of A will be Equal to the Whole B. Wherefore by the Definition above, A is Greater then B, which was to be proved.

CHAP.

CHAP. IX.
Of Cauſe *and* Effect.

1 *Action and Paſſion what they are.* 2 *Action and Paſſion Mediate and Immediate.* 3 *Cauſe ſimply taken. Cauſe without which no Effect follows, or Cauſe Neceſſary by Supposition.* 4 *Cauſe Efficient and Material.* 5 *An Entire Cauſe is alwayes ſufficient to produce its Effect. At the ſame inſtant that the Cauſe is Entire, the Effect is produced. Every Effect has a Neceſſary Cauſe.* 6 *The Generation of Effects is Continual. What is the Beginning in Cauſation.* 7 *No Cauſe of Motion but in a Body Contiguous and Moved.* 8 *The ſame Agents and Patients if alike diſpoſed, produce like Effects, though at different times.* 9 *All Mutation is Motion.* 10 *Contingent Accidents what they are.*

1 A Body is ſaid to Work upon or *Act*, that is to ſay, *Do* ſome thing to another Body, when it either generates or deſtroys ſome Accident in it; and the Body in which an Accident is generated or deſtroyed is ſaid to *Suffer*, that is, to have ſomething *Done* to it by another Body, As when one Body by putting forwards another Body generates Motion in it, it is called the AGENT; and the Body in which Motion is ſo generated, is called the PATIENT; ſo Fire that warms the Hand is the *Agent*, and the Hand which is warmed is the *Patient*. That Accident which is generated in the Patient is called the EFFECT.

2 When an Agent and Patient are Contiguous to one another, their Action and Reaſon are then ſaid to be *Immediate*, otherwiſe *Mediate*; and when another Body lying betwixt the Agent and Patient is Contiguous to them both, it is then it ſelf both an Agent and a Patient, an Agent in reſpect of the Body next after it, upon which it Works, and a Patient in reſpect of the Body next before it, from which it ſuffers. Alſo if many Bodies be ſo ordered that every two which are next to one another be contiguous, then all thoſe that are betwixt the firſt and the laſt are both Agents and Patients, and the firſt is an Agent onely, and the laſt a Patient onely.

3 An

3 An Agent is understood to *produce* its determined or certain Effect in the Patient, according to some certain Accident, or Accidents, with which both it and the Patient are affected; that is to say, the Agent hath its Effect precisely such, not because it is a Body, but because such a Body, or so Moved; For otherwise all Agents, seeing they are all Bodies alike, would produce like Effects in all Patients; and therefore the Fire (for example) does not warm, because it is a Body, but because it is Hot; nor does one Body put forward another Body because it is a Body, but because it is moved into the place of that other Body. The Cause therefore of all Effects consists in certain Accidents both in the Agents and in the Patient; which when they are all present, the Effect is produced; but if any one of them be wanting it is not produced; and that Accident either of the Agent or Patient, without which the Effect cannot be produced, is called *Causa sine qua non*, or *Cause Necessary by Supposition*, as also the *Cause Requisite for the Production of the Effect*. But a CAUSE simply, or *An Entire Cause, is the Aggregate of all the Accidents both of the Agents how many soever they be, and of the Patient, put together; which when they are all supposed to be present, it cannot be understood but that the Effect is produced at the same instant; and if any one of them be wanting, it cannot be understood but that the Effect is not produced.*

4 The Aggregate of Accidents in the Agent or Agents, requisite for the production of the Effect, the Effect being produced, is called the *Efficient Cause* thereof; and the Aggregate of Accidents in the Patient, the Effect being produced, is usually called the *Material Cause*; I say the Effect being produced; for where there is no Effect, there can be no Cause; for nothing can be called a Cause where there is nothing that can be called an Effect. But the Efficient and Material Causes, are both but Partial Causes, or Parts of that Cause which in the next precedent article I called an Entire Cause. And from hence it is manifest, that the Effect we expect, though the Agents be not defective on their part, may nevertheless be frustrated by a defect in the Patient; and when the Patient is sufficient, by a defect in the Agents.

5 An Entire Cause is alwayes sufficient for the production of its Effect, if the Effect be at all possible. For let any Effect whatsoever be propounded to be produced; if the same

ſame be produced, it is manifeſt that the Cauſe which produced it was a ſufficient Cauſe; but if it be not produced, and yet be poſſible, it is evident that ſomething was wanting either in ſome Agent, or in the Patient, without which it could not be produced; that is, that ſome Accident was wanting which was requiſite for its Production; and therefore that Cauſe was not *Entire*, which is contrary to what was ſuppoſed.

It follows alſo from hence, that in whatſoever inſtant the Cauſe is Entire, in the ſame inſtant the Effect is produced. For if it be not produced, ſomething is ſtill wanting, which is requiſite for the production of it; and therefore the Cauſe was not Entire, as was ſuppoſed.

And ſeeing a Neceſſary Cauſe is defined to be that, which being ſuppoſed, the Effect cannot but follow; this alſo may be collected, that whatſoever Effect is produced at any time, the ſame is produced by a Neceſſary Cauſe. For whatſoever is produced, in as much as it is produced, had an Entire Cauſe, that is, had all thoſe things, which being ſuppoſed, it cannot be underſtood but that the Effect follows; that is, it had a Neceſſary Cauſe. And in the ſame manner it may be ſhewn, that whatſoever Effects are hereafter to be produced, ſhall have a Neceſſary Cauſe; ſo that all the Effects, that have been or ſhall be produced, have their Neceſſity in things antecedent.

6 And from this, that whenſoever the Cauſe is Entire, the Effect is produced in the ſame inſtant, it is manifeſt, that Cauſation and the Production of Effects conſiſt in a certain continual Progreſs; ſo that as there is a continual Mutation in the Agent or Agents by the working of other Agents upon them, ſo alſo the Patient upon which they work is continually altered and changed. For example, as the Heat of the Fire encreaſes more and more, ſo alſo the Effects thereof, namely the Heat of ſuch Bodies as are next to it, & again of ſuch other Bodies as are next to them, encreaſes more & more accordingly; which is already no litle argument that all Mutation conſiſts in Motion onely; the truth whereof ſhall be further demonſtrated in the ninth Article. But in this Progreſs of Cauſation, that is, of Action and Paſſion, if any man comprehend in his imagination a part thereof, and divide the ſame into parts, the firſt part or Beginning of it cannot be conſidered otherwiſe then

as Action or Cauſe ; for if it ſhould be conſidered as Effect or Paſſion , then it would be neceſſary to conſider ſomething before it for its Cauſe or Action ; which cannot be ; for nothing can be before the Beginning. And in like manner ; the laſt part is conſidered onely as Effect ; for it cannot be called Cauſe if nothing follow it ; but after the laſt nothing follows. And from hence it is , that in all Action the Beginning and Cauſe are taken for the ſame thing. But every one of the intermediate parts are both Action and Paſſion, and Cauſe and Effect, according as they are compared with the antecedent or ſubſequent part.

7 There can be no Cauſe of Motion, except in a Body Contiguous, and Moved. For let there be any two Bodies which are not contiguous, and betwixt which the intermediate Space is empty, or if filled, filled with another Body which is at Reſt; and let one of the propounded Bodies be ſuppoſed to be at Reſt , I ſay it ſhall always be at Reſt. For if it ſhall be Moved, the Cauſe of that Motion (by the 8th. Chapter 19th. Article) will be in ſome external Body ; and therefore if between it and that external Body there be nothing but empty Space , then whatſoever the diſpoſition be of that external Body, or of the Patient it ſelf, yet if it be ſuppoſed to be now at Reſt, we may conceive it wil continue ſo til it be touched by ſome other Body; but ſeeing Cauſe (by the Definition) is the Aggregate of all ſuch Accidents, which being ſuppoſed to be preſent it cannot be conceived but that the Effect will follow , thoſe Accidents which are either in external Bodies, or in the Patient it ſelf, cannot be the Cauſe of future Motion; and in like manner, ſeeing we may conceive , that whatſoever is at Reſt , will ſtill be at Reſt , though it be touched by ſome other Body , except that other Body be moved , therefore in a contiguous Body which is at Reſt , there can be no Cauſe of Motion. Wherefore there is no Cauſe of Motion in any Body, except it be Contiguous and Moved.

The ſame reaſon may ſerve to prove, that whatſoever is Moved, will alwayes be Moved on in the ſame way and with the ſame Velocity , except it be hindered by ſome other Contiguous and Moved Body ; and conſequently that no Bodies either when they are at Reſt, or when there is an interpoſition of *Vacuum*, can generate or extinguiſh or leſſen Motion in other Bodies. There is one that has writ-

written, that things Moved are more reſiſted by things at Reſt, then by things contrarily Moved, for this reaſon, that he conceived Motion not to be ſo contrary to Motion as Reſt. That which deceived him was, that the words *Reſt* and *Motion* are but contradictory Names; whereas Motion indeed is not reſiſted by Reſt, but by contrary Motion.

8 But if a Body work upon another Body at one time, and afterwards the ſame Body work upon the ſame Body at another time ſo, that both the Agent and Patient, and all their parts, be in all things as they were; and there be no difference except onely in time, that is, that one Action be former the other later in time; it is manifeſt of it ſelf, that the Effects will be Equal and Like; as not differing in any thing beſides time. And as Effects themſelves proceed from their Cauſes; ſo the diverſity of them depends upon the diverſity of their Cauſes alſo.

9 This being true, it is neceſſary that *Mutation* can be nothing elſe, but Motion of the Parts of that Body which is Changed. For Firſt, we do not ſay any thing is Changed, but that which appears to our Senſes otherwiſe then it appeared formerly. Secondly, both thoſe Appearances are Effects produced in the Sentient; & therefore if they be differẽt, it is neceſſary (by the preceding article) that either ſome part of the Agent which was formerly at Reſt, is now Moved, and ſo the Mutation conſiſts in this Motion; or ſome part which was formerly Moved, is now otherwiſe Moved, and ſo alſo the Mutation conſiſts in this new Motion; or which being formerly Moved, is now at Reſt, which (as I have ſhewn above) cannot come to paſs without Motion, and ſo again Mutation is Motion; or, laſtly, it happens in ſome of theſe manners to the Patient or ſome of its parts; ſo that Mutation, howſoever it be made, will conſiſt in the Motion of the parts either of the Body which is perceived, or of the Sentient Body, or of both. Mutation therefore is Motion, (namely of the parts either of the Agent or of the Patient;) which was to be demonſtrated. And to this it is conſequent, that Reſt cannot be the Cauſe of any thing; nor can any Action proceed from it, ſeeing neither Motion nor Mutation can be cauſed by it.

10 Accidents, in respect of other Accidents which precede them, or are before them in time, & upon which they do not depend as upon their Causes, are called *Contingent* Accidents; I say in respect of those Accidents by which they are not generated; for in respect of their Causes all things come to pass with equal necessity, for otherwise, they would have no Causes at all; which of things generated is not intelligible.

CHAP.

CHAP. X.
Of Power *and* Act.

1 *Power and Cauſe are the ſame thing.* 2 *An Act is produced at the ſame inſtant in which the Power is Plenary.* 3 *Active and Paſſive Power are parts onely of Plenary Power.* 4 *An Act when ſaid to be Poſſible.* 5 *An Act Neceſſary and Contingent, what.* 6 *Active Power conſiſts in Motion.* 7 *Cauſe Formal and Final, what they are.*

1 COrreſpondent to *Cauſe* and *Effect* are POWER and ACT; Nay, thoſe and theſe are the ſame things, though for divers conſiderations they have divers names. For whenſoever any Agent has all thoſe Accidents which are neceſſarily requiſite for the production of ſome Effect in the Patient, then we ſay that Agent has *Power* to produce that Effect, if it be applyed to a Patient. But (as I have ſhewn in the precedent Chapter,) thoſe Accidents conſtitute the Efficient Cauſe; and therefore the ſame Accidents which conſtitute the Efficient Cauſe, conſtitute alſo the *Power* of the Agent. Wherefore the *Power of the Agent*, and the *Efficient Cauſe* are the ſame thing. But they are conſidered with this difference, that *Cauſe* is ſo called in reſpect of the Effect already produced, and *Power* in reſpect of the ſame Effect to be produced hereafter; ſo that *Cauſe* reſpects the Paſt, *Power* the Future time. Alſo the *Power of the Agent*, is that which is commonly called *Active Power*.

In like manner, whenſoever any Patient has all thoſe Accidents which it is requiſite it ſhould have for the production of ſome Effect in it, we ſay it is in the *Power* of that Patient to produce that Effect, if it be applyed to a fitting Agent. But thoſe Accidents (as is defined in the precedent Chapter) conſtitute the Material

Cauſes

Cauſe; and therefore the *Power of the Patient*, (commonly called *Paſſive Power*) and *Material Cauſe* are the ſame thing; but with this different conſideration, that in Cauſe the Paſt time, and in Power the Future is reſpected. Wherefore the Power of the Agent and Patient together, which may be called Entire or *Plenary Power*, is the ſame thing with *Entire Cauſe*; for they both conſiſt in the Sum or Aggregate of all the Accidents as well in the Agent as in the Patient, which are requiſite for the production of the Effect. Laſtly, as the Accident produced is in reſpect of the Cauſe called an *Effect*; ſo in reſpect of the Power it is called an *Act*.

2 As therefore the Effect is produced in the ſame inſtant in which the Cauſe is Entire; ſo alſo every Act that may be produced, is produced in the ſame inſtant, in which the Power is Plenary. And as there can be no Effect, but from a Sufficient and Neceſſary Cauſe; ſo alſo no Act can be produced, but by Sufficient Power, or that Power by which it could not but be produced.

3 And as it is manifeſt, (as I have ſhewn) that the Efficient and Material Cauſes are ſeverally and by themſelves parts onely of an Entire Cauſe, and cannot produce any Effect but by being joyned together; ſo alſo Power Active and Paſſive, are parts onely of Plenary and Entire Power; nor, except they be joyned, can any Act proceed from them; and therefore theſe Powers (as I ſaid in the firſt Article) are but conditionall, namely, *the Agent has Power, if it be applyed to a Patient*; *and the Patient has Power, if it be applyed to an Agent*; otherwiſe neither of them have Power, nor can the Accidents which are in them ſeverally be properly called Powers; nor any Action be ſaid to be Poſſible, for the Power of the Agent alone, or of the Patient alone.

4 For that is an Impoſſible Act, for the production of which there is no Power Plenary. For ſeeing Plenary Power is that in which all things concurre which are requiſite for the production of an Act, if the Power ſhall never be Plenary, there will always be wanting ſome of thoſe things, without which the Act cannot be produced; wherefore that Act ſhall never be produced, that is, that Act is IMPOSSIBLE: And every Act which is not Impoſſible, is POSSIBLE. Every Act therefore which is Poſſible ſhall at ſome time be produced; for if it ſhall never be produced, then thoſe things ſhall never concurre which are requiſite for the production of

of it; wherefore that Act is *Impossible* by the Definition; which is contrary to what was supposed.

5 A *Necessary Act* is that, the production whereof it is Impossible to hinder; and therefore every Act that shall be produced, shall necessarily be produced; for that it shall not be produced is Impossible, because (as is already demonstrated) every Possible Act shall at some time be produced; Nay, this Proposition, *What shall be, shall be*, is as necessary a Proposition, as this, *A Man is a Man*.

But here perhaps some man may ask, whether those Future things, which are commonly called *Contingents*, are Necessary. I say therefore that generally all Contingents, have their Necessary Causes, (as is shewn in the preceding Chapter,) but are called Contingents in respect of other Events upon which they do not depend; as the Rain which shall be to morrow, shall be Necessarily, (that is from necessary Causes;) but we think and say it happens by chance, because we doe not yet perceive the Causes thereof, though they exist now; for men commonly call that *Casuall* or *Contingent*, whereof they do not perceive the necessary Cause; and in the same manner they use to speake of things past, when not knowing whether a thing be done or no, they say it is possible it never was done.

Wherefore all Propositions concerning Future things contingent or not contingent, as this, *It will rayne to morrow*, or this, *To morrow the Sun will rise*, are either necessarily true, or necessarily false; but we call them Contingent because we doe not yet know whether they be true or false; whereas their Verity depends not upon our Knowledge, but upon the foregoing of their Causes. But there are some who though they confess this whole Proposition, *To morrow it will either rain or not rain*, to be true, yet they will not acknowledge the parts of it, as *To morrow it will rain*, or *To morrow it will not rain*, to be either of them true by it self, because they say neither this nor that is true *determinately*. But what is this *determinately true*, but true upon our knowledge, or evidently true? and therefore they say no more but that it is not yet known whether it be true or no; but they say it more obscurely, and darken the Evidence of the truth with the same words with which they endevour to hide their own ignorance.

6 In the 9th. Article of the precedent Chapter I have shewn, that

that the Efficient Cauſe of all Motion and Mutation conſiſts in the Motion of the Agent or Agents; And in the firſt Article of this Chapter, that the Power of the Agent is the ſame thing with the Efficient Cauſe. From whence it may be underſtood, that all Active Power conſiſts in Motion alſo; and that Power is not a certain Accident which differs from all Acts, but is indeed an Act, namely Motion, which is therefore called Power, becauſe another Act ſhall be produced by it afterwards. For example, if of three Bodies the firſt put forwards the ſecond, and this the third, the Motion of the ſecond in reſpect of the firſt which produceth it, is the Act of the ſecond Body, but in reſpect of the third it is the Active Power of the ſame ſecond Body.

7 The Writers of *Metaphyſiques* reckon up two other Cauſes beſides the *Efficient* and *Material*, namely the ESSENCE, which ſome call the *Formal Cauſe*; and the *End*, or *Final Cauſe*; both which are nevertheleſs Efficient Cauſes. For when it is ſaid, the Eſſence of a thing is the Cauſe thereof, *as to be Rational, is the Cauſe of Man*, it is not intelligible; for it is all one as if it were ſaid, *To be a Man is the Cauſe of Man*, which is not well ſaid. And yet the knowledge of the *Eſſence* of any thing, is the Cauſe of the knowledge of the thing it ſelfe; for if I firſt know that a thing is *Rational*, I know from thence that the ſame is *Man*; but this is no other then an Efficient Cauſe. A *Final Cauſe* has no place but in ſuch things as have Senſe and Will; and this alſo I ſhall prove hereafter to be an Efficient Cauſe.

CHAP.

CHAP. XI.

Of Identity *and* Difference.

1 *What it is for one thing to Differ from another.* 2 *To Differ in Number, Magnitude, Species and Genus, what.* 3 *What is Relation, Proportion, and Relatives.* 4 *Proportionals what.* 5 *The Proportion of Magnitudes to one another, wherein it consists.* 6 *Relation is no new Accident, but one of those that were in the Relative before the Relation or Comparison was made. Also the Causes of Accidents in the Correlatives are the Cause of Relation.* 7 *Of the Beginning of Individuation.*

1 

Hitherto I have spoken of Body simply, and of Accidents common to all Bodies as *Magnitude, Motion, Rest, Action, Passion, Power, Possible, &c.* And I should now descend to those Accidents by which one Body is distinguished from another, but that it is first to be declared what it is to be *Distinct*, and *not Distinct*, namely what are the SAME and DIFFERENT; for this also is common to all Bodies, that they may be distinguished and differenced from one another. Now two Bodies are said to *Differ* from one another, when something may be said of one of them, which cannot be said of the other at the same time.

2 And first of all, it is manifest that no Two Bodies are the *Same*; for seeing they are Two, they are in two places at the same time; as that which is the *Same*, is at the same time in one and the same place. All Bodies therefore differ from one another in *Number*, namely, as One and Another; so that the *Same* and *different in Number* are Names opposed to one another by Contradiction.

In *Magnitude* Bodies differ when One is greater then Another, as *a Cubit long*, and *two Cubits long*, *of two pound weight*, and of *three pound weight*. And to these, *Equals* are opposed.

Bodies which differ more then in Magnitude are called *Unlike*, and thoſe which differ onely in Magnitude, *Like*. Alſo of Unlike Bodies ſome are ſaid to differ in the *Species*, other in the *Genus*; in the *Species* when their difference is perceived by one and the ſame Senſe, as *White* and *Black*; and in the *Genus*, when their differenceis not perceived but by divers Senſes, as *White* and *Hot*.

3 And the *Likeneſs*, or *Unlikeneſs*, *Equality* or *Inequality* of one Body to another, is called their RELATION; and the Bodies themſelves *Relatives* or *Correlatives*; *Ariſtotle* calls them τὰ πρός τι; the firſt whereof is uſually named the *Antecedent*, and the ſecond the *Conſequent*; and the *Relation* of the Antecedent to the Conſequent according to Magnitude, namely, the Equality, the Exceſs or Defect thereof, is called the PROPORTION of the Antecedent to the Conſequent, ſo that *Proportion* is nothing but the Equality or Inequality of the Magnitude of the Antecedent compared to the Magnitude of the Conſequent by their difference only, or compared alſo with their difference. For Example, the *Proportion* of Three to Two conſiſts only in this, that Three *exceeds* Two by Unity; and the Proportion of Two to Five in this, that Two compared with Five is *deficient* of it by Three, either ſimply, or compared with the numbers different; and therefore in the Proportion of Unequals, the Proportion of the Leſſe to the Greater is called DEFECT, and that of the Greater to the Leſſe EXCESS.

4 Beſides, of Unequals, ſome are more, ſome leſſe, and ſome equally unequall; ſo that there is *Proportion of Proportions*, as well as of *Magnitudes*, namely, where two Unequals have relation to two other Unequals; as when the Inequality which is between 2 and 3 is compared with the Inequality which is between 4 and 5. In which Compariſon there are alwayes four Magnitudes, or (which is all one) if there be but three, the midlemoſt is twice numbred; and if the Proportion of the firſt to the ſecond be equal to the Proportion of the third to the fourth, then the four are ſaid to be *Proportionals*; otherwiſe they are not Proportionals.

5 The Proportion of the Antecedent to the Conſequent, conſiſts in their Difference, not onely ſimply taken, but alſo as compared with one of the Relatives; that is, either in that part of the greater by which it exceeds the leſſe, or in the Remainder after the

the leſſe is taken out of the greater; as the Proportion of Two to Five, conſiſts in the Three by which Five exceeds Two, not in Three, ſimply onely, but alſo as compared with Five or Two. For though there be the ſame difference between Two & Five, which is between Nine and Twelve, namely Three, yet there is not the ſame Inequality; and therefore the Proportion of Two to Five, is not in all Relation the ſame with that of Nine to Twelve, but onely in that which is called Arithmetical.

6 But we muſt not ſo think of Relation, as if it were an Accident differing from all the other Accidents of the Relative; but one of them; namely that, by which the Compariſon is made. For example, the likeneſs of one *White* to another *White*, or its Unlikeneſs to *Black*, is the ſame Accident with its *Whiteneſs*; and *Equality* and *Inequality*, the ſame Accident with the *Magnitude* of the thing compared, though under another Name; for that which is called *White* or *Great*, when it is not compared with ſomething elſe, the ſame when it is compared is called *Like* or *Unlike*, *Equal* or *Unequal*. And from this it follows, that the Cauſes of the Accidents which are in Relatives, are the Cauſes alſo of *Likeneſs*, *Unlikeneſs*, *Equality* and *Inequality*, namely, that he that makes two Unequal Bodies, makes alſo their Inequality; and he that makes a Rule and an Action, makes alſo, if the Action be congruous to the Rule, their Congruity; if Incongruous, their Incongruity. And thus much concerning *Compariſon* of one Body with another.

7 But the ſame Body may at different times be Compared with it ſelf. And from hence ſprings a great controverſie among Philoſophers about the *Beginning of Individuation*; namely, in what ſenſe it may be conceived that a Body is at one time the ſame, at another time not the ſame it was formerly. For example, whether a Man grown old be the ſame Man he was whileſt he was young, or another Man; or whether a City be in different Ages the ſame, or another City. Some place *Individuity* in the Unity of *Matter*; others in the Unity of *Form*; and one ſayes it conſiſts in the Unity of the *Aggregate of all the Accidents together*. For *Matter*, it is pleaded, that a lump of Wax, whether it be Spherical or Cubical, is the ſame Wax, becauſe the ſame Matter. For *Form*, that when a Man is grown from an Infant to be an Old Man, though his Matter be changed, yet he is ſtill the ſame Numerical Man; for that

Identity which cannot be attributed to the Matter, ought probably to be aſcribed to the Form. For the *Aggregate of Accidents* no Inſtance can be made; but becauſe when any new Accident is generated, a new Name is commonly impoſed on the Thing, therefore he that aſſigned this cauſe of *Individuity*, thought the thing it ſelf alſo was become another thing. According to the firſt Opinion, He that ſins, and he that is puniſhed ſhould not be the ſame Man, by reaſon of the perpetual flux and change of Mans Body; nor ſhould the City which makes Lawes in one Age, and abrogates them in another, be the ſame City; which were to confound all Civil Rights. According to the ſecond Opinion, two Bodies exiſting both at once, would be one and the ſame Numerical Body; for if (for example) that Ship of *Theſeus* (concerning the Difference whereof, made by continual reparation, in taking out the old Planks, and putting in new, the Sophiſters of *Athens* were wont to diſpute) were, after all the Planks were changed, the ſame Numerical Ship it was at the beginning; and if ſome Man had kept the Old Planks as they were taken out, and by putting them afterwards together in the ſame order, had again made a Ship of them, this without doubt had alſo been the ſame Numerical Ship with that which was at the beginning; and ſo there would have been two Ships Numerically the ſame, which is abſurd. But according to the third Opinion, Nothing would be the ſame it was; ſo that a Man ſtanding, would not be the ſame he was ſitting; nor the Water which is in the Veſſel, the ſame with that which is poured out of it. Wherefore the beginning of *Individuation* is not alwayes to be taken either from Matter alone, or from Form alone.

But we muſt conſider by what name any thing is called, when we enquire concerning the *Identity* of it; for it is one thing to ask concerning *Socrates* whether he be the ſame Man, and another to ask whether he be the ſame Body; for his Body when he is Old, cannot be the ſame it was when he was an Infant, by reaſon of the difference of Magnitude; for One Body has alwayes One and the ſame Magnitude; yet nevertheleſs he may be the ſame Man. And therefore whenſoever the Name by which it is asked whether a thing be the ſame it was, is given it for the Matter onely, then if the Matter be the ſame, the thing alſo is *Individually* the ſame; as the Water which was in the Sea, is the ſame which is afterwards in

in the Cloud; and any Body is the ſame, whether the parts of it be put together, or diſperſed, or whether it be congealed or diſſolved. Alſo if the Name be given for ſuch Form as is the beginning of Motion, then as long as that Motion remains it will be the ſame *Individual* thing; as that Man will be alwayes the ſame, whoſe Actions and Thoughts proceed all from the ſame beginning of Motion, namely, that which was in his generation; and that will be the ſame River, which flows from one and the ſame Fountain, whether the ſame Water, or other Water, or ſomething elſe then Water flow from thence; and that the ſame City, whoſe Acts proceed continually from the ſame Inſtitution, whether the Men be the ſame or no. Laſtly, if the Name be given for ſome Accident, then the *Identity* of the thing will depend upon the Matter; for by the taking away and ſupplying of Matter, the Accidents that were are deſtroyed, and other new ones are generated, which cannot be the ſame Numerically; ſo that a Ship, which ſignifies Matter ſo figured, will be the ſame, as long as the Matter remains the ſame; but if no part of the Matter be the ſame, then it is Numerically another Ship; and if part of the Matter remain, and part be changed, then the Ship will be partly the ſame, and partly not the ſame.

CHAP.

CHAP. XII.

Of Quantity.

1 *The Definition of Quantity.* 2 *The Expoſition of Quantity what it is.* 3 *How Line, Superficies and Solid are expoſed.* 4 *How Time is expoſed.* 5 *How Number is expoſed.* 6 *How Velocity is expoſed.* 7 *How Weight is expoſed* 8 *How the Proportion of Magnitudes is expoſed.* 9 *How the Proportion of Times and Velocities is expoſed.*

1 WHat, and how manifold *Dimenſion* is, has been ſaid in the 8th. Chapter, namely, that there are three Dimenſions, Line (or Length) Superficies and Solid; Every one of which, if it be determined, that is, if the limits of it be made known is commonly called *Quantity*; For by *Quantity* all men underſtand that which is ſignified by that word, by which anſwer is made to the queſtion *How much is it.* Whenſoever therefore it is asked (for example) *How long is the Journey*, it is not anſwered indefinitely *Length*; nor when it is asked, *How big is the Field* is it anſwered indefinitely *Superficies*; nor if a man ask *How great is the bulk*, indefinitely *Solid*; but it is anſwered determinately, The Journey is a hundred Miles; the Field is a hundred Acres; the Bulk is a hundred Cubical Feet; or at leaſt in ſome ſuch manner, that the Magnitude of the thing enquired after may by certain Limits be comprehended in the Mind. QUANTITY therefore cannot otherwiſe be defined, then to be *a Dimenſion determined* or *a Dimenſion, whoſe Limits are ſet out, either by their Place, or by ſome Compariſon.*

2 And *Quantity* is determined two wayes; One, by the Senſe, when ſome ſenſible Object is ſet before it; as when a Line, a Superficies or Solid, of a Foot or Cubit, marked out in ſome Matter, is objected to the Eyes; which way of Determining is called *Expoſition*, and the Quantity ſo known is called *Expoſed Quantity*; The Other, by Memory, that is, by Compariſon with ſome Expoſed Quantity. In the firſt manner, when it is asked of what Quantity a thing is, it is anſwered, of ſuch Quantity as you ſee

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Expoſed. In the ſecond manner, anſwer cannot be made but by Compariſon with ſome Expoſed Quantity; for if it be asked, How long is the Way; the anſwer is, ſo many thouſand Paces; that is, by Comparing the Way with a Pace, or ſome other Meaſure determined and known by Expoſition; or the Quantity of it is to ſome other Quantity known by Expoſition, as the Diameter of a Square is to the Side of the ſame, or by ſome other the like means. But it is to be underſtood that the Quantity Expoſed muſt be ſome ſtanding or permanent thing, ſuch as is marked out in conſiſtent or durable matter; or at leaſt ſomething which is revocable to ſenſe; for otherwiſe no Compariſon can be made by it. Seeing therefore (by what has been ſaid in the next preceding Chapter) Compariſon of one Magnitude with another, is the ſame thing with Proportion; it is manifeſt, that Quantity determined in the ſecond manner, is nothing elſe but the Proportion of a Dimenſion not Expoſed to another which is Expoſed; that is, the Compariſon of the Equality or Inequality thereof with an Expoſed Quantity.

3 *Lines*, *Superficies* and *Solids* are Expoſed, Firſt, by *Motion*, in ſuch manner, as (in the 8th Chapter) I have ſaid they are generated; but ſo, as that the Marks of ſuch Motion be permanent; as when they are deſigned upon ſome Matter, as a Line upon Paper; or graven in ſome durable Matter. Secondly, by *Appoſition*; as when one Line or Length is applyed to another Line or Length, one Breadth to another Breadth, and one Thickneſs to another Thickneſs; which is as much as to deſcribe a Line by Points, a Superficies by Lines, and a Solid by Superficies, ſaving that by Points in this place are to be underſtood very ſhort Lines, and by Superficies very Thin Solids. Thirdly, Lines and Superficies may be Expoſed by *Section*; namely a Line may be made by Cutting an Expoſed Superficies, and a Superficies by the Cutting of an Expoſed Solid.

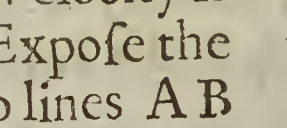
4 *Time* is Expoſed not onely by the Expoſition of a Line, but alſo of ſome Moveable thing, which is moved Uniformly upon that Line, or at leaſt is ſuppoſed ſo to be Moved. For ſeeing Time is an Idea of Motion in which we conſider Former and Later, that is, Succeſſion, it is not ſufficient for the Expoſition of Time that a Line be deſcribed, but we muſt alſo have in our Minde an Imagi-

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nation of ſome Moveable thing paſſing over that Line, and the Motion of it muſt be Uniform, that Time may be divided and compounded as often as there ſhall be need. And therefore when Philoſophers in their Demonſtrations draw a Line, and ſay, *Let that Line be Time*, it is to be underſtood as if they ſaid, *Let the Conception of Uniform Motion upon that Line, be Time.* For though the Circles in Dials be Lines, yet they are not of themſelves ſufficient to note Time by, except alſo there be, or be ſuppoſed to be a Motion of the Shadow or the Hand.

5 *Number* is Expoſed either by the Expoſition of Points, or of the Names of Number *One, Two, Three, &c.* and thoſe Points muſt not be contiguous, ſo as that they cannot be diſtinguiſhed by Notes, but they muſt be ſo placed that they may be *diſcerned* one from another; for from this it is that Number is called *Diſcrete Quantity*, whereas all Quantity which is deſigned by Motion, is called *Continual Quantity*. But that Number may be Expoſed by the Names of Number, it is neceſſary that they be recited by heart and in order, as One, Two, Three, &c. for by ſaying One, One, One and ſo forward, we know not what Number we are at beyond Two or Three, which alſo appear to Us in this manner not as Number, but as Figure.

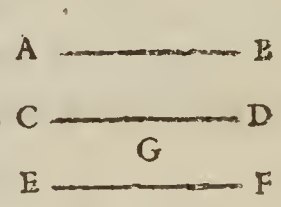
6 For the Expoſition of *Velocity* (which by the Definition therof is a Motion which in a certain Time paſſeth over a certain Space) it is requiſite not onely that Time be Expoſed, but that there be alſo Expoſed that Space which is tranſmitted by the Body whoſe Velocity we would determine, and that a Body be underſtood to be Moved in that Space alſo; ſo that there muſt be Expoſed two Lines, upon One of which Uniform Motion muſt be underſtood to be made, that the Time may be determined, and upon the other the Velocity is to be computed. As if we would Expoſe the Velocity of the Body A, we draw two lines A B and C D, and place a Body in C alſo, which done, we ſay, the Velocity of the Body A is ſo great, that it paſſeth over the Line A B in the ſame Time, in which the Body C paſſeth over the Line C D with Uniform Motion.



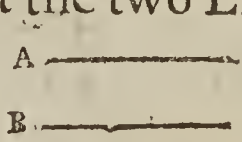
7 *Weight* is Expoſed by any Heavy Body, of what Matter ſoever, ſo it be alwayes alike Heavy.

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8 The *Proportion* of two Magnitudes is then Expoſed when the Magnitudes themſelves are Expoſed; namely, the Proportion of Equality, when the Magnitudes are Equal; and of Inequality, when they are Unequal. For ſeeing (by the 5th. Article of the precedent Chapter) the Proportion of two Unequal Magnitudes conſiſts in their Difference compared with either of them; and when two Unequal Magnitudes are Expoſed, their Difference is alſo Expoſed; it follows, that when Magnitudes which have Proportion to one another are Expoſed, their Proportion alſo is Expoſed with them; and in like manner, the Proportion of Equals (which conſiſts in this, that there is no Difference of Magnitude betwixt them) is Expoſed at the ſame time when the Equal Magnitudes themſelves are Expoſed. For Example, if the Expoſed Lines A B and C D be Equal, the Proportion of Equality is Expoſed in them; and if the Expoſed Lines E F & E G be Unequal, the Proportion which E F has to E G, and that which E G has to E F are alſo Expoſed in them; for not onely the Lines themſelves, but alſo their Difference G F is Expoſed. The Proportion of Unequals is Quantity; for the Difference G F, in which it conſiſts, is Quantity. But the Proportion of Equality is not Quantity, becauſe between Equals there is no Difference; nor is one Equality greater then another, as one Inequality is greater then another Inequality.



9 The Proportion of two *Times*, or of two *Uniform Velocities*, is then Expoſed, when two lines are expoſed, by which two Bodies are underſtood to be Moved Uniformly; and therefore the ſame two Lines ſerve to exhibit both their own Proportion, and that of the Times and Velocities, according as they are conſidered to be Expoſed for the Magnitudes themſelves, or for the Times or Velocities. For let the two Lines A and B be Expoſed; their Proportion therefore (by the laſt foregoing Article) is Expoſed; and if they be conſidered as drawn with equal and Uniform Velocity, then ſeeing their Times are greater, or equal, or leſſe, according as the ſame Spaces are tranſmitted in greater, or equal, or leſſe Time, the Lines A and B will exhibit the Equality or Inequality, that is, the Proportion of the Times. To conclude, If the



ſame Lines A and B be conſidered as drawn in the ſame Time, then ſeeing their Velocities are greater, or equal, or leſſe, according as they paſs over in the ſame time longer, or equal, or ſhorter Lines, the ſame Lines A and B will exhibit the Equality or Inequality, that is, the Proportion of their Velocities.

CHAP.

CHAP. XIII.
Of Analogiſme *or* the Same Proportion.

1, 2, 3, 4. *The Nature and Definition of Proportion Arithmetical and Geometrical.* 5 *The Definition and ſome properties of the Same Arithmetical Proportion.* 6, 7. *The Definition and Tranſmutations of Analogiſme, or The Same Geometrical Proportion.* 8, 9. *The Definitions of Hyperlogiſme and Hypologiſme, that is, of Greater and Leſſe Proportion, and their Tranſmutations.* 10, 11, 12. *Compariſon of Analogical quantities according to Magnitude.* 13, 14, 15. *Compoſition of Proportions.* 16, 17, 18, 19, 20, 21, 22, 23, 24, 25. *The Definition and Properties of Continual Proportion.* 26, 27, 28, 29. *Compariſon of Arithmetical and Geometrical Proportions.*

Note that in this Chapter, the ſigne +, *ſignifies that the quantities betwixt which it is put, are added together; and this ſigne* —, *the Remainder, after the later quantity is taken out of the former. So that* A+B *is equal to both* A *and* B *together; and where you ſee* A—B, *there* A *is the Whole,* B *the part taken out of it, and* A—B *the Remainder. Alſo two letters ſet together without any ſign, ſignifies (unleſſe they belong to a Figure) that one of the quantities is multiplyed by the other; as* A B *ſignifies the Product of* A *multiplyed by* B.

1 *Great* & *Little* are not intelligible but by Compariſon. Now that to which they are compared is ſomething Expoſed, that is, ſome Magnitude either perceived by Senſe, or ſo defined by Words, that it may be comprehended by the Mind. Alſo that to which any Magnitude is compared, is either Greater, or Leſs, or Equal to it. And therefore Proportion (which as I have ſhewn, is the Eſtimation, or Comprehenſion of Magnitudes by Compariſon) is threefold; namely Proportion of *Equality*, that is, of Equal to Equal; or of *Exceſſe*, which is of the Greater to the Leſſe; or of *Defect*, which is the Proportion of the Leſſe to the Greater.

Again, every one of theſe Proportions is twofold; For if it be asked concerning any Magnitude given, how Great it is, the anſwer may be made by Comparing it two wayes; Firſt, by ſaying it is greater or leſſe then another Magnitude by ſo much; as Seven is leſſe then Ten by three Unities; and this is called *Arithmetical Proportion*. Secondly, by ſaying it is greater or leſſe then another Magnitude by ſuch a part or parts thereof; as Seven is leſs then Ten by three tenth parts of the ſame Ten. And though this Proportion be not alwayes explicable by Number, yet it is a determinate Proportion, and of a different kind from the former, and called *Geometrical Proportion*, and moſt commonly *Proportion ſimply*.

2 Proportion, whether it be Arithmetical or Geometrical, cannot be Expoſed but in two Magnitudes (of which the former is cõmonly called the *Antecedent*, & the later the *Conſequent* of the Proportion) as I have ſhewn in the 8th. Article of the precedent Chapter. And therefore if two Proportions be to be compared, there muſt be four Magnitudes Expoſed, namely two Antecedents and two Conſequents; for though it happen ſometimes, that the Conſequent of the former Proportion be the ſame with the Antecedent of the later, yet in that double Compariſon it muſt of neceſſity be twice numbred; ſo that there will be alwayes four Terms.

3 Of two Proportions, whether they be Arithmetical or Geometrical, when the Magnitudes compared in both (which *Euclide* in the fifth Definition of his ſixth Book calls the *Quantities of Proportions*) are equal, then one of the Proportions cannot be either greater or leſſe then the other; For one Equality is neither greater nor leſſe then another Equality. But of two Proportions of Inequality, whether they be Proportions of Exceſſe or of Defect, one of them may be either greater or leſſe then the other, or they may both be equal; for though there be propounded two Magnitudes that are unequal to one another, yet there may be other two more unequal, and other two equally unequall, and other two leſs unequal then the two which were propounded. And from hence it may be underſtood that the Proportions of Exceſs and Defect are Quantity, being capable of More & Leſs; but the Proportion of Equality is not Quantity, becauſe not capable, neither of *More* nor of *Leſs*. And therefore Proportions of Inequality may be

be added together or ſubſtracted from one another, or be multiplyed or divided by one another, or by Number; but Proportions of Equality not ſo.

4 Two Equal Proportions are commonly called *The Same Proportion*; and it is ſaid, that the Proportion of the firſt Antecedent to the firſt Conſequent is the *ſame* with that of the ſecond Antecedent to the ſecond Conſequent. And when four Magnitudes are thus to one another in Geometrical Proportion, they are called *Proportionals*, and by ſome more briefly *Analogiſme*. And *Greater Proportion*, is the Proportion of a Greater Antecedent to the ſame Conſequent; or of the ſame Antecedent to a Leſs Conſequent; and when the Proportion of the firſt Antecedent to the firſt Conſequent, is greater then that of the ſecond Antecedent to the ſecond Conſequent, the four Magnitudes which are ſo to one another may be called *Hyperlogiſme*.

Leſs Proportion is the Proportion of a Leſs Antecedent to the ſame Conſequent, or of the ſame Antecedent to a Greater Conſequent; and when the Proportion of the firſt Antecedent to the firſt Conſequent, is leſs then that of the ſecond to the ſecond, the four Magnitudes may be called *Hypologiſme*.

5 One Arithmetical Proportion is the *Same* with another Arithmetical Proportion, when one of the Antecedents exceeds its Conſequent, or is exceeded by it, as much as the other Antecedent exceeds its Conſequent, or is exceeded by it. And therefore in four Magnitudes Arithmetically Proportional, the ſum of the Extremes is equal to the ſum of the Means. For if A. B :: C. D be Arithmetically Proportional, and the Difference on both ſides be the ſame Exceſs or the ſame Defect E; then B+C (if A be greater then B) will be equal to A−E+C; and A+D will be equal to A+C−E; But A−E+C and A+C−E are equal. Or if A be leſs then B, then B+C will be equal to A+E+C; and A+D will be equal to A+C+E; But A+E+C, and A+C+E are equal.

Alſo if there be never ſo many Magnitudes Arithmetically Proportional, the Sum of them all will be equal to the Product of half the number of the Terms multiplyed by the Sum of the Extremes. For if A. B :: C. D :: E. F be Arithmetically Proportional, the Couples A+F, B+E, C+D will be equal to one another; and

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their Sum will be equal to A+F multiplyed by the number of their Combinations, that is, by half the number of the Terms.

If of four Unequal Magnitudes, any two together taken be equal to the other two together taken, then the greateſt and the leaſt of them will be in the ſame Combination. Let the Unequal Magnitudes be A,B,C,D; and let A+B be equal to C+D; & let A be the greateſt of them all; I ſay B will be the leaſt. For if it may be, let any of the reſt, as D, be the leaſt. Seeing therefore A is greater then C, and B then D, A+B will be greater then C+D; which is contrary to what was ſuppoſed.

If there be any four Magnitudes, the Sum of the greateſt and leaſt, the Sum of the Means, the difference of the two greateſt, and the difference of the two leaſt will be Arithmetically Proportional. For let there be four Magnitudes, whereof A is the greateſt, D the leaſt, and B and C the Means; I ſay A+D. B+C :: A—B. C—D are Arithmetically Proportional. For the difference between the firſt Antecedent and its Conſequent is this, A+D—B—C; and the difference between the ſecond Antecedent and its Conſequent this, A—B—C+D; but theſe two Differences are equal, and therefore (by this 5th. Article) A+D. B+C :: A—B. C—D are Arithmetically Proportional.

If of four Magnitudes, two be equal to the other two, they will be in reciprocal Arithmetical Proportion. For let A+B be equal to C+D; I ſay A. C :: D. B are Arithmetically Proportional. For if they be not, let A. C :: D. E (ſuppoſing E to be greater or leſs then B) be Arithmetically Proportional, and then A+E will be equal to C+D; wherefore A+B and C+D are not equal; which is contrary to what was ſuppoſed.

6 One Geometrical Proportion is the ſame with another Geometrical Proportion, when the ſame Cauſe producing equal Effects in equal Times, determines both the Proportions.

If a Point Uniformly moved, deſcribe two Lines either with the ſame, or different Velocity, all the parts of them which are contemporary, that is, which are deſcribed in the ſame time, will be Two to Two in Geometrical Proportion, whether the Antecedents be taken in the ſame Line, or not. For, from the point A (in the 10 Figure at the end of the 14 Chapter) let the two Lines A D, A G, be deſcribed with Uniform Motion; and let there be taken

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in them two parts A B, A E, and again two other parts A C, A F; in such manner, that A B, A E, be contemporary, and likewise A C, A F contemporary. I say first (taking the Antecedents A B, A C in the Line A D, and the Consequents A E, A F in the Line A G) that A B. A C :: A E. A F are Proportionals. For seeing (by the 8th. Chapter and the 15 Article) Velocity is Motion considered as determined by a certain Length or Line, in a certain Time transmitted by it, the quantity of the Line A B will be determined by the Velocity and Time by which the same A B is described; and for the same reason, the quantity of the Line A C will be determined by the Velocity and Time, by which the same A C is described; and therefore the proportion of A B to A C, whether it be Proportion of Equality, or of Excess or Defect, is determined by the Velocities and Times by which A B, A C are described; But seeing the Motion of the Point A upon A B and A C is Uniform, they are both described with equal Velocity; and therefore whether one of them have to the other the Proportion of Majority or of Minority, the sole cause of that Proportion is the difference of their Times; and by the same reason it is evident, that the proportion of A E to A F is determined by the difference of their Times onely. Seeing therefore A B, A E, as also A C, A F are contemporary, the difference of the Times in which A B and A C are described, is the same with that in which A E and A F are described. Wherfore the proportion of A B to A C, and the proportion of A E to A F are both determined by the same Cause. But the Cause which so determines the proportion of both, works equally in equal Times, for it is Uniform Motion; and therefore (by the last precedent Definition) the proportion of A B to A C is the same with that of A E to A F; and consequently A B. A C :: A E. A F are Proportionals; which is the first.

Secondly (taking the Antecedents in different Lines,) I say, A B. A E :: A C. A F are Proportionals; For seeing A B, A E are described in the same Time, the difference of the Velocities in which they are described are the sole Cause of the proportion they have to one another. And the same may be said of the proportion of A C to A F. But seeing both the Lines A D and A G are passed over by Uniform Motion, the difference of the Velocities in which A B, A E are described, will be the same with the

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difference of the Velocities, in which A C, A F are described. Wherefore the Cauſe which determines the proportion of A B to A E, is the ſame with that which determines the proportion of A C to A F; and therefore A B. A E : : A C. A F, are Proportionals; which remained to be proved.

1 *Corollary.* If four Magnitudes be in Geometrical Proportion, they will alſo be Proportionals by *Permutation*, (that is, by tranſpoſing the Middle Terms.) For I have ſhewn, that not onely A B. A C : : A E. A F, but alſo that (by *Permutation*) A B. A E : : A C. A F are Proportionals.

2 *Coroll.* If there be four Proportionals, they will alſo be Proportionals by *Inverſion* or *Converſion*, that is, by turning the Antecedents into Conſequents. For if in the laſt *Analogiſme*, I had for A B, A C, put by Inverſion A C, A B, and in like manner converted A E, A F into A F, A E, yet the ſame Demonſtration had ſerved. For as well A C, A B, as A B, A C are of equal Velocity; and A C, A F, as well as A F, A C are Contemporary.

3 *Coroll.* If Proportionals be added to Proportionals, or taken from them, the Aggregates, or Remainders will be Proportionals. For Contemporaries whether they be added to Contemporaries, or taken from them; make the Aggregates or Remainders Contemporary, though the Addition or Subſtraction be of all the Terms, or of the Antecedents alone, or of the Conſequents alone.

4 *Coroll.* If both the Antecedents of four Proportionals, or both the Conſequents, or all the Terms, be multiplyed or divided by the ſame Number or Quantity, the Products or Quotients will be Proportionals. For the Multiplication and Diviſion of Proportionals, is the ſame with the Addition and Subſtraction of them.

5 *Coroll.* If there be four Proportionals, they will alſo be Proportionals by *Compoſition* (that is, by compounding an Antecedent of the Antecedent and Conſequent put together, and by taking for Conſequent either the Conſequent ſingly, or the Antecedent ſingly). For this Compoſition is nothing but Addition of Proportionals, namely, of Conſequents to their own Antecedents, which by Suppoſition are Proportionals.

6 *Coroll.* In like manner, if the Antecedent ſingly, or Conſequent ſingly be put for Antecedent, and the Conſequent be made of both put together, theſe alſo will be Proportionals. For it is the *Inverſion of Porportion by Compoſition.* 7 Co-

7 *Coroll.* If there be four Proportionals, they will alſo be Proportionals by *Diviſion* (that is, by taking the Remainder after the Conſequent is ſubſtracted from the Antecedent, or the Difference between the Antecedent and Conſequent for Antecedent, and either the Whole or the Remainder for Conſequent;) As if A.B :: C.D be Proportionals, they will by Diviſion be A—B. B :: C—D. D, and A—B. A :: C—D. C; and when the Conſequent is greater then the Antecedent, B—A. A :: D—C. C, and B—A B :: D—C. D. For in all theſe Diviſions, Proportionals are (by the very ſuppoſition of the Analogiſme A. B :: C. D) taken from A and B, and from C and D.

8 *Coroll.* If there be four Proportionals, they will alſo be Proportionals by the *Converſion of Proportion*, (that is, by Inverting the Divided Proportion, or by taking the Whole for Antecedent, & the Difference or Remainder for Conſequent.) As, if A. B :: C. D be Proportionals, then A. A—B :: C. C—D, as alſo B. A—B :: D. C—D will be Proportionals. For ſeeing theſe Inverted be Proportionals, they are alſo themſelves Proportionals.

9 *Coroll.* If there be two Analogiſmes which have their quantities equal, the ſecond to the ſecond, and the fourth to the fourth, then either the Sum or Difference of the firſt quantities will be to the ſecond, as the Sum or Difference of the third quantities is to the fourth. Let A.B :: C.D and E. B :: F.D be Analogiſmes; I ſay C+E.B :: F.D are Proportionals. For the ſaid Analogiſmes will by Permutation be A.C :: B.D, and E.F :: B.D; and therefore A.C :: E.F will be Proportionals (for they have both the proportion of B to D common.) Wherefore if in the Permutation of the firſt Analogiſme, there be added E and F to A and C, which E and F are proportional to A and C, then (by the 3d *Coroll.*) A+E.B :: C+F.D will be Proportionals, which was to be proved.

Alſo in the ſame manner it may be ſhewn, that A—E.B :: C + F.D are Proportionals.

7 If there be two Analogiſmes, where foure Antecedents make an Analogiſme, their Conſequents alſo ſhall make an Analogiſme; as alſo the Sums of their Antecedents will be proportionall to the Sums of their Conſequents. For if A.B :: C.D and E. F :: G.H be two Analogiſmes; and A. E :: C. G be Proportionals, then by Permutation A.C :: E.G, and E.G :: F.H, and A.C :: B.D will be Proportionals; Wherefore B.D :: E.G, that is, B.D :: F.H, and by

Permutation B. F :: D.H are Proportionals; which is the firſt. Secondly, I ſay A+E.B+F :: C+G.D+H will be Proportionalls. For ſeeing A.E :: C.G are Proportionals, A+E.E :: C+G. G will alſo by Compoſition be Proportionals, and by Permutation A+E. C+G :: E.G will be Proportionals; Wherefore alſo A+E.C+G :: F.H will be Proportionals. Again, ſeeing (as is ſhewn above) B.F :: D.H are Proportionals, B+F. F :: D+H. H will alſo by Compoſition be Proportionals; and by Permutation B+F. D+H :: F. H will alſo be Proportionals ; Wherefore A+E. C+G :: B+F. D+H are Proportionals; which remained to be proved.

Coroll. By the ſame reaſon, if there be never ſo many Analogiſmes, and the Antecedents be proportional to the Antecedents, it may be demonſtrated alſo that the Conſequents will be proportional to the Conſequents, as alſo the Sum of the Antecedents to the Sum of the Conſequents.

8 In an Hyperlogiſme, that is, where the Proportion of the firſt Antecedent to its Conſequent, is greater then the proportion of the ſecond Antecedent to its Conſequent, the Permutation of the Proportionals, and the Addition of Proportionals to Proportionals, & Subſtraction of them from one another ; as alſo their Compoſition & Diviſion, & their Multiplication & Diviſion by the ſame Number, produce always an Hyperlogiſme. For ſuppoſe A.B :: C.D & A.C :: E.F be Analogiſmes, A+E.B :: C+F.D will be alſo an Analogiſme; But A+E.B :: C. D will be an Hyperlogiſme; Wherefore by Permutation, A+E. C :: B. D is an Hyperlogiſme, becauſe A.B :: C.D is an Analogiſme. Secondly, if to the Hyperlogiſme A+E. B :: C.D the Proportionals G and H be added, A+E+G. B ::, C+H. D will be an Hyperlogiſme, by reaſon A+E+G. B :: C+F+H. D is an Analogiſme. Alſo if G and H be taken away, A+E−G. B :: C−H.D will he an Hyperlogiſme ; for A+E−G. B :: C+F−H. D are an Analogiſme. Thirdly, by Compoſition A+E+B. B :: C+D. D will be an Hyperlogiſme, becauſe A+E+B.B :: C+F+D.D is an Analogiſme, & ſo it will be in all the varieties of Compoſition. Fourthly, by Diviſion A+E−B.B :: C−D.D will be an Hyperlogiſme, by reaſon A+E−B. B :: C+F−D. D is an Analogiſme. Alſo A+E−B. A+E :: C−D. C is an Hyperlogiſme ; for A+E−B. A+E :: C+F−D. C is an Analogiſme. Fifthly, by Multiplication 4 A+E.B :: 4 C.D is an Hyperlogiſme, becauſe 4 A. B :: 4C. D is an Analogiſme;

giſme; and by Diviſion $\frac{1}{4}$A+$\frac{1}{4}$E. B::$\frac{1}{4}$C.D is an Hyperlogiſme, becauſe $\frac{1}{4}$A.B::$\frac{1}{4}$C.D is an Analogiſme.

9 But if A+E.B::C.D be an Hyperlogiſme, then by Inverſion B.A+E::D.C will be an Hypologiſme, becauſe B.A::D.C being an Analogiſme, the firſt Conſequent will be too great. Alſo by Converſion of Proportion A+E.A+E—B::C.C—D is an Hypologiſme, becauſe the Inverſion of it, namely A+E—B.A+E::C—D.C is an Hyperlogiſme (as I have ſhewn but now). So alſo B.A+E—B::D.C—D is an Hypologiſme, becauſe (as I have newly ſhewn) the Inverſion of it, namely A+E—B.B::C—D.D is an Hyperlogiſme. Note that this Hypologiſme A+E.A+E—B::C.C—D is commonly thus expreſſed; If the proportion of the Whole (A+E) to that which is taken out of it (B), be greater then the proportion of the Whole (C) to that which is taken out of it (D,) then the proportion of the whole (A+E) to the Remainder (A+E—B) will be leſs then the proportion of the whole (C) to the Remainder (C—D.)

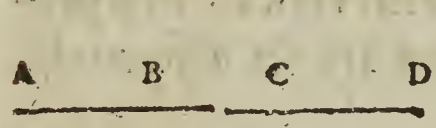
10 If there be four Proportionals, the Difference of the two firſt, to the Difference of the two laſt will be as the firſt Antecedent is to the ſecond Antecedent, or as the firſt Conſequent to the ſecond Conſequent. For if A.B::C.D be Proportionals, then by Diviſion A—B.B::C—D.D will be Proportionals; and by Permutation A—B.C—D::B.D; that is, the Differences are proportional to the Conſequents, and therefore they are ſo alſo to the Antecedents.

11 Of four Proportionals, if the firſt be greater then the ſecond, the third alſo ſhall be greater then the fourth. For ſeeing the firſt is greater then the ſecond, the proportion of the firſt to the ſecond is the proportion of Exceſs; But the proportion of the third to the fourth is the ſame with that of the firſt to the ſecond; and therefore alſo the proportion of the third to the fourth is the Proportion of Exceſs; Wherefore the third is greater then the fourth. In the ſame manner it may be proved, that whenſoever the firſt is leſs then the ſecond, the third alſo is leſs then the fourth; and when thoſe are equal, that theſe alſo are equal.

12 If there be four Proportionals whatſoever A.B::C.D, and the firſt and third be multiplyed by any one number, as by 2; and again the ſecond and fourth be multiplyed by any one number, as

by 3; and the product of the first 2 A, be greater then the product of the second 3 B; the product also of the third 2 C, will be greater then the product of the fourth 3 D. But if the product of the first be less then the product of the second, then the product of the third will be less then that of the fourth. And lastly, if the products of the first and second be equal, the products of the third and fourth shall also be equal. Now this Theoreme is all one with *Euclides* Definition of *The Same Proportion*; and it may be demonstrated thus. Seeing A. B :: C. D are Proportionals, by Permutation also (Art. 6. Corol. 1.) A. C :: B. D will be Proportionals; Wherefore (by the 4 Corol. of the same 6 Article) 2 A. 2 C :: 3 B. 3 D will be Proportionals; and again by Permutation 2 A. 3 B :: 2 C. 3 D will be Proportionals; and therefore (by the last Article) If 2 A be greater then 3 B, then 2 C will be greater then 3 D; if less, less; and if equal, equal; which was to be demonstrated.

13 If any three Magnitudes be propounded, or three things whatsoever that have any proportion one to another, as three Nũbers, three Times, three Degrees, &c; the proportions of the first to the second, and of the second to the third together taken, are equal to the proportion of the first to the third. Let there be three Lines (for any proportion may be reduced to the proportion of Lines) A B, A C, A D; And in the first place, let the proportion as well of the first A B to the second A C, as of the second A C to the third A D be the proportion of Defect, or of Less to Greater; I say the proportions together taken of A B to A C, and of A C to A D, are equal to the proportion of A B to A D. Suppose the Point A to be moved over the whole Line A D with Uniform Motion; then the proportions as well of A B to A C, as of A C to A D are determined by the difference of the Times in which they are described; that is, A B has to A C such proportion as is determined by the different Times of their description; and A C to A D such proportion as is determined by their Times. But the proporrion of A B to A D is such as is determined by the difference of the Times in which A B and A D are described; and the difference of the Times in which A B and A C are described, together with the difference of the Times in which A C and A D are described, is the same with the difference of the Times in which A B and A D are de-



described. And therefore the same Cause which determines the two proportions of A B to A C and of A C to A D, determines also the proportion of A B to A D. Wherefore (by the Definition of the Same Proportion delivered above in the 6 Art.) the proportion of A B to A C together with the proportion of A C to A D, is the same with the proportion of A B to A D.

In the second place, let A D be the first, A C the second, & A B the third, and let their proportion be the Porportion of Excess, or of Greater to Less; then, as before, the proportions of A D to A C, and of A C to A B, and of A D to A B will be determined by the difference of their Times; which in the description of A D and A C, and of A C and A B together taken, is the same with the difference of the Times in the description of A D and A B. Wherefore the proportion of A D to A B is equal to the two proportions of A D to A C and of A C to A B.

In the last place. If one of the proportions, namely of A D to A B be the Proportion of Excess; and another of them, as of A B to A C be the Proportion of Defect, thus also the proportion of A D to A C will be equal to the two proportions together taken of A D to A B, and of A B to A C. For the difference of the Times in which A D and A B are described is Excess of Time; for there goes more time to the description of A D then of A B; and the difference of the Times in which A B and A C are described is Defect of Time, for less Time goes to the description of A B then of A C; but this Excess, and Defect being added together make D B — B C, which is equal to D C, by which the first A D exceeds the third A C; and therefore the proportions of the first A D to the second A B, and of the second A B to the third A C, are determined by the same Cause which determines the Proportion of the first A D to the third A C. Wherefore, If any three Magnitudes, &c.

1 *Coroll.* If there be never so many Magnitudes having proportion to one another, the proportion of the first to the last is compounded of the proportions of the first to the second, of the second to the third, & so on till you come to the last; or, the proportion of the first to the last, is the same with the Sum of all the intermediate proportions. For any Number of Magnitudes having proportion to one another, as A, B, C, D, E being propounded, the proportion

of

of A to E (as is newly ſhewn) is compounded of the Proportions of A to D and of D to E; and again the proportion of A to D, of the proportions of A to C, and of C to D; and laſtly, the proportion of A to C, of the proportions of A to B, and of B to C.

2 *Coroll.* From hence it may be underſtood how any two proportions may be compounded. For if the proportions of A to B, and of C to D be propounded to be added together, let B have to ſomething elſe, as to E, the ſame proportion which C has to D, and let them be ſet in this order A, B, E; for ſo the proportion of A to E will evidently be the Sum of the two Proportions of A to B, and of B to E, that is, of C to D. Or let it be as D to C, ſo A to ſomething elſe, as to E, and let them be ordered thus E, A, B; for the proportion of E to B will be compounded of the proportions of E to A (that is, of C to D), and of A to B. Alſo it may be underſtood how one Proportion may be taken out of another. For if the proportion of C to D be to be ſubſtracted out of the proportion of A to B, let it be as C to D, ſo A to ſomething elſe, as E, and ſetting them in this order, A, E, B; and taking away the proportion of A to E, that is, of C to D, there will remain the proportion of E to B.

3 *Coroll.* If there be two Orders of Magnitudes which have proportion to one another beginning and ending with the ſame Magnitudes, and the ſeveral proportions of the firſt Order be the ſame and equal in number with the proportions of the ſecond Order; then, whether the proportions in both Orders be ſucceſſively anſwerable to one another, which is called *Ordinate Proportion*, or not ſucceſſively anſwerable, which is called *Perturbed Proportion*, the firſt and the laſt in both will be Proportionals. For the Proportion of the firſt to the laſt is equal to all the intermediate proporons; which being in both Orders the ſame, and equal in number, the Aggregates of thoſe proportions will alſo be equal to one another; but to their Aggregates the proportions of the firſt to the laſt are Equal; and therefore the proportion of the firſt to the laſt in one Order, is the ſame with the proportion of the firſt to the laſt in the other Order. Wherefore the firſt and the laſt in both are Proportionals.

14 If any two quantities be made of the mutual Multiplication of many quantities which have proportion to one another, and the

the Efficient quantities on both ſides be equal in number, the proportion of the Products will be compounded of the ſeveral proportions which the Efficient quantities have to one another.

Firſt, let the two Products be A B, and C D, whereof one is made of the Multiplication of A into B, and the other of the Multiplication of C into D. I ſay the porportion of A B to C D is componnded of the proportions of the Efficient A to the Efficient C, and of the Efficient B to the Efficient D. For let A B, C B and C D be ſet in Order; and as B is to D, ſo let C be to another quantity as E; and let A, C, E, be ſet alſo in Order. Then by (the 4 *Coroll.* of the 6 Art.) it will be as A B the firſt quantity, to C B the ſecond quantity in the firſt Order, ſo A to C in the ſecond Order; and again, as C B to C D in the firſt Order, ſo B to D, that is, (by Conſtruction) ſo C to E in the ſecond Order; and therefore (by the laſt *Corollary*) A B. C D :: A. E will be Proportionals. But the proportion of A to E is compounded of the Proportions of A to C, and of B to D; Wherefore alſo the proportion of A B to C D is compounded of the ſame.

A B	A
C B	C
C D	E

Secondly, let the two Products be A B F, and C D G, each of them made of three Efficients, the firſt of A, B and E, and the ſecond of C, D and G; I ſay, the proportion of A B F to C D G is compounded of the proportions of A to C, of B to D and of F to G. For let them be ſet in Order as before; and as B is to D, ſo let C be to another quantity E; and again, as F is to G, ſo let E be to another, H; and let the firſt Order ſtand thus, A B F, C B F, C D F and C D G; and the ſecond Order thus, A, C, E, H, Then the proportion of A B F to C B F in the firſt Order, will be as A to C in the ſecond; and the Proportion of C B F to C D F in the firſt Order, as B to D, that is, as C to E (by Conſtruction) in the ſecond Order; and the Proportion of C D F to C D G in the firſt, as F to G, that is, as E to H (by Conſtruction) in the ſecond Order; and therefore A B F. C D G :: A. H will be Proportionals. But the Proportion of A to H is compounded of the Proportions of A to C, B to D, and F to G, Wherefore the Proportion of the Product A B F to C D G is alſo com-

A B F	A
C B F	C
C D F	E
C D G	H

compo unded of the ſame. And this operation ſerves, how many ſoever the Efficients be that make the Quantities given.

From hence ariſeth another way of Compounding many Proportions into One, namely, that which is ſuppoſed in the 5 Definition of the 6 Book of *Euclide*; which is, by multiplying all the Antecedents of the Proportions into one another, and in like manner all the Conſequents into one another. And from hence alſo it is evident, in the firſt place, That the Cauſe why Parallelograms, which are made by the Duction of two ſtraight Lines into one anther, and all Solids which are equal to Figures ſo made, have their proportions compounded of the proportions of the Efficients; And in the ſecond place, why the Multiplication of two or more Fractions into one another, is the ſame thing with the Compoſition of the proportions of their ſeveral Numerators to their ſeveral Denominators. For example, if theſe Fractions $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}$ be to be multiplyed into one another, the Numerators 1, 2, 3 are firſt to be multiplyed into one another, which make 6; and next the Denominators 2, 3, 4, which make 24; and theſe two Products make the Fraction $\frac{6}{24}$. In like manner, if the proportions of 1 to 2, of 2 to 3, and of 3 to 4 be to be compounded, by working as I have ſhewn above, the ſame proportion, of 6 to 24 will be produced.

15 If any Proportion be compounded with it ſelf inverted, the Compound will be the Proportion of Equality. For let any Proportion be given, as of A to B, and let the Inverſe of it be that of C to D; and as C to D, ſo let B be to another quantity; for thus they will be compounded (by the 2 *Coroll.* of the 12 Art.) Now ſeeing the proportion of C to D is the Inverſe of the proportion of A to B, it will be as C to D, ſo B to A; and therefore if they be placed in Order A, B, A, the proportion compounded of the proportions of A to B, and of C to D will be the proportion of A to A, that is, the proportion of Equality. And from hence the cauſe is evident, why two equal products have their Efficients reciprocally proportional. For, for the making of two products equal, the proportions of their Efficients muſt be ſuch, as being compounded may make the proportion of Equality, which cannot be, except one be the Inverſe of the other; for if betwixt A and A any other quantity as C be interpoſed, their order will be A, C, A, and the later proportion of C to A will be the Inverſe of the former proportion of A to C.

16 A

16 A Proportion is ſaid to be multiplied by a Number when it is ſo often taken as there be Unities in that Number; and if the Proportion be of the Greater to the Leſs, then ſhall alſo the quantity of the Proportion be increaſed by the Multiplication; but when the Proportion is of the Leſs to the Greater, then as the Number increaſeth, the quantity of the Proportion diminiſheth; as in theſe three Numbers 4, 2, 1, the Proportion of 4 to 1, is not onely the Duplicate of 4 to 2, but alſo twice as great; but inverting the order of thoſe Numbers thus 1, 2, 4, the Proportion of 1 to 2 is greater then that of 1 to 4; and therefore though the proportion of 1 to 4 be the Duplicate of 1 to 2, yet it is not twice ſo great as that of 1 to 2, but contrarily the half of it. In like manner, a Proportion is ſaid to be Divided, when between two quantities are interpoſed one or more Means in continual Proportion, and then the Proportion of the firſt to the ſecond is ſaid to be Subduplicate of that of the firſt to the third, and Subtriplicate of that of the firſt to the fourth, &c.

This mixture of Proportions, where ſome are Proportions of Exceſs, others of Defect (as in a Merchants accompt of *Debitor* and *Creditor*) is not ſo eaſily reckoned as ſome think; but maketh the Compoſition of Proportions ſometimes to be Addition, ſometimes Subſtraction; which foundeth abſurdly to ſuch as have alwayes by Compoſition underſtood Addition, and by Diminution Subſtraction. Therefore to make this account a little clearer, we are to conſider (that which is commonly aſſumed, and truly) that if there be never ſo many Quantities, the Proportion of the firſt to the laſt is compounded of the Proportions of the firſt to the ſecond, and of the ſecond to the third, and ſo on to the laſt, without regarding their Equality, Exceſs or Defect; So that if two Proportions, one of Inequality, the other of Equality be added together, the Proportion is not thereby made Greater nor Leſs; as for example, if the Proportions of A to B and of B to B be compounded, the Proportion of the firſt to the ſecond is as much as the Sum of both, becauſe Proportion of Equality (being not quantity) neither augmenteth quantity nor leſſeneth it. But if there be three quantities A, B, C, unequal, and the firſt be greateſt, the laſt leaſt, then the Proportion of B to C is an addition to that of A to B, and

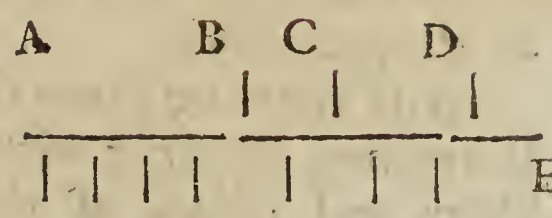
makes it greater; and on the contrary, if A be the leaſt, and C the greateſt quantity, then doth the addition of the Proportion of B to C make the cõpounded Proportion of A to C leſs then the Proportion of A to B, that is, the Whole leſs then the Part. The Compoſition therefore of Proportions is not in this caſe the Augmentation of them, but the Diminution; for the ſame quantity (*Euclide* the 5, 8.) compared with two other quantities hath a greater Proportion to the leſſer of them then to the greater. Likewiſe, when the Proportions compounded are one of Exceſs, the other of Defect, if the firſt be of Exceſs, as in theſe numbers 8,6,9, the Proportion compounded, namely, of 8 to 9, is leſs then the Proportion of one of the parts of it, namely of 8 to 6; but if the Proportion of the firſt to the ſecond be of Defect, and that of the ſecond to the third be of Exceſs, as in theſe Numbers 6,8,4, then ſhall the Proportion of the firſt to the third be greater then that of the firſt to the ſecond, as 6 hath a greater Proportion to 4 then to 8; the reaſon whereof is manifeſtly this, that the leſs any quantity is deficient of another, or the more one exceedeth another, the proportion of it to that other is the greater.

Suppoſe now three quantities in continual Proportion A B 4, A C 6, A D 9. Becauſe therefore A D is greater then A C, but not greater then A D, the proportion of A D to A C will be (by *Euclide* 5, 8.) greater then that of A D to A D; and likewiſe, becauſe the Proportions of A D to A C, and of A C to A B are the ſame, the proportions of A D to A C and of A C to A B (being both Proportions of Exceſs) make the whole Proportion of A D to A B (or of 9 to 4) not onely the Duplicate of A D to A C (that is, of 9 to 6) but alſo the Double, or twice ſo great. On the other ſide, becauſe the proportion of A D to A D (or 9 to 9) being Proportion of Equality, is no quantity, & yet greater then that of A C to A D (or 6 to 9) it will be as 0—9 to 0—6, ſo A C to A D; and again, as 0—9 to 0—6, ſo 0—6 to 0—4; but 0—4, 0—6, 0—9 are in continual proportion; and becauſe 0—4 is greater then 0—6, the proportion of 0—4 to 0—6 will be Double to the proportion of 0—4 to 0—9, Double I ſay, and yet not Duplicate, but Subduplicate.

If any be unſatisfied with this ratiocination, let him firſt conſider that (by *Euclide* 5, 8) the proportion of A B to A C is greater then

that

that of A B to A D whereſoever D be placed in the line A C prolonged; and the further off the point D is from C, ſo much the greater is the proportion of A B to A C then that of A B to A D.



There is therefore ſome point (which ſuppoſe be E) in ſuch diſtance from C, as that the proportion of A B to A C will be twice as great as that of A B to A E. That conſidered, let him determine the length of the line A E, and demonſtrate (if he can) that A E is greater or leſs then A D.

By the ſame method, if there be more quantities then three, as A, B, C, D in continual Proportion, and A be the leaſt, it may be made appear, that the Proportion of A to B is Triple Magnitude (though Subtriple in Multitude) to the Proportion of A to D.

17 If there be never ſo many quantities, the number whereof is odd, and their Order ſuch, that from the middlemoſt quantity both wayes they proceed in continual Proportion, the proportion of the two which are next on either ſide to the middlemoſt, is Subduplicate to the proportion of the two which are next to theſe on both ſides, and Subtriplicate of the proportion of the two which are yet one place more remote, &c. For let the Magnitudes be C, B, A, D, E, and let A, B, C, as alſo A, D, E be in Continual Proportion; I ſay the proportion of D to B is Subduplicate of the proportion of E to C. For the proportion of D to B is compounded of the proportions of D to A, and of A to B once taken; But the proportion of E to C is compounded of the ſame twice taken; and therefore the proportion of D to B is Subduplicate of the proportion of E to C. And in the ſame manner, if there were three Terms on either ſide, it might be demonſtrated that the proportion of D to B would be Subtriplicate of that of the Extremes, &c.

18 If there be never ſo many continual Proportionals, as the firſt, ſecond, third, &c. their Differences will be Proportional to them. For the ſecond, third, &c. are ſeverally Conſequents of the preceding, and Antecedents of the following Proportion. But (by the 10 *Art.*) the Difference of the firſt Antecedent and Conſequent, to Difference of the ſecond Antecedent and Conſequent, is as the firſt Antecedent to the ſecond Antecedent, that is, as the firſt

Term to the ſecond, or as the ſecond to the third, &c. in continuall Proportionals.

19 If there be three Continual Proportionals, The Sum of the Extremes, together with the Mean twice taken; The Sum of the Mean and either of the Extremes; and the ſame Extreme, are Continual Proportionals. For let A. B. C ÷ be Continual Proportionals. Seeing therefore A. B :: B. C are Proportionals; by Compoſition alſo A+B.B :: B+C. C will be Proportionals; and by Permutation A+B. B+C :: B. C will alſo be Proportionals; and again by Compoſition A+2 B+C. B+C :: B +C. C; which was to be proved.

20 In four Continual Proportionals, the greateſt and the leaſt put together, is a greater quantity then the other two put together. Let A.B :: C.D be Continual Proportionals; whereof let the greateſt be A, and the leaſt be D; I ſay A+D is greater then B+C. For (by the 10 *Art.*) A—B.C—D :: A.C are Proportionals; and therefore A—B is (by the 11 *Art.*) greater then C—D. Add B on both ſides, and A will be greater then C+B—D. And again, add D on both ſides, and A+D will be greater then B+C; which was to be proved.

21 If there be four Proportionals, the Extremes multiplyed into one another, & the Means multiplied into one another wil make equal Products. Let A.B :: C.D be Proportionals; I ſay A D is equal to B C. For the Proportion of A D to B C is compounded (by the 13 *Art.*) of the Proportions of A to B, and D to C, that is, its Inverſe B to A; and therefore (by the 14 *Art*) this Compounded Proportion is the Proportion of Equality; and therefore alſo the Proportion of A D to B C is the Proportion of Equality. Wherefore they are equal.

22 If there be four quantities, and the Proportion of the firſt to the ſecond be Duplicate of the Proportion of the third to the fourth, the Product of the Extremes to the Product of the Means will be as the third to the fourth. Let the four Quantities be A, B C and D; and let the Proportion of A to B be Duplicate of the Proportion of C to D; I ſay A D, that is, the Product of A into D, is to BC, that is, to the Product of the Means, as C to D. For ſeeing the Proportion of A to B is Duplicate of the proportion of C to D, if it be as C to D, ſo D to another E, then A.B :: C.E will be Pro-

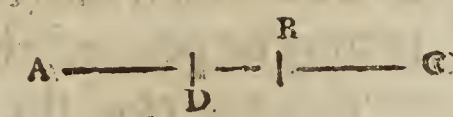
por-

portionals; for the proportion of A to B is by ſuppoſition duplicate of the Proportion of C to D; and C to E duplicate alſo of that of C to D by the Definition *(15 Art.)* Wherefore (by the laſt *Article)* A E or A into E, is equal to B C or B into C; But (by the 4 *Coroll.* of the 6 *Art.)* A D is to A E as D to E, that is, as C to D; and therefore A D is to B C (which as I have ſhewn, is equal to A E) as C to D; which was to be proved.

Moreover, If the proportion of the firſt A, to the ſecond B be triplicate of the proportion of the third C to the fourth D, the Product of the Extremes to the product of the Means will be duplicate of the Proportion of the third to the fourth. For if it be as C to D ſo D to E, and again, as D to E ſo E to another F, then the proportion of C to F will be triplicate of the proportion of C to D; and conſequently A. B : : C. F will be proportionals, and A F equal to B C. But as A D to A F, ſo is D to F; and therefore alſo as A D to B C, ſo D to F, that is, ſo C to E; But the proportion of C to E is duplicate of the proportion of C to D; Wherefore alſo the proportion of A D to B C is duplicate of that of C to D, as was propounded.

23 If there be four Proportionals, and a Mean be interpoſed betwixt the firſt and ſecond, and another betwixt the third and fourth, the firſt of theſe Means will be to the ſecond, as the firſt of the Proportionals is to the third, or as the ſecond of them is to the fourth. For let A. B : : C. D be Proportionals, and let E be a Mean betwixt A and B, & F a Mean betwixt C and D; I ſay A. C : : E. F are Proportionals. For the proportion of A to E is Subduplicate of the proportion of A to B, or of C to D. Alſo the proportion of C to F is Subduplicate of that of C to D; and therefore A. E : : C. F are Proportionals; and by Permutation A. C : : E. F are alſo Proportionals; which was to be proved.

24 Any thing is ſaid to be divided into *Extreme* and *Mean Proportion*, when the Whole and the Parts are in Continual Proportion. As (for example) when A + B. A. B ∴ are Continual Proportionals; or when the ſtraight line A C is ſo divided in B, that A C. A B. B C ∴ are in Continual Proportion. And if the ſame Line A C be again divided in D, ſo as that A C. C D. A D ∴ be continual Proportionals; then alſo A C. A B. A D ∴ will be continual Proportionals; and in like manner, though in



con-

contrary order, CA.CD.CB ∴ will be continual Proportionals; which cannot happen in any Line otherwiſe divided.

25 If there be three continual Proportionals, and again, three other continual Proportionals which have the ſame Middle Term, their Extremes will be in reciprocal Proportion. For let A. B.C ∴ and D. B. E ∴ be continal Proportionals, I ſay A.D : : E. C ſhall be Proportionals. For the Proportion of A to D is compounded of the Proportions of A to B, and of B to D; and the Proportion of E to C is compounded of thoſe of E to B, that is, of B to A, and of B to C, that is, of A to B. Wherefore (by Equality) A.D : : E.C. are Proportionals.

26 If any two unequal quantities be made Extremes, and there be interpoſed betwixt them any Number of Means in Geometrical Proportion, and the ſame Number of Means in Arithmetical Proportiõ, the ſeveral Means in Geometrical Proportion wil be leſs then the ſeveral Means in Arithmetical proportion. For betwixt A the leſſer, & E the greater Extreme, let there be interpoſed three Means B,A,D in Geometrical Proportion, & as many more F,G,H in Arithmetical Proportion; I ſay B will be leſs then F, A then G, & D then H. For firſt, the difference between A & F is the ſame with that betwcen F &G,& with that between G &H (by the Definition of Arithmetical Proportion;) & therefore the difference of the pro portionals which ſtand next to one another, to the difference of the Extremes, is, when there is but one Mean, half their difference; when two, a 3d part of it; when three, a quarter, &c. ſo that in this example it is a quarter. But the difference between D and E (by the 17. *Art.*) is more then a quarter of the difference between the Extremes, becauſe the Proportion is Geometrical; & therefore the difference between A and D is leſs then 3 quarters of the ſame difference of the Extremes. In like manner, if the difference betweenA&D be underſtood to be divided into three equall parts,

A	A
B	F
C	G
D	H
E	E

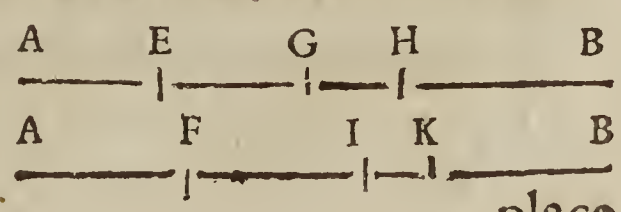
it may be proved, that the difference between A and C is leſs then two quarters of the difference of the Extremes A & E. And laſtly, if the difference betwe[illegible] A and C be divided into two equal parts, that the difference between A and B is leſs then a quarter of the difference of the Extremes A and E.

From

From the conſideration hereof it is manifeſt, that B, that is, A together with ſomething elſe which is leſs then a fourth part of the difference of the Extremes A and E, is leſs then F, that is, then the ſame A with ſomething elſe which is equal to the ſaid fourth part. Alſo, that C, that is A with ſomething elſe which is leſs then two fourth parts of the ſaid difference, is leſs then G, that is, then A together with the ſaid two fourths. And laſtly, that D which exceeds A by leſs then three fourths of the ſaid difference, is leſs then H, which exceeds the ſame A by three entire fourths of the ſaid difference. And in the ſame manner it would be if there were four Means, ſaving that in ſtead of fourths of the difference of the Extremes, we are to take fifth parts; and ſo on.

27 *Lemma.* If a quantity being given, firſt one quantity be both added to it and ſubſtracted from it, and then another greater or leſſe, the proportion of the Remainder to the Aggregate, is greater where the leſs quantity is added and ſubſtracted, then where the greater quantity is added and ſubſtracted. Let B be added to, and ſubſtracted from the quantity A; ſo that A—B be the Remainder, and A+B the Aggregate; And again let C, a greater quantity then B be added to, and ſubſtracted from the ſame A, ſo that A—C be the Remainder and A+C the Aggregate; I ſay A—B. A+B :: A—C. A+C will be an Hyperlogiſme. For A—B. A :: A—C. A is an Hyperlogiſme of a leſs Antecedent to the ſame Conſequent; and therefore A—B. A+B :: A—C. A+C is a much greater Hyperlogiſme, being made of a leſs Antecedent to a greater Conſequent.

28 If unequal parts be taken from two equal quantities, and betwixt the whole and the part of each there be interpoſed two Means, One in Geometrical, the other in Arithmetical proportion; The Difference betwixt the two Means will be greateſt, where the Difference betwixt the Whole and its part is greateſt. For let A B & A B be two equal quantities; from which let two unequal parts be taken, namely, A E the leſs, and A F the greater; and betwixt A B and A E let A G be a Mean in Geometrical Proportion, and A H, a Mean in Arithmetical Proportion. Alſo betwixt A B and A F let A I be a Mean in Geometrical Porportion, and A K a Mean in Arithmetical Proportion; I ſay H G is greater then K I. For in the firſt



place

place we have this Analogiſme AB. AG :: BG. GE (by the 18 *Art.)*

Then by Compoſition we have this AB+AG. AB :: BG+GE (that is BE). BG.

And by taking the halves of the Antecedents this third $\frac{1}{2}$AB+$\frac{1}{2}$AG. AB :: $\frac{1}{2}$BG+$\frac{1}{2}$GE (that is BH). BG.

And by Converſion a fourth AB. $\frac{1}{2}$AB+$\frac{1}{2}$AG :: BG. BH

And by Diviſion this fifth $\frac{1}{2}$AB—$\frac{1}{2}$AG. $\frac{1}{2}$AB+$\frac{1}{2}$AG :: HG. BH.

And by doubling the firſt Antecedent and the firſt Conſequent } AB—AG. AB+AG :: HG. BH.

Alſo by the ſame method may be found out this Analogiſme } AB—AI. AB+AI :: KI. BK.

Now ſeeing the proportion of AB to AE is greater then that of AB to AF, the proportion of AB to AG, which is half the greater proportion, is greater then the proportion of AB to AI the half of the leſs Proportion; and therefore AI is greater then AG. Wherefore the proportion of AB—AG to AB+AG (by the precedent *Lemma)* will be greater then the proportion of AB—AI to AB+AI; & therefore alſo the proportion of HG to BH will be greater then that of KI to BK, and much greater then the proportion of KI to BH, which is greater then BK, (for BH is the half of BE, as BK is the half of BF, which (by ſuppoſition) is leſs leſs then BE). Wherefore HG is greater then KI; which was to be proved.

Coroll. It is manifeſt from hence, that if any quantity be ſuppoſed to be divided into equal parts infinite in number, the difference between the Arithmetical and Geometrical Means will be infinitely little, that is, none at all. And upon this foundation chiefly, the Art of making thoſe Numbers which are called *Logarithmes* ſeems to have been built.

29 If any number of quantities be propounded, whether they be unequall, or equall to one another; and there be another quantity which multiplied by the number of the propounded quantities is equall to them all; that other Quantity is a mean in Arithmeticall Proportion to all thoſe propounded quantities.

CHAP.

CHAP. XIV.
Of Straight *and* Crooked, Angle *and* Figure.

1 *The Definition and Properties of a Straight Line* 2 *The Definition and Properties of a Plain Superficies.* 3 *Several sorts of Crooked Lines.* 4 *The Definition and Properties of a Circular Line.* 5 *The Properties of a Straight Line taken in a Plain.* 6 *The Definition of Tangent Lines.* 7 *The Definition of an Angle, and the kindes thereof.* 8 *In Concentrick Circles, Arches of the same Angle are to one another as the whole Circumferences are.* 9 *The Quantity of an Angle in what it consists.* 10 *The Distinction of Angles simply so called.* 11 *Of Straight Lines from the Center of a Circle to a Tangent of the same.* 12 *The general Definition of Parallels; and the Properties of Straight Parallels.* 13 *The Circumferences of Circles are to one another as their Diameters are.* 14 *In Triangles, Straight Lines parallel to the Bases, are to one another, as the parts of the Sides which they cut off from the Vertex.* 15 *By what Fraction of a Straight Line the Circumference of a Circle is made.* 16 *That an Angle of Contingence is Quantity, but of a Different kinde from that of an Angle simply so called; and that it can neither add nor take away any thing from the same.* 17 *That the Inclination of Plains is Angle simply so called.* 18 *A Solid Angle what it is.* 19 *What is the Nature of Asymptotes.* 20 *Situation, by what it is determined.* 21 *What is like Situation; What is Figure; and what are like Figures.*

1 BEtween two points given, the shorteſt Line is that, whoſe extreme points cannot be drawn further aſunder, withour altering the quantity, that is, without altering the proportion of that line to any other line given. For the Magnitude of a Line is computed by the greateſt diſtance which may be between its extreme points; So that any one Line, whether it be extended, or bowed, has alwayes one and the ſame Length, becauſe it can have but one greateſt diſtance between its extreme points.

And seeing the action by which a Straight Line is made Crooked, or contrarily a Crooked Line is made Straight, is nothing but the bringing of its extreme points neerer to one another, or the setting of them further asunder; a CROOKED *Line* may rightly be defined to be *That, whose extreme points may be understood to be drawn further asunder*; and a STRAIGHT *Line* to be *That, whose extreme points cannot be drawn further asunder*; *and comparatively, A more Crooked, to be That line whose extreme points are neerer to one another then those of the other, (supposing both the Lines to be of equal Length.)* Now howsoever a Line be bowed, it makes alwayes a *Sinus* or Cavity, sometimes on one side, sometimes on another; So that the same Crooked Line may either have its whole Cavity on one Side onely, or it may have it part on one side and part on other sides. Which being well understood, it will be easie to understand the following Comparisons of Straight and Crooked *Lines*.

First, If a Straight & a Crooked Line have their Extreme points common, the Crooked Line is longer then the Straight Line. For if the extreme points of the Crooked Line be drawn out to their greatest distance, it wil be made a straight line, of which that which was a Straight Line from the beginning will be but a part; and therefore the Straight Line was shorter then the Crooked Line which had the same extreme points. And for the same reason, if two Crooked Lines have their extreme points common, and both of them have all their cavity on one and the same side, the outermost of the two will be the longest Line.

Secondly, A Straight Line and a perpetually Crooked Line cānot be coincident, no not in the least part. For if they should, then not onely some Straight Line would have its extreme points common with some Crooked Line, but also they would by reason of their coincidence, be equal to one another; which, as I have newly shewn, cannot be.

Thirdly, Between two points given there can be understood but one straight Line; because there cannot be more then one least Interval or Length between the same points. For if there may be two, they will either be coincident, and so both of them will be one Straight Line; or if they be not coincident, then the application of one to the other by extension, will make the extended Line

have

have its extreme points at greater distance then the other; and consequently it was Crooked from the beginning.

Fourthly, From this last it follows, that two Straight Lines cannot include a Superficies. For if they have both their extreme points common, they are coincident; and if they have but one, or neither of them common, then at one, or both ends, the extreme points will be disjoyned, and include no Superficies, but leave all open and undetermined.

Fifthly, Every part of a Straight Line is a Straight Line. For seeing every part of a Straight Line is the least that can be drawn between its own extreme points, if all the parts should not constitute a Straight Line, they would all together be longer then the whole Line.

2 A PLAIN, or a *Plain Superficies, is that which is described by a Straight Line so moved that all the several points thereof describe several Straight Lines.* A straight line therefore is necessarily all of it in the same Plain which it describes. Also the Straight Lines which are made by the points that describe a Plain, are all of them in the Same Plain. Moreover, if any Line whatsoever be moved in a Plain, the Lines which are described by it are all of them in the same Plain.

All other *Superficies* which are not Plain, are Crooked, that is, are either *Concave* or *Convex*. And the same Comparisons which were made of Straight and Crooked Lines: may also be made of Plain and Crooked Superficies. For

First, If a Plain and a Crooked Superficies be terminated with the same Lines, the Crooked Superficies is greater then the Plain Superficies. For if the Lines of which the Crooked Superficies consists be extended, they will be found to be longer then those of which the Plain Superficies consists, which cannot be extended because they are Straight.

Secondly, Two Superficies, wherof the one is Plain, and the other continually Crooked, cannot be coincident, no not in the least part. For if they were coincident they would be equal; nay, the same Superficies would be both Plain and Crooked, which is impossible.

Thirdly, Within the same terminating Lines, there can be no more then one Plain Superficies; because there can be but one least Superficies within the same.

Fourthly, No number of Plain Superficies can include a Solid, unleſs more then two of them end in a Common *Vertex*. For if two Plains have both the ſame terminating Lines, they are coincident, that is, they are but one Superficies; and if their terminating Lines be not the ſame, they leave one or more ſides open.

Fifthly, Every part of a Plain Superficies is a Plain Superficies. For ſeeing the whole Plain Superficies is the leaſt of all thoſe that have the ſame terminating Lines, and alſo every part of the ſame Superficies is the leaſt of all thoſe that are terminated with the ſame Lines; if every part ſhould not conſtitute a Plain Superficies, all the parts put together would not be equal to the whole.

3 Of *Straightneſs*, whether it be in Lines, or in Superficies there is but one kinde; but of *Crookedneſs* there are many kindes; for of Crooked Magnitudes, ſome are *Congruous*, that is, are coincident when they are applyed to one another; others are *Incongruous*. Again, ſome are ὁμοιομερεῖς, or *Uniform*, that is, have their parts howſoever taken, congruous to one another; others are ἀνομοιομερεῖς or of *ſeveral Forms*. Moreover, of ſuch as are Crooked, ſome are *Continually Crooked*, others *have parts which are not Crooked*.

4 If a Straight Line be moved in a Plain, in ſuch manner, that while one end of it ſtands ſtill, the whole Line be carried round about, til it come again into the ſame place from whence it was firſt moved, it will deſcribe a plain Superficies, which will be terminated every way by that Crooked Line which is made by that end of the Straight Line which was carried round. Now this Superficies is called a CIRCLE; and of this Circle, the Unmoved Point, is the the *Center*; the Crooked Line which terminates it, the *Perimeter*; and every part of that Crooked Line, a *Circumference* or *Arch*; the ſtraight Line which generated the *Circle*, is the *Semidiameter* or *Radius*; and any ſtraight Line which paſſeth through the Center, and is terminated on both ſides in the Circumference is called the *Diameter*. Moreover, every point of the *Radius* which deſcribes the Circle, deſcribes in the ſame time its own Perimeter, terminating its own Circle, which is ſaid to be *Concentrick* to all the other Circles, becauſe this and all thoſe have one common Center.

Wherefore in every Circle, all Straight Lines from the Center

ter to the Circumference are equal. For they are all coincident with the Radius which generates the Circle.

Alſo the Diameter divides both the Perimeter and the Circle it ſelf into two equal parts. For if thoſe two parts be applyed to one another, and the Semiperimeters be coincident, then ſeeing they have one common Diameter, they will be equal; and the Semicircles will be equal alſo; for theſe alſo will be coincident. But if the Semiperimeters be not coincident, then ſome one ſtraight Line which paſſes through the Center (which Center is in the Diameter) will be cut by them in two points. Wherefore, ſeeing all the ſtraight Lines from the Center to the Circumference are equal, a part of the ſame ſtraight Line will be equal to the whole; which is impoſſible.

For the ſame reaſon the Perimeter of a Circle will be Uniform, that is, any one part of it will be coincident with any other equal part of the ſame.

5 From hence may be collected this property of a Straight Line, namely, that it is all conteined in that Plain which conteins both its extreme points. For ſeeing both its extreme points are in the Plain, that Straight Line which deſcribes the Plain will paſs through them both; and if one of them be made a Center, and at the diſtance between both, a Circumference be deſcribed, whoſe Radius is the Straight Line which deſcribes the Plain, that Circumference will paſs through the other point. Wherefore between the two propounded points, there is one ſtraight line (by the Definition of a Circle) conteined wholly in the propounded Plain; and therefore if another ſtraight Line might be drawn between the ſame points, and yet not be conteined in the ſame Plain, it would follow, that between two points two ſtraight lines may be drawn; which has been demonſtrated to be impoſſible.

It may alſo be collected, That if two Plains cut one another, their common ſection will be a ſtraight Line. For the two extreme points of the interſection are in both the interſecting Plains; and between thoſe points a ſtraight Line may be drawn; but a ſtraight Line between any two points is in the ſame Plain in which the Points are; and ſeeing theſe are in both the Plains, the ſtraight line which connects them will alſo be in both the ſame Plains, and therefore it is the cõmon ſection of both. And every other Line that

can

can be drawn between thoſe points will be either coincident with that Line; that is, it will be the Same Line; or it will not be coincident; and then it wil be in neither, or but in one of thoſe Plains.

As a ſtraight Line may be underſtood to be moved round about whileſt one end thereof remains fixed, as the Center; ſo in like manner it is eaſie to underſtand, that a Plain may be circumduced about a ſtraight line, whileſt the ſtraight line remaines ſtill in one and the ſame place, as the *Axis* of that motion. Now from hence it is manifeſt, that any three Points are in ſome one Plain. For as any two Points, if they be connected by a ſtraight Line, are underſtood to be in the ſame Plaine in which the ſtraight Line is; ſo, if that Plaine be circumduced about the ſame ſtraight Line, it will in its revolution take in any third Point, howſoever it be ſituate; and then the three Points will be all in that Plaine; and conſequently the three ſtraight Lines which connect thoſe Points, will alſo be in the ſame Plain.

6 Two Lines are ſaid to *Touch* one another, which being both drawne to one and the ſame point, will not cut one another, though they be produced, produced I ſay in the ſame manner in which they were generated. And therefore if two ſtraight Lines touch one another in any one point, they wil be contiguous through their whole length. Alſo two Lines continually crooked wil do the ſame, if they be congruous, and be applyed to one another according to their congruity; otherwiſe, if they be incongruouſly applyed, they will, as all other crooked Lines, touch one another (where they touch) but in one point onely. Which is manifeſt from this, that there can be no congruity between a ſtraight line and a line that is continually crooked; for otherwiſe the ſame line might be both ſtraight and crooked. Beſides when a ſtraight line touches a crooked line, if the ſtraight line be never ſo little moved about upon the point of contact, it will cut the crooked line; for ſeeing it touches it but in one point, if it incline any way, it will do more then touch it; that is, it will either be congruous to it, or it will cut it; but it cannot be congruous to it; and therefore it will cut it.

7 An Angle, according to the moſt general acception of the word, may be thus defined; *When two Lines, or many Superficies concurre in one ſole point, and diverge every where elſe, the quantity of*

that

that divergence is an ANGLE. And an Angle is of two ſorts; for firſt it may be made by the concurrence of Lines, and then it is a *Superficiall Angle*; or by the concurrence of Superficies, and then it is called a *Solid Angle*.

Again, from the two wayes by which two lines may diverge from one another, Superficial Angles are divided into two kindes. For two ſtraight lines which are applyed to one another, and are contiguous in their whole length, may be ſeparated or pulled open in ſuch manner, that their concurrence in one point will ſtill remain; And this Separation or Opening may be either by Circular Motion, the Center whereof is their point of concurrence, and the Lines will ſtill retein their ſtraightneſs, the quantity of which Separation, or Divergence is an *Angle* ſimply ſo called; Or they may be ſeparated by continual Flexion or Curvation in every imaginable point; and the quantity of this Separation is that which is called an *Angle of Contingence*.

Beſides of Superficial Angles ſimply ſo called, thoſe which are in a plain Superficies are Plain; and thoſe which are not plain, are denominated from the Superficies in which they are-

Laſtly, thoſe are *Straight-lined Angles* which are made by ſtraight lines; as thoſe which are made by crookcd lines are *Crooked-lined*; and thoſe which are made both of ſtraight and crookcd lines, are *Mixed Angles*.

8 Two Arches intercepted between two *Radii* of Concentrick Circles, have the ſame proportion to one another, which their whole Perimeters have to one another. For let the point A (in the firſt Figure) be the Center of the two Circles B C D and E F G; in which the *Radii* A E B and A F C intercept the Arches B C and E F; I ſay the proportion of the Arch B C to the Arch E F, is the ſame with that of the Perimet B C D to the Perimerer er E F G. For if the Radius A F C be underſtood to be moved about the Center A with Circular & Uniform Motion, that is, with equal Swiftneſs every where, the point C will in a certain time deſcribe the Perimeter B C D, and in a part of that time the Arch B C; and becauſe the Velocities are equal by which both the Arch and the whole Perimeter are deſcribed, the proportion of the magnitude of the Perimeter B C D to the magnitude of the Arch B C, is determined by no-

thin

thing but the difference of the times in which the Perimeter and the Arch are deſcribed. But both the Perimeters are deſcribed in one and the ſame time, and both the Arches in one and the ſame time; and therefore the proportions of the Perimeter B C D to the ArchB C , and of the Perimeter E F G to the Arch E F , are both determined by the ſame cauſe. Wherefore B C D. B C : : E F G. E F are Proportionals (by the *6 Art.* of the laſt Chapter), and by Permutation B C D. E F G : : B C. E F will alſo be Proportionals, which was to be demonſtrated.

9 Nothing is contributed towards the Quantity of an Angle, neither by the Length, nor by the Equality , nor by the Inequality of the lines which comprehend it. For the lines A B and A C comprehend the ſame Angle which is comprehended by the lines A E and A F, or A B and A F. Nor is an Angle either increaſed or diminiſhed by the abſolute quantity of the Arch which ſubtends the ſame; for both the greater Arch B C, and the leſſer Arch E F are ſubtended to the ſame Angle. But the quantity of an Angle is eſtimated by the quantity of the ſubtending Arch compared with the quantity of the whole Perimeter. And therefore the Quantity of an Angle ſimply ſo called may be thus defined.

The Quantity of an Angle, is an Arch or Circumference of a Circle , determined by its proportion to the whole Perimeter. So that when an Arch is intercepted between two ſtraight lines drawn from the Center, look how great a portion that Arch is of the whole Perimeter, ſo great is the Angle. From whence it may be underſtood, that when the lines which contein an Angle are ſtraight lines, the quantity of that Angle may be taken at any diſtance from the Center. But if one, or both of the conteining lines be crooked , then the quantity of the Angle is to be taken in the leaſt diſtance from the Center, or from their concurrence ; for the leaſt diſtance is to be conſidered as a ſtraight line , ſeeing no crooked line can be imagined ſo little, but that there may be a leſs ſtraight line. And although the leaſt ſtraight line cannot be given, becauſe the leaſt given line may ſtill be divided, yet we may come to a part ſo ſmall, as is not at all conſiderable; which we call a Point. And this point may be underſtood to be in a ſtraight line which touches a crooked line ; for an Angle is generated by ſeparating by circular motion one ſtraight line from another which touches it, (as has been ſaid above in the

7th

7th *Article).* Wherefore an Angle which two crooked lines make is the ſame with that which is made by two ſtraight lines which touch them.

10 From hence it follows, that *Vertical Angles*, ſuch as are ABC, D B F (in the ſecond Figure) are equal to one another. For if from the two Semiperimeters D A C, F D A which are equal to one another, the common Arch D A be taken away, the remayning Arches A C, D F will be equal to one another.

Another diſtinction of Angles is, into *Right* and *Oblique*. *A Right Angle is that, whoſe quantity is the fourth part of the Perimeter.* And the lines which make a Right Angle are ſaid to be *Perpendicular* to one another. Alſo of Oblique Angles, that which is greater then a Right, is called an *Obtuſe Angle*; and that which is leſs, an *Acute Angle*. From whence it follows, that all the Angles that can poſſibly be made at one and the ſame Point, together taken, are equal to four Right Angles; becauſe the quantities of them all put together make the whole Perimeter. Alſo, that all the Angles which are made on one ſide of a ſtraight line from any one point taken in the ſame, are equal to two Right Angles; for if that point be made the Center, that ſtraight line will be the Diameter of a Circle, by whoſe Circumference the quantity of an Angle is determined; and that Diameter will divide the Perimeter into two equal parts.

11 If a Tangent be made the Diameter of a Circle whoſe Center is the point of Contact, a ſtraight line drawn from the Center of the former Circle to the Center of the later Circle, will make two Angles with the Tangent (that is, with the Diameter of the later Circle) equal to two Right Angles (by the laſt *Article).* And becauſe (by the 6th *Article)* the Tangent has on both ſides equal inclination to the Circle, each of them will be a Right Angle; as alſo the Semidiameter will be perpendicular to the ſame Tangent. Moreover, the Semidiameter, in as much as it is the Semidiameter, is the leaſt ſtraight line which can be drawn fro the Center to the Tangent; and every other ſtraight line that reaches the Tangent will paſs out of the Circle, and will therefore be greater then the Semidiameter. In like manner, of all the ſtraight lines which may be drawn from the Center to the Tangent, that is the greateſt which makes the greateſt Angle with the Perpenpicular; which

will be manifeſt, if about the ſame Center another Circle be deſcribed, whoſe Semidiameter is a ſtraight line taken neerer to the Perpendicular, and there be drawn a Perpendicular (that is, a Tangent) to the ſame.

From whence it is alſo manifeſt, that if two ſtraight lines which make equal Angles on either ſide of the Perpendicular, be produced to the Tangent, they will be equal.

12 There is in *Euclide* a Definition of Straight-lined Parallels; but I do not find that Parallels in general, are any where defined; and therefore for an Univerſal definition of them, I ſay, that *Any two lines whatſoever (Straight or Crooked) as alſo any two Superficies, are* PARALLEL, *when two equal ſtraight lines whereſoever they fall upon them, make always equal Angles with each of them.* From which Definition it follows.

Firſt, that any two ſtraight lines not enclined oppoſite wayes, falling upon two other ſtraight lines which are Parallel, and intercepting equal parts in both of them, are themſelves alſo equal and Parallel. As if A B and C D (in the third Figure) enclined both the ſame way (fall upon the Parallels A C and B D, and A C and B D be equal, A B and C D will alſo be equal and Parallel. For the Perpendiculars B E and D F being drawn, the Right Angles E B D and F D H will be equal. Wherefore ſeeing E F and B D are parallel, the angles E B A and F D C will be equal. Now if D C be not equal to B A, let any other ſtraight line equal to B A be drawn from the point D; which ſeeing it cannot fall upon the point C, let it fall upon G. Whereore A G will be either greater or leſs then B D; and therefore the angles E B A and F D G are not equal, as was ſuppoſed. Wherefore A B and C D are equal; which is the firſt.

Again, becauſe they make equal Angles with the Perpendiculars B E and D F; therefore the Angle C D H will be equal to the Angle A B D, and (by the Definition of Parallels) A B and C D will be parallel; which is the ſecond.

That Plain which is included both wayes within parallel lines is called a PARALLELOGRAM.

1 *Corollary.* From this laſt it follows, That the Angles A B D and C D H are equal; that is, that a ſtraight line (as B H) falling upon two Parallels (as A B and C D) makes the internal Angle A B D equal to the external and oppoſite Angle C D H. 2 *Co-*

2 *Coroll.* And from hence again it follows, that a ſtraight line falling upon two Parallels, makes the alternate Angles equal; that is, the Angle A G F (in the fourth figure) equal to the Angle G F D. For ſeeing G F D is equal to the external oppoſite Angle E G B, it will be alſo equal to its vertical Angle A G F, which is alternate to G F D.

3 *Coroll.* That the internal Angles on the ſame ſide of the line F G are equal to two Right Angles. For the Angles at F, namely G F C and G F D are equal to two Right Angles. But G F D is equal to its alternate Angle A G F. Wherefore both the Angles G F C and A G F which are internal on the ſame ſide of the line F G, are equal to two Right Angles.

4 *Coroll.* That the three Angles of a ſtraight-lined plain Triangle are equal to two Right Angles; and any ſide being produced, the external Angle will be equal to the two oppoſite internal Angles. For if there be drawn by the Vertex of the plain Triangle A B C (figure 5.) a Parallel to any of the ſides, as to A B, the Angles A and B will be equal to their alternate Angles E & F, & the Angle C is common. But (by the 10th *Article*) the three Angles E, C and F are equal to two Right Angles; and therefore the three Angles of the Triangle are equal to the ſame; which is the firſt. Again, the two Angles B and D are equal to two Right Angles (by the 10th *Article*). Wherefore taking away B, there will remain the Angles A and C equal to the Angle D; which is the ſecond.

5 *Coroll.* If the Angles A and B be equal, the ſides A C and C B will alſo be equal, becauſe A B and E F are parallel; And on the contrary, if the ſides A C and C B be equal, the Angles A and B will alſo be equal. For if they be not equal, let the Angles B and G be equal. Wherefore ſeeing G B and E F are parallels, and the Angles G and B equal, the ſides G C and C B will alſo be equal; and becauſe C B and A C are equal by ſuppoſition, C G and C A will alſo be equal; which cannot be (by the 11th *Article.*)

6 *Coroll.* From hence it is manifeſt, that if two *Radii* of a Circle be connected by a ſtraight Line, the Angles they make with that connecting Line will be equal to one another; and if there be added that ſegment of the Circle which is ſubtended by the ſame line which connects the *Radii*, then the Angles which thoſe *Radii* make with the circumference wil alſo be equal to one another. For

a ſtraight line which ſubtends any Arch, makes equal Angles with the ſame, becauſe if the Arch and the Subtenſe be divided in the middle, the two halves of the ſegment wil be congruous to one another, by reaſon of the Uniformity both of the Circumference of the Circle and of the ſtraight Line.

13 Perimeters of Circles are to one another as their Semidiameters are. For let there be any two Circles, as (in the firſt figure) B C D the greater, and E F G the leſſer, having their common Center at A; and let their Semidiameters be A C and A E. I ſay A C has the ſame proportion to A E which the Perimeter B C D has to the Perimeter E F G. For the magnitude of the Semidiameters A C and A E is determined by the diſtances of the points C and E from the Center A; and the ſame diſtances are acquired by the uniform motion of a point from A to C in ſuch manner that in equal times the diſtances acquired be equal. But the Perimeters B C D and E F G are alſo determined by the ſame diſtances of the points C and E from the Center A; and therefore the Perimeters B C D and E F G, as well as the Semidiameters A C and A E have their magnitudes determined by the ſame cauſe, which cauſe makes in equal times equal ſpaces. Wherefore (by the 13 Chapter and 6th *Article*) the Perimeters of Circles and their Semidiameters are Proportionals; which was to be proved.

14 If two ſtraight Lines w^ch cõſtitute an Angle, be cut by ſtraight-lined Parallels, the intercepted Parallels will be to one another, as the parts w^ch they cut off frõ the Vertex. Let the ſtraight lines A B and A C (in the 6 figure) make an Angle at A, & be cut by the two ſtraight-lined Parallels B C and D E, ſo that the parts cut off from the Vertex in either of thoſe Lines (as in A B) may be A B and A D. I ſay the Parallels B C and D E are to one another as the parts A B and A D. For let A B be divided into any number of equal parts, as into A F, F D, D B; and by the points F and D, let F G and D E be drawn Parallel to the baſe B C, and cut A C in G and E; and again by the points G and E let other ſtraight lines be drawn Parallel to A B, and cut B C in H and I. If now the point A be underſtood to be moved uniformly over A B, and in the ſame time B be moved to C, and all the points F, D and B be moved uniformly and with equal Swiftneſs over F G, D E, and B C; then ſhall B paſs over B H (equal to F G) in the ſame time that A paſſes over A F; and A F and F G will be to one another as their Velocities

locities are; and when A is in F, D will be in K; when A is in D, D will be in E; and in what manner the point A passes by the points F, D and B, in the same manner the point B will pass by the points H, I and C; & the straight lines F G, D K, K E, B H, H I & I C are equal by reason of their Parallelisme; and therefore, as the velocitie in A B is to the velocity in B C, so is A D to D E; but as the velocity in A B is to the velocity in B C, so is A B to B C; that is to say, all the Parallels will be severally to all the parts cut off from the Vertex, as A F is to F G. Wherefore, A F. G F: : A D. D E: : A B. B C are Proportionals.

The Subtenses of equal Angles in different Circles (as the straight lines B C and F E (in the 1 figure) are to one another as the Arches which they subtend. For (by the 8th *Article*) the Arches of equal Angles are to one another as their Perimeters are; and (by the 13th *Art.*) the Perimeters as their Semidiameters; But the the Subtenses B C and F E are parallel to one another by reason of the equality of the Angles which they make with the Semidiameters; and therefore the same Subtenses (by the last precedent *Article*) will be proportional to the Semidiameters, that is, to the Perimeters, that is, to the Arches which they subtend.

15 If in a Circle any number of equal Subtenses be placed immedia ly after one another, and straight lines be drawn from the extreme point of the first Subtense to the extreme points of all the rest, The first Subtense being produced will make with the second Subtense an external Angle double to that which is made by the same first Subtense and a Tangent to the Circle touching it in the extreme point thereof; and if a straight line which subtends two of those Arches be produced, it will make an external Angle with the third Subtense, triple to the Angle which is made by the Tangent with the first Subtense; and so continually. For with the Radius A B (in the 7th figure) let a circle be described, & in it let any number of equal Subtenses B C, C D & D E be placed; also let B D & B E be drawn; & by producing B C, B D & B E to any distance in G, H and I, let them make Angles with the Subtenses which succeed one another, namely the external Angles G C D, and H D E. Lastly, let the Tangent K B be drawn, making with the first Subtense the Angle K B C. I say the Angle G C D is double to the Angle K B C, and the Angle H D E triple to the same Angle K B C. For if A C be drawn cutting B D in M, and from the point C there

there be drawn L C perpendicular to the ſame A C, then C L and M D will be parallel by reaſon of the right Angles at C and M; and therefore the alterne Angles L C D and B D C wil be equal; as alſo the Angles B D C and C B D will be equal becauſe of the equality of the ſtraight lines B C and C D. Wherefore the Angle G C D is double to either of the Angles C B D or C D B; and therefore alſo the Angle G C D is double to the Angle L C D, that is, to the Angle K B C. Again, C D is parallel to B E by reaſon of the equality of the Angles C B E and D E B, and of the ſtraight lines C B and D E; and therefore the Angles G C D and G B E are equal; and conſequently G B E, as alſo D E B is double to the Angle K B C. But the external Angle H D E is equal to the two internal D E B and D B E; and therefore the Angle H D E is triple to the Angle K B C, &c. which was to be proved.

1 *Corollary.* From hence it is manifeſt, that the Angles K B C and C B D, as alſo, that all the Angles that are comprehended by two ſtraight lines meeting in the circumference of a Circle and inſiſting upon equal Arches, are equal to one another.

2 *Coroll.* If the Tangent B K be moved in the Circumference with Uniform motion about the Center B, it will in equal times cut off equal Arches; and will paſs over the whole Perimeter in the ſame time in which it ſelf deſcribes a ſemiperimeter about the Center B.

3 *Coroll.* From hence alſo we may underſtand what it is that determines the bending or *Curvation* of a ſtraight line into the circumference of a Circle; namely, that it is *Fraction* continually encreaſing in the ſame manner as Nūbers from *One* upwards encreaſe by the continual addition of *Unity*. For the indefinite ſtraight Line K B being *broken* in B according to any Angle, as that of KBC, & again in C according to a double Angle, and in D according to an Angle which is triple, and in E according to an Angle which is quadruple to the firſt Angle, and ſo continually, there will be deſcribed a figure which will indeed be *rectilineal* if the broken parts be conſidered as having magnitude; but if they be underſtood to be the leaſt that can be, that is, as ſo many Points; then the figure deſcribed will not be rectilineal; but a Circle, whoſe Circumference will be the broken line.

4 *Coroll.* From what has been ſaid in this preſent *Article* it may alſo be demonſtrated, that *An Angle in the center is double to an Angle in*

in the Circumference of the ſame Circle, if the intercepted Arches be equal. For ſeeing that ſtraight Line by whoſe motion an Angle is determined, paſſes over equal Arches in equal times, as well from the Center as from the Circumference; and while that which is from the Circumference is paſſing over half its own Perimeter, it paſſes in the ſame time over the whole Perimeter of that which is from the Center, the Arches w^ch it cuts off in the Perimeter whoſe Center is A, wil be double to thoſe which it makes in its own Semiperimeter whoſe Center is B. But in equal Circles, as Arches are to one another, ſo alſo are Angles.

It may alſo be demonſtrated that the external Angle made by a Subtenſe produced and the next equal Subtenſe, is equal to an Angle from the Center inſiſting upon the ſame Arch; As in the laſt Diagram, the Angle G C D is equal to the Angle C A D; For the external Angle G C D is double to the Angle C B D; and the Angle C A D inſiſting upon the ſame Arch C D, is alſo double to the ſame Angle C B D or K B C.

16 An Angle of Contingence, if it be compared with an Angle ſimply ſo called how little ſoever, has ſuch proportion to it as a Point has to a Line; that is, no proportion at all, nor any quantity. For firſt, an Angle of cõtingence is made by cõtinual flexion, ſo that in the generation of it there is no circular motion at all, in which conſiſts the nature of an Angle ſimply ſo called; and therefore it cannot be compared with it according to Quantity. Secondly, ſeeing the external Angle made by a Subtenſe produced and the next Subtenſe, is equal to an Angle from the Center inſiſting upon the ſame Arch (as in the laſt figure the Angle G C D is equal to the Angle C A D) the Angle of Contingence wil be equal to that Angle from the Center which is made by A B and the ſame A B; for no part of a Tangent can ſubtend any Arch; but as the point of Contact is to be taken for the Subtenſe, ſo the Angle of Contingence is to be accounted for the external Angle, and equal to that Angle whoſe Arch is the ſame point B.

Now ſeeing an Angle in general is defined to be the Opening or Divergence of two lines which concurre in one ſole point; & ſeeing one Opening is greater then another, it cãnot be denied but that by the very generation of it an Angle of Contingence is Quantity; for whereſoever there is Greater and Leſs, there is alſo Quantity;

but

but this quantity conſiſts in greater and leſs Flexion; for how much the greater a Circle is, ſo much the neerer comes the Circumference of it to the nature of a ſtraight Line; for the Circumference of a Circle being made by the curvation of a ſtraight line, the leſs that ſtraight line is, the greater is the curvation; & therfore when one ſtraight line is a Tangent to many Circles, the Angle of Contingence which it makes with a leſs Circle is greater then that which it makes with a greater Circle.

Nothing therefore is added to, or taken from an Angle ſimply ſo called, by the addition to it or taking from it of never ſo many Angles of Contingence. And as an Angle of one ſort can never be equal to an Angle of the other ſort, ſo they cannot be either greater or leſs then one another.

From whence it follows, that an Angle of a Segment, that is, the Angle which any ſtraight line makes with any Arch, is equal to the Angle which is made by the ſame ſtraight line, & another which touches the Circle in the point of their Concurrence; as in the laſt figure, the Angle which is made between G B and B K is equal to that which is made between G B and the Arch B C.

17 An Angle which is made by two Plains, is commonly called the *Inclination* of thoſe Plains; And becauſe Plains have equal Inclination in all their parts, inſtead of their Inclination an Angle is taken which is made by two ſtraight lines, one of which is in one, the other in the other of thoſe Plains, but both perpendicular to the common Section.

18 A Solid Angle may be conceived two wayes. Firſt, for the aggregate of all the Angles which are made by the motion of a ſtraight line, while one extreme point thereof remayning fixed, it is carried about any plain figure in which the fixed point of the ſtraight line is not conteined. And in this ſenſe it ſeems to be underſtood by *Euclide*. Now it is manifeſt that the quantity of a Solid Angle ſo conceived is no other then the aggregate of all the Angles in a Superficies ſo deſcribed, that is, in the Superficies of a Pyramidal Solid. Secondly, when a Pyramis or Cone has its Vertex in the Center of a Sphere, a Solid Angle may be underſtood to be the proportion of a Spherical Superficies ſubtending that Vertex, to the whole Superficies of the Sphere. In which ſenſe ſolid Angles are to one another as the

the Spherical Baſes of Solids which have their Vertex in the Center of the ſame Sphere.

19 All the wayes by which two lines reſpect one another, or all the variety of their poſition may be comprehended under four kindes; For any two lines whatſoever are either *Parallels* or being produced (if need be) or moved one of them to the other parallelly to it ſelf, they make an *Angle*; or elſe (by the like production and motion) they *Touch* one another; or laſtly, they are *Aſymptotes*. The nature of Parallels, Angles and Tangents has been already declared. It remains that I ſpeak briefly of the nature of *Aſymptotes*.

Aſymptoſy depends upon this, that Quantity is infinitly diviſible. And from hence it follows, that any line being given, and a Body ſuppoſed to be moved from one extreme thereof towards the other, it is poſſible (by taking degrees of Velocity alwayes leſſe and leſſe in ſuch proportion as the parts of the Line are made leſſe by continual diviſion) that the ſame Body may be alwayes moved forwards in that Line, and yet never reach the end of it. For it is manifeſt that if any ſtraight Line, as A F (in the 8th figure) be cut any where in B, and again B F be cut in C, and C F in D, and D F in E, and ſo eternally, and there be drawn from the point F the ſtraight Line F F at any Angle A F F; and laſtly, if the ſtraight Lines A F, B F, C F, D F, E F, &c. having the ſame proportion to one another with the Segments of the Line A F, be ſet in order and parallel to the ſame A F, the crooked Line A B C D E and the ſtraight Line F F will be *Aſymptotes*, that is, they will alwayes come neerer and neerer together, but never touch one another. Now becauſe any Line may be cut eternally according to the proportions which the Segments have to one another, therefore the divers kindes of *Aſymptotes* are infinite in number, and not neceſſary to be further ſpoken of in this place. In the nature of Aſymptotes in general there is no more, then that they come ſtill neerer and neerer but never touch. But in ſpecial in the Aſymptoſie of Hyperbolique Lines, it is underſtood they ſhould approach to a diſtance leſſe then any given quantity.

20 SITUATION *is the relation of one place to another*; & where there are many places, their *Situation* is determined by four things; By

their Diſtances from one another; By *ſeveral Diſtances from a place aſsigned*; By *the order of ſtraight lines drawn from a place aſsigned to the places of them all*; and by *the Angles which are made by the lines ſo drawn*. For if their Diſtances, Order, and Angles be given, that is, be certainly known, their ſeveral places will alſo be ſo certainly known, as that they can be no other.

21 Points, how many ſoever they be, have *Like Situation* with an equal number of other Points, when all the ſtraight lines that are drawn from ſome one point to all theſe, have ſeverally the ſame proportion to thoſe that are drawn in the ſame order and at equal Angles from ſome one point to all thoſe. For let there be any number of Points as A, B and C, (in the 9 figure), to which from ſome one point D let the ſtraight Lines D A, D B and D C be drawn; and let there be an equal number of other Points as E, F and G, and from ſome point H let the ſtraight Lines H E, H F and H G be drawn, ſo that the Angles A D B and B D C be ſeverally and in the ſame order equal to the Angles E H F and F H G, and the ſtraight Lines D A, D B and D C proportional to the ſtraight Lines H E, H F and H G; I ſay the three points A, B and C, have Like Situation with the three points E, F & G, or are placed Alike. For if H E be underſtood to be layed upon D A, ſo that the point H be in D, the point F will be in the ſtraight Line D B by reaſon of the equality of the Angles A D B and E H F; and the point G will be in the ſtraight Line D C by reaſon of the equality of the Angles B D C aud F H G; and the ſtrright Lines A B and E F, as alſo B C and F G will be parallel, becauſe A D. E D :: B H. F H :: C D. G H are Proportionals by conſtruction; and therefore the diſtances between the points A and B, and the points B and C, will be proportional to the diſtances between the points E and F, and the points F and G. Wherefore in the ſituation of the points A, B and C, and the ſituation of the points E, F and G the Angles in the ſame order are equal, ſo that their ſituations differ in nothing but the inequality of their diſtances from one another, and of their diſtances from the points D and H. Now in both the orders of Points thoſe inequalities are equal; for A B. B C :: E F. F G, which are their diſtances from one another, as alſo D A. D B. D C :: H E. H F. H G which are their diſtances from the aſſumed points D and H, are Proportionals. Their difference therefore con-

consists solely in the magnitude of their distances. But by the definition of *Like* (*Chap.*11.*Art*,2) those things which differ onely in Magnitude are Like. Wherefore the points A,B and C have to one another Like Situation with the points E,F and G, or are placed Alike; which was to be proved.

FIGURE, *is quantity determined by the Situation, or placing of all its extreme Points.* Now I call those points Extreme which are contiguous to the place which is without the figure. In Lines therefore and Superficies all Points may be called Extreme, but in Solids onely those which are in the Superficies that includes them.

Like Figures, are those, whose extreme points in one of them, are all placed like all the extreme points in the other; for such Figures differ in nothing but Magnitude.

And like Figures are *alike placed*, when in both of them the homologal straight lines, that is, the straight lines which connect the points which answer one another are parallel, and have their proportional sides enclined the same way.

And seeing every Straight Line is like every other Straight Line, and every Plain like every other Plain when nothing but Plainness is considered; if the Lines which include Plains, or the Superficies which include Solids have their proportions known, it will not be hard to know whether any Figure be like or unlike to another propounded Figure.

And thus much concerning the *First Grounds of Philosophy*. The next place belongs to *Geometry*; in which the Quantities of Figures are sought out from the Proportions of Lines and Angles. Wherefore it is necessary for him that would study Geometry to know first what is the nature of Quantity, Proportion, Angle and Figure. Having therefore explained these in the three last Chapters, I thought fit to add them to this Part; and so passe to the next.

AD CAP. XIV

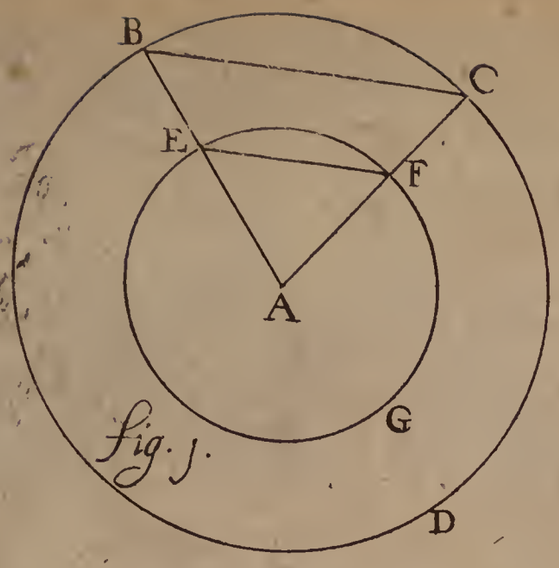


fig. 1.

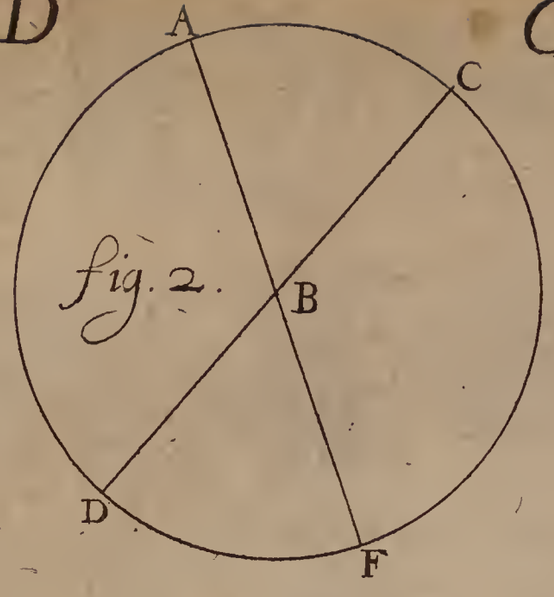


fig. 2.

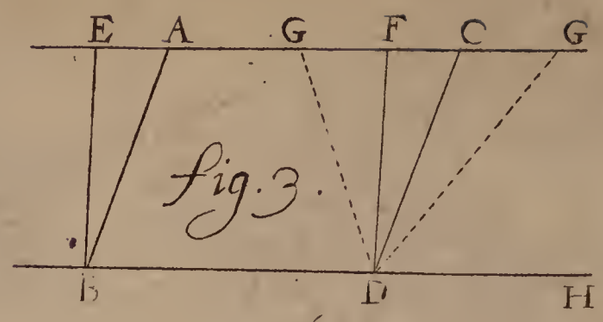


fig. 3.

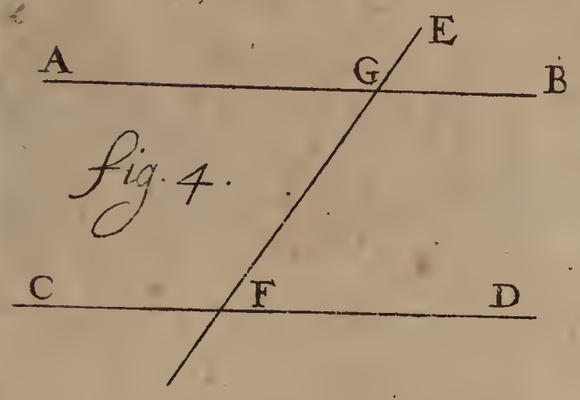


fig. 4.

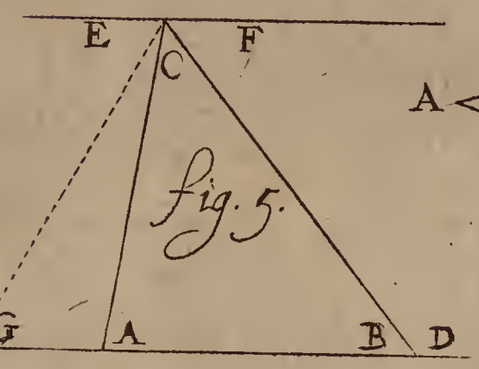


fig. 5.

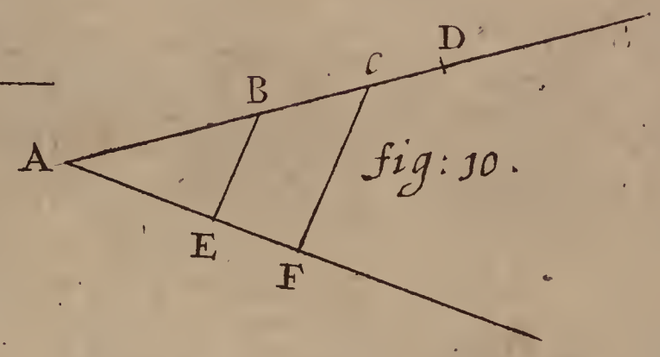


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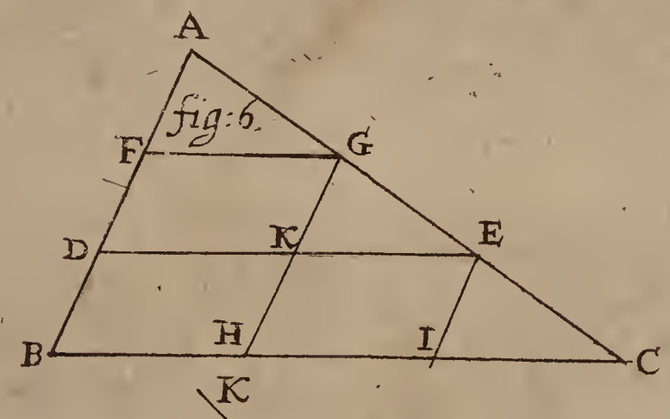


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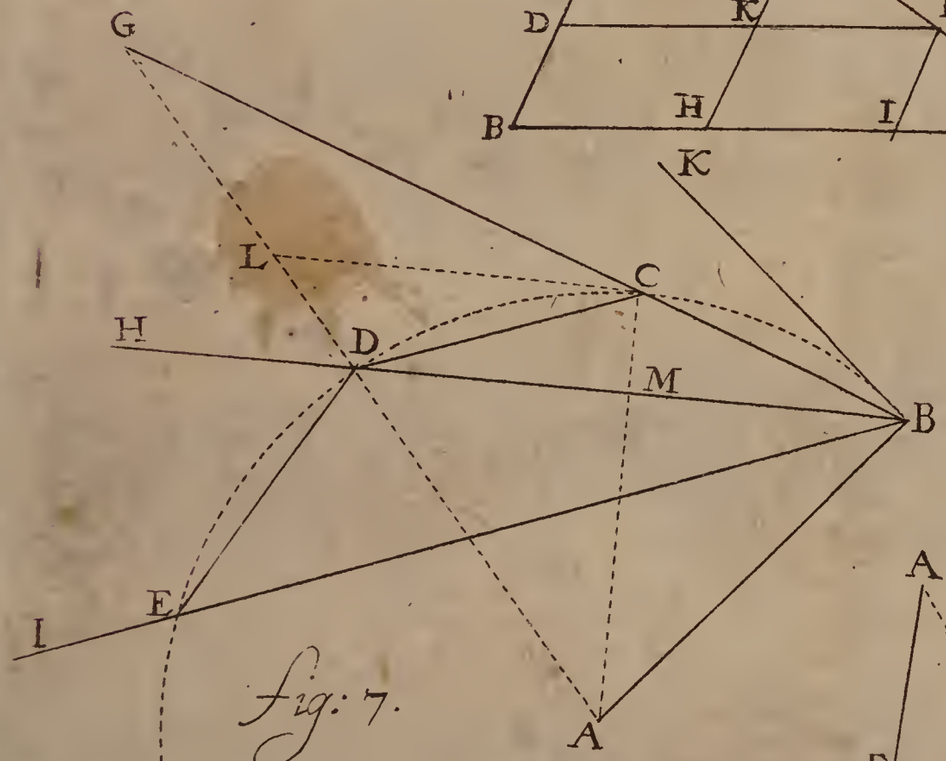


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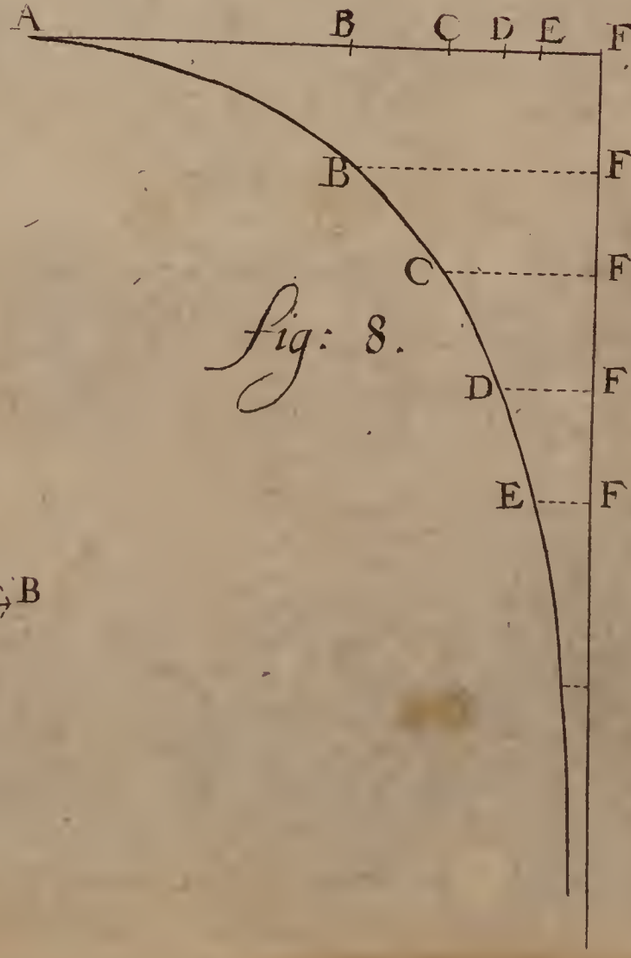


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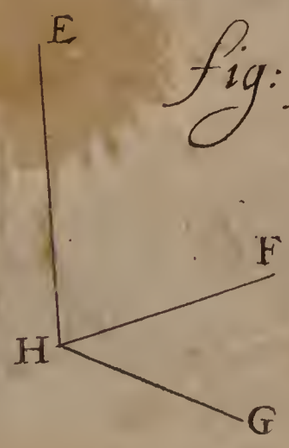
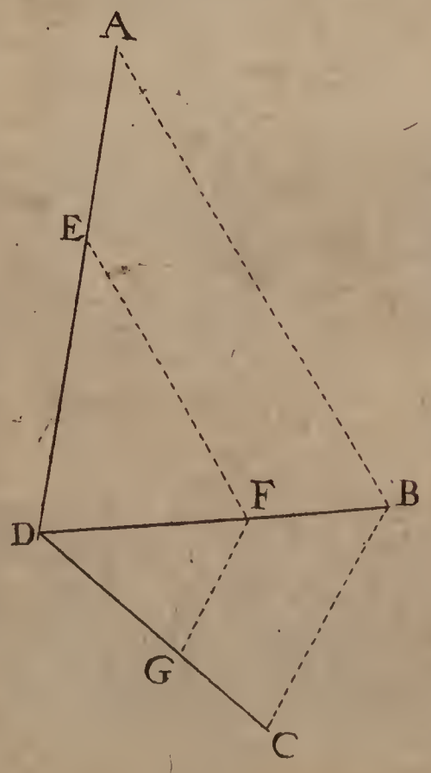


fig: 9.



OF THE PROPORTIONS OF MOTIONS AND MAGNITVDES.

CHAP. XV.

Of the Nature, Properties, and diverſe Conſiderations of Motion *and* Endeavour.

1 *Repetition of ſome Principles of the doctrine of Motion formerly ſet down.* 2 *Other Principles added to them.* 3 *Certain Theoremes concerning the nature of Motion.* 4 *Diverſe Conſiderations of Motion,* 5 *The way by which the firſt Endeavour of Bodies Moved tendeth.* 6 *In Motion which is made by Concourſe, one of the Movents ceaſing, the Endeavour is made by the way by which the reſt tend.* 7 *The Endeavour of any Moved Body, which having its Motion in the Circumference of a Circle, parts from the ſame, proceeds afterwards in a ſtraight line which toucheth the Circle.* 8 *How much greater, the Velocity or Magnitude is of a Movent, ſo much greater is the Efficacy thereof upon any other Body in its way.*

1. THe next things in order to be treated of are MOTION and MAGNITUDE, which are the moſt common Accidents of all Bodies. This place therefore moſt properly belongs to the Elements of Geometry. But becauſe this part of Philoſophy having been improved by the beſt Wits of all Ages has afforded greater plenty of matter then can well be thruſt together within the narrow limits of this diſcourſe; I thought fit to admoniſh the Reader, that before he pro-

proceed further, he take into his hands the Works of *Euclide*, *Archimedes*, *Apollonius* and other as well Ancient as Modern Writers. For to what end is it to do over again that which is already done? The little therefore that I shall say concerning Geometry in some of the following Chapters, shall be such onely as is new, and conducing to Natural Philosophy.

I have already delivered some of the Principles of this doctrine in the 8 & 9 Chapters, which I shall briefly put together here, that the Reader in going on may have their light neerer at hand.

First therefore in the 8th Chap. and 10 *Article*, *Motion* is defined to be *the continual privation of one place, and acquisition of another.*

Secondly, it is there shewn, that *Whatsoever is Moved is Moved in Time.*

Thirdly, in the same Chap. 11. *Article*, I have defined *Rest to be when a Body remains for some time in one place.*

Fourthly, it is there shewn, that *Whatsoever is Moved is not in any determined place*; as also that the same *has been Moved, is still Moved, and will yet be Moved*; So that in every part of that Space in which Motion is made, we may consider three Times, namely the *Past*, the *Present*, and the *Future Time.*

Fiftly, in the 15 *Article* of the same Chapter, I have defined *Velocity* or *Swiftness to be Motion considered as Power, namely, that Power by which a Body Moved may in a certain Time transmit a certain Length*; which also may more briefly be enunciated thus, *Velocity is the quantity of Motion determined by Time and Line.*

Sixthly, in the same Chap. 16. *Article*, I have shewn that *Motion is the Measure of Time.*

Seventhly, in the same Chap. 17th *Art.* I have defined Motions to be Equally Swift, when in Equal Times Equal Lengths are transmitted by them.

Eighthly, in the 18 *Article* of the same Chapter, *Motions* are defined to be *Equal, when the Swiftness of one Moved Body computed in every part of its magnitude, is equal to the Swiftness of another computed also in every part of its magnitude.* From whence it is to be noted, that *Motions Equal to one another, and Motions Equally Swift*, do not signifie the same thing; for when two horses draw abrest, the Motion of both is greater then the Motion of either of them singly; but the Swiftness of both together is but Equal to that of either.

Ninthly, in the 19 *Article* of the same Chapter, I have shewn, that

that *Whatsoever is at Rest will alwayes be at Rest, unless there be some other Body besides it, which by getting into its place, suffers it no longer to remain at Rest.* And that *Whatsoever is Moved, will alwayes be Moved, unless there be some other Body besides it, which hinders its Motion.*

Tenthly, In the 9 Chapter and 7 *Article*, I have demonstrated, that *When any Body is moved which was formerly at Rest, the immediate efficient cause of that Motion is in some other Moved and Contiguous Body.*

Eleventhly, I have shewn in the same place, that *Whatsoever is Moved, will always be Moved in the same way, and with the same Swiftness, if it be not hindered by some other Moved and Contiguous Body.*

2 To which Principles I shall here add these that follow. First, I define ENDEAVOUR *to be Motion made in less Space and Time then can be given*; that is, *less then can be determined or assigned by Exposition or Number*; that is, *Motion made through the length of a Point, and in an Instant or Point of Time.* For the explayning of which Definition it must be remembred, that by a Point is not to be understood that which has no quantity, or which cannot by any means be divided (for there is no such thing in Nature); but that whose quantity is not at all considered, that is, whereof neither quantity nor any part is computed in demonstration; so that a Point is not to be taken for an Indivisible, but for an Undivided thing; as also an Instant is to be taken for an Undivided, and not for an Indivisible Time.

In like manner Endeavour is to be conceived as Motion; but so, as that neither the quantity of the Time in which, nor of the Line in which it is made may in demonstration be at all brought into comparison with the quantity of that Time, or of that Line of which it is a part. And yet, as a Point may be compared with a Point, so one Endeavour may be compared with another Endeavour, and one may be found to be greater or lesse then another. For if the Vertical points of two Angles be compared, they will be equal or unequal in the same proportion which the Angles themselves have to one another. Or if a straight Line cut many Circumferences of Concentrick Circles, the inequality of the points of intersection will be in the same proportion which the Perimeters have to one another. And in the same manner, if two Motions begin and end both together, their Endeavours will be Equal or Unequal according to the proportion of their Velocities; as we

see

ſee a bullet of Lead deſcend with greater Endeavour then a ball of Wooll,

Secondly, I define IMPETUS *or Quickneſs of Motion, to be the Swiftneſs or Velocity of the Body moved, but conſidered in the ſeveral points of that time in which it is moved; In which ſenſe* Impetus *is nothing elſe but the quantity or velocity of Endeavour. But conſidered with the whole time, it is the whole velocity of the Body moved, taken together throughout all the time, and equal to the Product of a Line repreſenting the time multiplyed into a Line repreſenting the arithmetically mean* Impetus *or Quickneſs.* Which Arithmetical Mean what it is, is defined in the 29th *Article* of the 13th Chapter.

And becauſe in equal times the wayes that are paſſed are as the Velocities, and the *Impetus* is the Velocity they go withal reckoned in all the ſeveral points of the times, it followeth that during any time whatſoever, howſoever the *Impetus* be encreaſed or decreaſed, the length of the way paſſed over ſhall be encreaſed or decreaſed in the ſame proportion; and the ſame Line ſhall repreſent both the way of the Body moved, and the ſeveral *Impetus* or degrees of Swiftneſs wherewith the way is paſſed over.

And if the Body moved be not a point, but a ſtraight line moved ſo as that every point thereof make a ſeveral ſtraight line, the Plain deſcribed by its motion, whether Uniform, Accelerated or Retarded, ſhall be greater or leſs (the time being the ſame) in the ſame proportion with that of the *Impetus* reckoned in one motion to the *Impetus* reckoned in the other. For the reaſon is the ſame in Parallelograms and their Sides,

For the ſame cauſe alſo if the Body moved be a Plain, the Solid deſcribed ſhall be ſtill greater or leſs in the proportions of the ſeveral *Impetus* or Quickneſſes reckoned through one Line, to the ſeveral *Impetus* reckoned through another.

This underſtood, let A B C D (in the firſt figure of the 17th Chapter) be a Parallelogram; in which ſuppoſe the ſide A B to be moved parallelly to the oppoſite ſide C D, decreaſing al the way till it vaniſh in the point C, and ſo deſcribing the figure A B E F C; the point B as A B decreaſeth, will therefore deſcribe the Line B E F C; and ſuppoſe the time of this motion deſigned by the line C D; and in the ſame time C D ſuppoſe the ſide A C to be moved parallelly and uniformly to B D. From the point O taken at adven-

ture

ture in the Line C D, draw O R parallel to B D, cutting the Line B E F C in E, and the ſide A B in R. And again from the point Q taken alſo at adventure in the Line C D, draw Q S parallel to B D, cutting the Line B E F C in F, and the ſide A B in S; and draw E G and F H parallel to C D, cutting A C in G and H. Laſtly, ſuppoſe the ſame conſtruction done in all the points poſſible of the Line B E F C. I ſay, that as the proportions of the Swiftneſſes wherewith Q F, O E, D B, and all the reſt ſupopſed to be drawn parallel to D B, and terminated in the Line B E F C, are to the proportions of their ſeveral Times deſigned by the ſeveral parallels H F, G E, A B and all the reſt ſuppoſed to be drawn parallel to the Line of time C D, and terminated in the Line B E F C (the aggregate to the aggregate) ſo is the Area or Plain D B E F C to the Area or Plain A C F E B. For as A B decreaſing continually by the line B E F C vaniſheth in the time C D into the point C, ſo in the ſame time the line D C continually decreaſing vaniſheth by the ſame line C F E B into the point B; and the point D deſcribeth in that decreaſing motion the line D B equall to the line A C deſcribed by the point A in the decreaſing motion of A & B; & their ſwiftneſſes are therefore equal. Again, becauſe in the time G E the point O deſcribeth the line O E, and in the ſame time the point R deſcribeth the line RE, the line OE ſhall be to the line R E, as the ſwiftneſs wherewith O E is deſcribed to the ſwiftneſs wherwith RE is deſcribed. In like mãner, becauſe in the ſame time H F the point Q deſcribeth the Line Q F, and the point S the Line S F, it ſhall be as the ſwiftneſs by which Q F is deſcribed to the ſwiftneſs by which S F is deſcribed, ſo the Line it ſelf Q F to the Line it ſelf S F; and ſo in all the Lines that can poſſibly be drawn parallel to B D in the points where they cut the Line B E F C. But all the parallels to B D, as R E, S F, A C and the reſt that can poſſibly be drawn from the Line A B to the Line B E F C make the Area of the Plain A B E F C, and all the parallels to the ſame B D, as Q F, O E, D B & the reſt drawn to the points where they cut the ſame Line B E F C make the Area of the Plain B E F C D. As therefore the aggregate of the Swiftneſſes wherwith the Plain B E F C D is deſcribed is to the aggregate of the Swiftneſſes wherewith the Plain A C F E B is deſcribed, ſo is the Plain it ſelf B E F C D to the

Plain it ſelf ACFE B. But the aggregate of the Times repreſented by the parallels A B, G E, H F and the reſt, maketh alſo the Area A C F E B. And therefore as the aggregate of all the Lines Q F, O E, D B and all the reſt of the Lines parallel to B D and terminated in the Line B E F C is to the aggregate of all the Lines H F, G E, A B and all the reſt of the Lines parallel to C D & terminated in the ſame Line B E F C; that is, as the aggregate of the Lines of Swiftneſs to the aggregate of the lines of Time, or as the whole Swiftneſs in the parallels to D B to the whole Time in the parallels to C D, ſo is the Plain B E F C D to the Plain A C F E B. And the proportions of Q F to F H, and of O E to E G, and of D B to B A, and ſo of all the reſt taken together, are the proportion of the Plain D B E F C to the Plain A B E F C. But the Lines Q F, O E, D B and the reſt are the Lines that deſigne the Swifneſs; and the Lines H F, G E, A B & the reſt are the Lines that deſigne the Times of the motions; and therefore the proportion of the Plain D B E F C to the Plain A B E F C is the proportions of all the Velocities taken together, to all the Times taken together. Wherefore as the proportions of the Swiftneſſes, &c. which was to be demonſtrated.

The ſame holds alſo in the diminution of the Circles whereof the lines of Time are the Semidiameters, as may eaſily be conceived by imagining the whole Plain A B C D turned round upon the Axis B D; for the Line B E F C will be every where in the Superficies ſo made, and the Lines H F, G E, A B which here are Parallelograms will be there Cylinders, the Diameters of whoſe baſes are the lines H F, G E, A B, &c. and the Altitude a point, that is to ſay, a quantity leſs then any quantity that can poſſibly be named; and the Lines Q F, O E, D B, &c. ſmall ſolids whoſe lengths and breadths are leſs then any quantity that can be named.

But this is to be noted, that unleſs the proportion of the ſumme of the Swiftneſſes to the proportion of the ſumme of the Times be determined, the proportion of the Figure D B E F C to the Figure A B E F C cannot be determined.

Thirdly, I define RESISTANCE *to be the endeavour of one moved Body, either wholly or in part contrary to the endeavour of another moved Body, which toucheth the ſame.* I ſay wholly contrary, when the endeavour of two Bodies proceeds in the ſame ſtraight Line from the oppoſite extremes, and contrary in part, when two Bodies have

have their endeavour in two Lines, which proceeding from the extreme points of a ſtraight Line, meet without the ſame.

Fourthly, that I may define what it is to PRESSE, I ſay tha *Of two moved Bodies one Preſſes the other, when with its Endeavour it make either all or part of the other Body to go out of its place.*

Fifthly, *A Body which is preſſed and not wholly removed is ſaid to* RESTORE *it ſelf, when (the preſſing Body being taken away) the parts which were moved, do by reaſon of the internal conſtitution of the preſſed Body, return every one into its own place.* And this we may obſerve in Springs, in blown Bladders, and in many other Bodies, whoſe parts yeild more or leſs to the Endeavour which the preſſing Body makes at the firſt arrival; but afterwards (when the preſſing Body is removed) they do by ſome force within them *Reſtore* themſelves, and give their whole Body the ſame figure it had before.

Sixthly, I define FORCE *to be the* Impetus *or Quickneſs of Motion multiplyed either into it ſelf, or into the Magnitude of the Movent, by means whereof the ſaid Movent works more or leſs upon the Body that reſiſts it.*

3 Having premiſed thus much, I ſhal now demonſtrate, Firſt, That if a point moved come to touch another point which is at reſt, how little ſoever the *Impetus* or quickneſs of its motion be, it ſhall move that other point. For if by that *Impetus* it do not at all move it out of its place, neither ſhall it move it, with double the ſame *Impetus*; for nothing doubled is ſtill nothing; and for the ſame reaſon it ſhall never move it with that *Impetus* how many times ſoever it be multiplyed, becauſe nothing how ſoever it be multiplyed will for ever be nothing. Wherefore when a point is at reſt, if it do not yeild to the leaſt *Impetus*, it will yeild to none, and conſequently it will be impoſſible that that which is at reſt ſhould ever be moved.

Secondly, that when a point moved, how little ſoever the *Impetus* thereof be, falls upon a point of any Body at reſt, how hard ſoever that Body be, it will at the firſt touch make it yeild a little. For if it do not yeild to the *Impetus* which is in that point, neither will it yeild to the *Impetus* of never ſo many points, which have all their *Impetus* ſeverally equal to the *Impetus* of that point. For ſeeing all thoſe points together work equally, if any one of them have no effect, the aggregate of them all together ſhall have no effect as many times told as there are points in the whole Body, that is, ſtill no effect at all; and by con-

ſequent there would be ſome Bodies ſo hard that it would be impoſſible to break them; that is, a finite hardneſſe, or a finite force would not yeild to that which is infinite; which is abſurd.

Corollary. It is therefore manifeſt, that Reſt does nothing at all, nor is of any efficacy; and that nothing but Motion gives Motion to ſuch things as be at Reſt, and takes it from things moved,

Thirdly, that Ceſſation in the Movent does not cauſe Ceſſation in that which was moved by it. For (by the 11th Number of the 1 *Article* of this Chapter) whatſoever is moved, perſevers in the ſame way, & with the ſame Swiftneſs, as long as it is not hindered by ſome thing that is moved againſt it. Now it is manifeſt, that Ceſſation is not contrary Motion; and therefore it follows, that the ſtanding ſtill of the Movent, does not make it neceſſary that the thing moved ſhould alſo ſtand ſtill.

Corollary. They are therefore deceived, that reckon the taking away of the impediment or reſiſtance, for one of the cauſes of Motion.

4 Motion is brought into account for divers reſpects; Firſt, as in a Body *Undivided*, (that is, conſidered as a point); or, as in a *Divided* Body. In an Undivided Body, when we ſuppoſe the way by which the Motion is made, to be a Line; and in a Divided Body, when we compute the Motion of the ſeveral parts of that Body, as of Parts.

Secondly, From the diverſity of the regulation of Motion, it is in a Body conſidered as Undivided, ſometimes *Uniform*, and ſometimes *Multiform*. *Uniform* is that by which equal Lines are alwayes tranſmitted in equal times; & *Multiform*, when in one time more, in another time leſs ſpace is tranſmitted. Again, of Multiform Motions, there are ſome in which the degrees of Acceleration and Retardation proceed in the ſame proportions which the Spaces tranſmitted have, whether duplicate, or triplicate, or by whatſoever number multiplyed; and others in which it is otherwiſe.

Thirdly, from the number of the Movents; that is, one Motion is made by one Movent onely, and another by the concourſe of many Movents.

Fourthly, from the poſition of that Line in which a Body is moved, in reſpect of ſome other Line; and from hence one Motion is called *Perpendicular*, another *Oblique*, another *Parallel*.

Fifthly,

Fifthly, from the position of the Movent in respect of the Moved Body; from whence one Motion is *Pulsion* or Driving; another *Traction* or Drawing. PULSION, when the Movent makes the Moved Body goe before it; and TRACTION, when it makes it follow. Again, there are two sorts of *Pulsion*; one, when the motions of the Movent and Moved Body begin both together, which may be called TRUSION or *Thrusting* and VECTION; the other, when the Movent is first moved, and afterwards the Moved Body, which Motion is called PERCUSSION or *Stroke*.

Sixthly, Motion is considered sometimes from the Effect onely which the Movent works in the Moved Body, which is usually called *Moment*. Now MOMENT *is the Excess of Motion which the Movent has, above the Motion or Endeavour of the Resisting Body.*

Seventhly, it may be considered from the diversity of the *Medium*; as one Motion may be made in *Vacuity* or *empty Place*; another in a *fluid*; another in a *consistent Medium*, that is, a *Medium* whose parts are by some power so *consistent* and *cohering*, that no part of the same will yeild to the Movent, unless the whole yeild also.

Eighthly, when a Moved Body is considered as having parts, there arises another distinction of Motion into *Simple* and *Compounded*. *Simple*, when all the several parts describe several equal lines; *Compounded*, when the lines described are Unequal.

5 All Endeavour tends towards that part, that is to say, in that way which is determincd by the Motion of the Movent, if the Movent be but one; or, if there be many Movents, in that way which their concourse determines. For example, if a Moved Body have direct Motion, its first Endeavour will be in a Straight line; if it have Circular Motion, its first Endeavour will be in the Circumference of a Circle; & whatsoever the line be in which a Body has its Motion from the concourse of two Movents, as soon as in any point thereof the force of one of the Movents ceases, there immediately the former Endeavour of that Body will be changed into an Endeavour in the line of the other Movent.

6 Wherefore, when any Body is carried on by the concourse of two Winds, one of those Winds ceasing, the Endeavour and Motion of that Body will be in that line, in which it would have been carried by that Wind alone which blows still. And in the describing

ſcribing of a Circle, where that which is moved has its Motion determined by a Movent in a Tangent, and by the Radius which keeps it in a certain diſtance from the Center, if the retention of the Radius ceaſe, that Endeavour which was in the Circumference of the Circle, will now be in the Tangent, that is, in a Straight line. For ſeeing Endeavour is computed in a leſſe part of the Circumference then can be given, that is, in a point, the way by which a Body is moved in the Circumference is compounded of innumerable Straight lines; of which every one is leſs then can be given, which are therefore called Points. Wherefore when any Body which is moved in the Circumference of a Circle, is freed from the retention of the Radius, it will proceed in one of thoſe Straight lines, that is, in a Tangent.

7 All Endeavour, whether ſtrong or weak, is propagated to infinite diſtance; for it is Motion. If therefore the firſt Endeavour of a Body be made in Space which is empty, it will alwayes proceed with the ſame Velocity; for it cannot be ſuppoſed that it can receive any reſiſtance at all from empty Space; and therefore (by the 7 *Article* of the 9 Chapter) it will alwayes proceed in the ſame way and with the ſame Swiftneſs. And if its Endeavour be in Space which is filled, yet ſeeing Endeavour is Motion, that which ſtands next in its way ſhall be removed, and endeavour further, and again remove that which ſtands next, & ſo infinitely. Wherefore the propagation of Endeavour from one part of full Space to another, proceeds infinitely. Beſides, it reaches in any inſtant to any diſtance, how great ſoever; For in the ſame inſtant in which the firſt part of the full *Medium* removes that which is next it, the ſecond alſo removes that part which is next to it; and therefore all Endeavour, whether it be in empty or in full Space, proceeds not onely to any diſtance how great ſoever, but alſo in any time how little ſoever, that is, in an inſtant. Nor makes it any matter, that Endeavour by proceeding growes weaker and weaker, till at laſt it can no longer be perceived by Senſe; for Motion may be inſenſible; and I do not here examine things by Senſe and Experience, but by Reaſon.

8 When two Movents are of equal Magnitude, the ſwifter of them works with greater force then the ſlower upon a Body that re-

reſiſts their Motion. Alſo if two Movents have equal Velocity, the greater of them works with more force then the leſs. For where the Magnitude is equal, the Movent of greater Velocity makes the greater impreſſion upon that Body upon which it falls; and where the Velocity is equal, the Movent of greater Magnitude falling upon the ſamepoint, or an eqnal part of another Body, loſes leſs of its Velocity, becauſe the reſiſting Body works onely upon that part of the Movent which it touches, and therefore abates the *Impetus* of that part onely, whereas in the mean time the parts which are not touched proceed, and retein their whole force till they alſo come to be touched, and their force has ſome effect. Wherfore (for example) in Batteries, a longer then a ſhorter piece of Timber of the ſame thickneſs and velocity, and a thicker then a ſlenderer piece of the ſame length and velocity, works a greater effect upon the Wall.

CHAP.



CHAP. XVI.

Of Motion Accelerated *and* Vniform, *and of* Motion *by* Concourſe.

1 *The Velocity of any Body, in what Time ſoever it be computed, is that which is made of the multiplication of the* Impetus, *or Quickneſs of its Motion into the Time.* 2, *&c. In all Motion, the Lengths which are paſſed through, are to one another, as the Products made by the* Impetus *multipyed into the* Time. 6 *If two Bodies be moved with Uniform Motion through two Lengths, the proportion of thoſe Lengths to one another will be compounded of the proportions of Time to Time, and* Impetus *to* Impetus, *directly taken.* 7 *If two Bodies paſs through two Lengths with Uniform Motion, the proportion of their Times to one another will be compounded of the proportions of Length to Length and* Impetus *to* Impetus, *reciprocally taken; alſo the proportion of their* Impetus *to one another will be compounded of the proportions of Length to Length and Time to Time, reciprocally taken.* 8 *If a Body be carried on with Uniform Motion by two Movents together, which meet in an Angle, the line by which it paſſes will be a ſtraight line ſubtending the complement of that Angle to two right Angles.* 9, *&c. If a Body be carried by two Movents together, one of them being moved with Uniform, the other with Accelerated Motion, and the proportion of their Lengths to their Times being explicable in numbers, How to find out what line that Body deſcribes.*

1 THe *Velocity* of any Body, in whatſoever Time it be moved, has its quantity determined by the ſum of all the ſeveral Quickneſſes or *Impetus* which it hath in the ſeveral points of the Time of the Bodies Motion. For ſeeing Velocity (by the Definition of it *Chap.* 8. *Art.* 15.) is that Power by which a Body can in a certain time paſs through a certain length; and Quickneſs of *Motion*, or *Impetus* (by the 15 *Chap. Artic.* 2. *Numb.* 2.) is Velocity taken in one point of time onely; all the *Impetus* together taken in all the points of time, will be the ſame thing with the Mean *Impetus* multiplyed into the whole Time, or which is all one, will be the Velocity of the whole Motion.

Corol-

Corollary. If the *Impetus* be the ſame in every point, any ſtraight line repreſenting it may be taken for the meaſure of Time; and the Quickneſſes or *Impetus* applyed ordinately to any ſtraight line making an Angle with it, and repreſenting the way of the Bodies motion, will deſigne a parallelogram which ſhall repreſent the Velocity of the whole Motion. But if the *Impetus* or Quickneſs of Motion begin from Reſt, and increaſe Uniformly, that is, in the ſame proportion continually with the times which are paſſed, the whole Velocity of the Motion ſhall be repreſented by a Triangle, one ſide whereof is the whole time, and the other the greateſt *Impetus* acquired in that time; or elſe by a parallelogram, one of whoſe ſides is the whole time of Motion, and the other, half the greateſt *Impetus*; or laſtly by a parallelogram having for one ſide a mean proportional between the whole time & the half of that time, & for the other ſide the half of the greateſt *Impetus*. For both theſe parallelograms are equal to one another, & ſeverally eqnal to the triangle which is made of the whole line of time, and the greateſt acquired *Impetus*; as is demonſtrated in the Elements of Geometry.

2. In all Uniform Motions the Lengths which are tranſmitted are to one another, as the product of the mean *Impetus* multiplyed into its time, to the product of the mean *Impetus* multiplyed alſo into its time.

For let A B (in the firſt Figure) be the Time, and A C the *Impetus* by which any Body paſſes with Uniform Motion through the Length D E; & in any part of the time A B, as in the time A F, let another Body be moved with Uniform Motion, firſt, with the ſame *Impetus* A C. This Body therefore in the time A B with the *Impetus* A C will paſs through the length A F. Seeing therefore, when Bodies are moved in the ſame Time, & with the ſame Velocity & *Impetus* in every part of their motion, the proportion of one Length tranſmitted to another Length trāſmitted, is the ſame wth that of Time to Time, it followeth, that the Length tranſmitted in the time A B with the *Impetus* A C will be to the Length tranſmitted in the time A F with the ſame *Impetus* A C, as A B it ſelf is to A F, that is, as the parallelogram A I is to the parallelogram A H, that is, as the product of the time A B into the mean *Impetus* A C is to the product of the time A F into the ſame *Impetus* A C. Again, let it be ſuppoſed that a Body be moved in the time A F,

not with the ſame but with ſome other Uniform *Impetus*, as A L. Seeing therfore one of the Bodies has in all the parts of its motion the *Impetus* A C, and the other in like manner the *Impetus* A L, the Length trãſmitted by the Body moved with the *Impetus* A C will be to the Length tranſmitted by the Body moved with the *Impetus* A L, as A C it ſelf is to A L, thatis, as the parallelogram A H is to the parallelogram F L. Wherefore, by ordinate proportion it will be, as the parallelogram A I to the parallelogram F L, that is, as the product of the mean *Impetus* into the Time is to the product of the mean I*mpetus* into the Time, ſo the Length tranſmitted in the time A B with the *Impetus* A C, to the length tranſmitted in the time A F with the *Impetus*, A L; which was to be demonſtrated.

Corollary. Seeing therefore in Uniform Motion (as has been ſhewn) the Lengths tranſmitted are to one another as the parallelograms which are made by the multiplication of the mean *Impetus* into the Times, that is, (by reaſon of the equality of the *Impetus* all the way) as the Times themſelves, it will alſo be by permutation, as to Time to Length, ſo Time to Length; and in general, to this place are applicable all the properties and tranſmutations of Analogiſmes which I have ſet down and demonſtrated in the 13 Chapter.

3 In Motion begun from Reſt, and Uniformly Accelerated (that is, where the *Impetus* encreaſeth continually according to the proportion of the Times) it will alſo be, as one product made by the Mean *Impetus* multiplyed into the Time, to another product made likewiſe by the Mean *Impetus* multiplyed into the Time, ſo the Length tranſmitted in the one Time, to the Length tranſmitted in the other Time.

For let A B (in the ſame 1 figure) repreſent a Time; in the beginning of which Time A, let the *Impetus* be as the point A; but as the Time goes on, ſo let the *Impetus* encreaſe Uniformly till in the laſt point of that Time AB, namely in B, the *Impetus* acquired be BI. Again, let A F repreſent another Time, in whoſe beginning A, let the *Impetus* be as the point it ſelf A; but as the Time proceeds, ſo let the I*mpetus* encreaſe Uniformly till in the laſt point F of the Time A F the *Impetus* acquired be F K; and let D E be the Length paſſed through in the Time A B with *Impetus* Uniformly encreaſed. I ſay the Length D E, is to the Length tranſmitted in the Time A F, as the Time A B multiplyed into the Mean of the *Impetus* encreaſing

fing through the time A B, is to the Time A F multiplyed into the Mean of the *Impetus* encreasing through the time A F.

For seeing the Triangle A B I is the whole Velocity of the Body moved in the Time A B till the *Impetus* acquired be B I; and the Triangle A F K the whole Velocity of the Body moved in the Time A F with *Impetus* encreasing till there be acquired the *Impetus* F K; the Length D E to the Length acquired in the Time A F with *Impetus* encreasing from Rest in A till there be acquired the Impetus F K, will be as the Triangle A B I to the Triangle A F K, that is, if the Triangles A B I and A F K be like, in duplicate proportion of the Time A B to the Time A F; but if unlike, in the proportion compounded of the proportions of A B to B I, & of A K to A F. Wherefore, as A B I is to A F K, so let D E be to D P; for so, the Length transmitted in the Time A B with *Impetus* encreasing to B I, will be to the Length transmitted in the Time A F with *Impetus* encreasing to F K, as the triangle A B I is to the triangle A F K; But the triangle A B I is made by the multiplication of the Time A B into the Mean of the *Impetus* encreasing to B I, and the triangle A F K is made by the multiplication of the Time A F into the Mean of the *Impetus* encreasing to F K; and therefore the Length D E which is transmitted in the Time A B with *Impetus* encreasing to B I, to the Length D P which is transmitted in the Time A F with *Impetus* encreasing to F K, is as the product which is made of the Time A B multiplyed into its mean *Impetus*, to the product of the Time A F multiplyed also into its mean *Impetus*; which was to be proved.

Corol. 1 In Motion Uniformly accelerated, the proportion of the Lengths transmitted, to that of their Times, is compounded of the proportions of their Times to their Times and *Impetus* to *Impetus*.

Corol. 2 In Motion Uniformly accelerated, the Lengths transmitted in equal times taken in continual succession from the beginning of Motion, are as the differences of square numbers beginning from Unity, namely, as 3, 5, 7, &c. For if in the first time the Length transmitted be as 1, in the first and second times the Length transmitted will be as 4, which is the Square of 2, and in the three first times, it will be as 9, which is the Square of 3, and in the four first times as 16, and so on. Now the differences of these Squares are 3, 5, 7, &c.

Corol. 3 In Motion Uniformly accelerated from Rest, the Length

tranſmitted, is to another Length tranſmitted vniformly in the ſame Time, but with ſuch *Impetus* as was acquired by the accelerated Motion in the laſt point of that Time, as a triangle to a parallelogram which have their altitude and baſe common. For ſeeing the Length D E (in the ſame 1 figure) is paſſed through with Velocity as the triangle A B I, it is neceſſary that for the paſſing through of a Length which is double to DE, the Velocity be as the parallelogram A I; for the parallelogram A I is double to the triangle A B I.

4 In Motion which beginning from Reſt, is ſo accelerated, that the *Impetus* thereof encreaſe continually in proportion duplicate to the proportion of the times in which it is made, a Length tranſmitted in one time will be to a Length tranſmitted in another time, as the product made by the Mean *Impetus* multiplyed into the time of one of thoſe Motions, to the product of the Mean *Impetus* multiplyed into the time of the other Motion.

For let A B (in the 2d. figure) repreſent a Time, in whoſe firſt inſtant A let the *Impetus* be as the point A; but as the time proceeds, ſo let the *Impetus* encreaſe continually in duplicate proportion to that of the times, till in the laſt point of time B the *Impetus* acquired be B I; then taking the point F any where in the time A B, let the *Impetus* F K acquired in the time A F be ordinately applyed to that point F. Seeing therefore the proportion of F K to B I is ſuppoſed to be duplicate to that of A F to A B, the proportion of A F to A B will be ſubduplicate to that of F K to B I; and that of A B to A F will be (by Chap. 13. *Article* 16) duplicate to that of B I to F K, and conſequently the point K will be in a parabolical line whoſe diameter is A B and baſe B I; and for the ſame reaſon, to what point ſoever of the time A B the *Impetus* acquired in that time be ordinately applyed, the ſtraight line deſigning that *Impetus* will be in the ſame parabolical line A K I. Wherefore the mean *Impetus* multiplyed into the whole time A B will be the Parabola A K I B, equal to the parallelogram A M, which parallelogram has for one ſide the line of time A B and for the other the line of the *Impetus* A L, which is two thirds of the *Impetus* B I; for every Parabola is equal to two thirds of that parallelogram with which it has its altitude and baſe common. Wherefore the whole Velocity in A B will be the parallelogram A M, as being made by the multiplication of the *Impetus* A L into the time A B. And in like manner, if F N

be

be taken, which is two thirds of the *Impetus* F K, and the parallelogram F O be completed, F O will be the whole Velocity in the time A F, as being made by the Uniform Impetus A O or F N multiplyed into the time A F. Let now the length transmitted in the time A B and with the Velocity A M be the straight line D E; and lastly, let the Length transmitted in the time A F with the Velocity A N, be D P; I say that as A M is to A N, or as the Parabola A K I B to the Parabola A F K, so is D E to D P. For as A M is to F L (that is, as A B is to A F) so let D E be to D G. Now the proportion of A M to A N is compounded of the proportions of A M to F L, and of F L to A N. But as A M to F L, so (by construction) is D E to D G; and as F L is to A N (seeing the time in both is the same, namely, A F), so is the Length D G to the Length D P; for Lengths transmitted in the same time are to one another as their Velocities are. Wherefore by ordinate proportion, as A M is to A N, that is, as the mean *Impetus* A L multiplyed into its time A B, is to the mean *Impetus* A O multiplyed into A F, so is D E to D P; which was to be proved.

Corol. 1 Lengths transmitted with Motion so accelerated that the *Impetus* encrease continually in duplicate proportion to that of their times, if the base represent the *Impetus*, are in triplicate proportion of their *Impetus* acquired in the last point of their times. For as the Length D E is to the Length D P, so is the parallelogram A M to the parallelogram A N, and so the Parabola A B I K to the Parabola A F K. But the proportion of the Parabola A B I K to the Parabola A F K is triplicate to the proportion which the base B I has to the base F K. Wherefore also the proportion of D E to D P, is triplicate to that of B I to F K.

Corol. 2 Lengths transmitted in equal Times succeeding one another from the beginning, by Motion so accelerated, that the proportiō of the *Impetus* be duplicate to the proportiō of the times, are to one another as the differences of Cubique Numbers beginning at Unity, that is, as 7, 19, 37, &c. For if in the first time the Length transmitted be as 1, the Length at the end of the second time will be as 8, at the end of the third time as 27, and at the end of the fourth time as 64, &c. which are Cubique Numbers, whose differences are 7, 19, 37, &c.

Corol. 3 In Motion so accelerated, as that the Length transmitted be

be alwayes to the Length tranſmitted in duplicate proportion to their Times, the Length Uniformly tranſmitted in the whole time and with *Impetus* all the way equal to that which is laſt acquired, is as a Parabola to a parallelogram of the ſame altitude & baſe, that is, as 2 to 3. For the Parabola A B I K is the *Impetus* encreaſing in the time A B; and the parallelogram A I is the greateſt Uniform *Impetus* multiplyed into the ſame time A B. Wherefore the Lengths tranſmitted will be as a Parabola to a parallelogram &c. that is, as 2 to 3.

5 If I ſhould proceed to the explication of ſuch Motions as are made by *Impetus* encreaſing in proportion triplicate, quadruplicate, quintuplicate, &c. to that of their times, it would be a labour infinite and unneceſſary. For by the ſame method by which I have computed ſuch Lengths as are tranſmitted with *Impetus* encreaſing in ſingle and duplicate proportion, any man may compute ſuch as are tranſmitted with *Impetus* encreaſing in triplicate, quadruplicate or what other proportion he pleaſes.

In making which computation he ſhall finde, that where the *Impetus* encreaſe in proportion triplicate to that of the times, there the whole Velocity will be deſigned by the firſt Parabolaſter (of which ſee the next Chapter); and the Lengths tranſmitted will be in proportion quadruplicate to that of the times. And in like manner, where the *Impetus* encreaſe in quadruplicate proportion to that of the times, that there the whole Velocity will be deſigned by the ſecond Parabolaſter, and the Lengths tranſmitted will be in quintuplicate proportion to that of the times; and ſo on continually.

6 If two Bodies with Uniform Motion tranſmit two Lengths, each with its own *Impetus* and Time, the proportion of the Lengths tranſmitted will be compounded of the proportions of Time to Time, and *Impetus* to *Impetus*, directly taken.

Let two Bodies be moved Uniformly (as in the 3d figure) One in the time A B with the I*mpetus* A C, the other in the time A D with the I*mpetus* A E. I ſay the Lengths tranſmitted have their proportion to one another compounded of the proportions of A B to A D, and of A C to A E. For let any Length whatſoever, as Z, be tranſmitted by one of the Bodies in the time A B with the I*mpetus* A C; and any other Length, as X, be tranſmitted by the other Body in the time A D with the I*mpetus* A E; and let the parallelograms

grams A F and A G be completed. Seeing now Z is to X (by the 2d *Article*) as the *Impetus* A C multiplyed into the time A B is to the *Impetus* A E multiplyed into the time A D, that is, as A F to A G; the proportion of Z to X will be compounded of the ſame proportions, of which the proportion of A F to A G is compounded; But the proportion of A F to A G is compounded of the proportions of the ſide A B to the ſide A D, and of the ſide A C to the ſide A E (as is evident by the Elements of *Euclide*), that is, of the proportions of the time A B to the time A D, and of the *Impetus* A C to the *Impetus* A E. Wherefore alſo the proportion of Z to X is compounded of the ſame proportions of the time A B to the time A D, and of the *Impetus* A C to the *Impetus* A E; which was to be demonſtrated.

Corol. 1 When two Bodies are moved with Uniform Motion, if the Times and *Impetus* be in reciprocal proportion, the Lengths tranſmitted ſhall be equal. For if it were as A B to A D (in the ſame 3d figure) ſo reciprocally A E to A C, the proportion of A F to A G would be compounded of the proportions of A B to A D and of A C to A E, that is, of the proportions of A B to A D and of A D to A B. Wherefore, A F would be to A G as A B to A B, that is equal; and ſo the two products made by the multiplication of *Impetus* into Time would be equal; and by conſequent, Z would be equal to X.

Corol. 2 If two Bodies be moved in the ſame Time, but with different *Impetus*, the Lengths tranſmitted will be as *Impetus* to *Impetus*. For if the Time of both of them be A D, and their different *Impetus* be A E and A C, the proportion of A G to D C will be compounded of the proportions of A E to A C and of A D to A D, that is, of the proportions of A E to A C and of A C to A C; and ſo the proportion of A G to D C, that is, the proportion of Length to Length will be as A E to A C, that is, as that of *Impetus* to *Impetus*. In like manner, if two Bodies be moved Uniformly, and both of them with the ſame *Impetus*, but in different times, the proportion of the Lengths tranſmitted by them will be as that of their times. For if they have both the ſame *Impetus* A C, and their different times be A B & A D, the proportion of A F to D C will be compounded of the proportions of A B to A D and of A C to A C;

A C; that is, of the proportions of A B to A D and of A D, to A D and therefore the proportion of A F to D C, that is, of Length to Length, will be the same with that of A B to A D, which is the proportion of Time to Time.

7 If two Bodies pass through two Lengths with Uniform Motion, the proportion of the Times in which they are moved will be compounded of the proportions of Length to Length and *Impetus* to *Impetus* reciprocally taken.

For let any two Lengths be given, as (in the same 3d figure) Z and X, and let one of them be transmitted with the *Impetus* A C, the other with the *Impetus* A E. I say the proportion of the Times in which they are transmitted, will be compounded of the proportions of Z to X, and of A E (which is the *Impetus* with which X is transmitted) to A C (the *Impetus* with which Z is transmitted.) For seeing A F is the product of the *Impetus* A C multiplyed into the Time A B, the time of Motion through Z will be a line wch is made by the applicatiõ of the parallelogram A F to the straight line A C, which line is A B; and therefore A B is the time of motion through Z. In like manner, seeing A G is the product of the *Impetus* A E multiplied into the Time A D, the time of motion through X wil be a line which is made by the application of A G to the straight line A D; but A D is the time of motiõ through X. Now the proportion of A B to A D is cõpounded of the proportions of the parallelogram A F to the parallelogram A G, and of the *Impetus* A E to the *Impetus* A C; which may be demonstrated thus. Put the parallelograms in order A F, A G, D C; and it will be manifest that the proportion of A F to D C is compounded of the proportions of A F to A G and of A G to D C; but A F is to D C as A B to A D; wherefore also the proportion of A B to A D is compounded of the propotrions of A F to A G & of A G to D C. And because the Length Z is to the Length X as A F is to A G, & the *Impetus* A E to the *Impetus* A C as A G to D C, therefore the proportion of A B to A D will be compounded of the proportions of the Length Z to the Length X, and of the *Impetus* A E to the *Impetus* A C; which was to be demonstrated.

In the same manner it may be proved, that in two Uniform Motions the proportion of the *Impetus* is compounded of the proporti-

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ons of Length to Length, and of Time to Time reciprocally taken.

For if we ſuppoſe A C (in the ſame 3d figure) to be the Time, and A B the *Impetus* with which the Length Z is paſſed through; and A E to be the Time, and A D the *Impetus* with which the Length X is paſſed through, the demonſtration will proceed as in the laſt Article.

8 If a Body be carried by two Movents together which move with ſtraight and Uniform Motion, and concurre in any given angle, the line by which that Body paſſes will be a ſtraight line.

Let the Movent A B (in the 4th figure) have ſtraight and Uniform Motion, and be moved till it come into the place C D; and let another Movent A C, having likewiſe ſtraight and Uniform Motion, and making with the Movent A B any given angle C A B, be underſtood to be moved in the ſame time to D B; and let the Body be placed in the point of their concourſe A. I ſay the line which that Body deſcribes with its Motion is a ſtraight line. For let the parallelogram A B D C be completed, and its diagonal A D be drawn; and in the ſtraight line A B let any point E be taken; and from it let E F be drawn parallel to the ſtraight lines A C and B D, cutting A D in G; and through the point G let H I be drawn parallel to the ſtraight lines A B and C D; and laſtly, let the meaſure of the time be A C. Seeing therefore both the Motions are made in the ſame time, when A B is in C D, the Body alſo will be in C D; and in like manner, when A C is in B D, the Body will be in B D. But A B is in C D at the ſame time when A C is in B D; and therefore the Body will be in C D and B D at the ſame time; Wherefore it will be in the common point D. Again, ſeeing the Motion from A C to B D is Uniform, that is, the Spaces tranſmitted by it are in proportion to one another as the Times in which they are tranſmitted, when A C is in E F, the proportion of A B to A E will be ſame with that of E F to E G, that is, of the Time A C to the Time A H. Wherefore A B will be in H I in the ſame time in which A C is in E F, ſo that the Body will at the ſame time be in E F and in H I, and therefore in their common point G. And in the ſame manner it will, be whereſoever the point E be taken between A and B. Wherefore the Body will alwayes be in the Diagonal A D; which was to be demonſtrated.

Corollary. From hence it is manifeſt, that the Body will be carried through the ſame ſtraight line AD, though the Motion be not Uniform, provided it have like acceleration; for the proportion of AB to AE will alwayes be the ſame with that of AC to AH.

9 If a Body be carried by two Movents together, which meet in any given angle, and are moved, the one Uniformly; the other with Motion Uniformly accelerated from Reſt (that is, that the proportion of their *Impetus* be as that of their Times) that is, that the proportion of their Lēgths be duplicate to that of the lines of their Times, till the line of greateſt *Impetus* acquired by acceleration be equal to that of the line of Time of the Uniform Motion; the line in which the Body is carried will be the crooked line of a Semiparabola, whoſe baſe is the *Impetus* laſt acquired, and Vertex the point of Reſt.

Let the ſtraight line AB (in the 5th Figure) be underſtood to be moved with Uniform Motion to CD; and let another Movent in the ſtraight line AC be ſuppoſed to be moved in the ſame time to BD, but with motion Uniformly accelerted, that is, with ſuch motion, that the proportion of the Spaces which are tranſmitted be alwayes duplicate to that of the Times, till the *Impetus* acquired be BD equal to the ſtraight line AC; and let the Semiparabola AGDB be deſcribed. I ſay that by the concourſe of thoſe two Movents, the Body will be carried through the Semiparabolical crooked line AGD. For let the parallelelogram ABDC be completed; & from the point E taken any where in the ſtraight line AB let EF be drawn parallel to AC, and cutting the crooked line in G; and laſtly, through the point G let AI be drawn parallel to the ſtraight lines AB and CD. Seeing therefore the proportion of AB to AE is by ſuppoſition duplicate to the proportion of EF to EG, that is, of the time AC to the time AH, at the ſame time when AC is in EF, AB will be in HI; and therefore the moved Body will be in the common point G. And ſo it will alwayes be in what part ſoever of AB the point E be taken. Wherefore the moved Body will always be found in the parabolical line AGD; which was to be demonſtreted.

10 If a Body be carried by two Movents together, which meet in any given angle, and are moved, the one Uniformly, the other with *Impetus* encreaſiug from Reſt till it be equal to that of the Uniform

niform Motion, and with ſuch acceleration, that the proportion of the Lengths tranſmitted be every where triplicate to that of the Times in which they are tranſmitted, The line in which that Body is moved, will be the crooked line of the firſt Semiparabolaſter of two Means, whoſe baſe is the *Impetus* laſt acquired.

Let the ſtraight line A B (in the 6th. Figure) be moved Uniformly to C D; and let another Movent A C be moved at the ſame time to B D with motion ſo accelerated, that the proportion of the Lengths tranſmitted by every where triplicate to the proportion of their Times; and let the *Impetus* acquired in the end of that motion be B D, equal to the ſtraight line A C; & laſtly, let A D be the crooked line of the firſt Semiparabolaſter of two Means. I ſay that by the concourſe of the two Movents together, the Body will be alwayes in that crooked line A D. For let the parallelogram A B D C be completed; and from the point E taken any where in the ſtraight line A B let E F be drawn parallel to A C and cutting the crooked line in G; and through the point G let H I be drawn parallel to the ſtraight lines A B and C D. Seeing therefore the proportion of A B to A E is (by ſuppoſition) triplicate to the proportion of E F to E G, that is, of the time A C to the time A H, at the ſame time when A C is in E F, A B will be in H I; and therefore the moved Body will be in the common point G. And ſo it will alwayes be in what part ſoever of A B the point E be taken; and by conſeqnent the Body will always be in the crooked line A G D; which was to be demonſtrated.

11 By the ſame method it may be ſhewn what line it is that it made by the motion of a Body carried by the concourſe of any two Movents, which are moved, one of them Uniformly, the other with acceleration, but in ſuch proportions of Spaces and Times as are explicable by Numbers, as *duplicate*, *triplicate* &c. or ſuch as may be deſigned by any broken number whatſover. For which this is the Rule.

Let the two numbers of the Length & Time be added together; & let their Sum be the Denominator of a Fraction, whoſe Numerator muſt be the number of the Length. Seek this Fraction in the Table of the third Article of the 17th Chapter; and the line ſought will be that which denominates the three-ſided Figure noted on the left hand, and the kind of it will be that which is num-

red above over the Fraction. For example, Let there be a concourſe of two Movents, whereof one is moved Uniformly, the other with motion ſo accelerated that the Spaces are to the Times as 5 to 3. Let a Fraction be made whoſe Denominator is the Sum of 5 and 3, and the Numerator 5, namely the Fraction $\frac{5}{8}$. Seek in the Table, and you will find $\frac{5}{8}$ to be the third in that row which belongs to the three-ſided Figure of four Means. Wherfore the line of Motion made by the concourſe of two ſuch Movents as are laſt of all deſcribed, will be the crooked line of the third Parabolaſter of four Means.

12. If Motion be made by the concourſe of two Movents whereof one is moved Uniformly, the other beginning from Reſt in the Angle of concourſe with any acceleration whatſoever; the Movent which is Moved Uniformly ſhall put forward the moved Body in the ſeveral parallel Spaces, leſſe, then if both the Movents had Uniform motion; and ſtill leſſe and leſſe, as the Motion of the other Movent is more and more accelerated.

Let the Body be placed in A (in the 7th figure) and be moved by two Movents, by one with Uniform Motion from the ſtraight line A B to the ſtraight line C D parallel to it; and by the other with any acceleration from the ſtraight line A C to the ſtraight line B D parallel to it; and in the parallelogram A B D C let a Space be taken between any two parallels E F and G H. I ſay, that whileſt the Movent A C paſſes through the latitude which is between E F and G H, the Body is leſſe moved forwards from A B towards C D, then it would have been, if the Motion from A C to B D had been Uniform.

For ſuppoſe that whileſt the Body is made to deſcend to the parallel E F by the power of the Movent from A C towards B D, the ſame Body in the ſame time is moved forwards to any point F in the line E F by the power of the Movent from A B towards C D; and let the ſtraight line A F be drawn and produced indeterminately, cutting G H in H. Seeing therefore it is as A E to A G, ſo E F to G H; if A C ſhould deſcend towards B D with uniform Motion, the Body in the time G H (for I make A C and its parallels the meaſure of time) would be found in the point H. But becauſe A C is ſuppoſed to be moued towards B D which motion continually accelerated, that is, in greater proportion of Space to

Space

Space then of Time to Time, in the time G H the Body will be in ſome parallel beyond it, as between G H and B D. Suppoſe now that in the end of the time G H it be in the parallel I K, & in I K let I L be taken equal to G H. When therefore the Body is in the parallel I K, it will be in the point L. Wherefore when it was in the parallel G H, it was in ſome point between G and H, as in the point M; but if both the Motions had been Uniform it had been in the point H; and therefore whileſt the Movent A C paſſes over the latitude which is between E F and G A, the Body is leſs moved forwards from A B towards C D, then it would have been if both the Motions had been Uniform; which was to be demonſtrated.

13 Any Length being given which is paſſed through in a given time with uniform motion, To find out what Length ſhall be paſſed through in the ſame time with Motion uniformly accelerated, that is, with ſuch Motion, that the proportion of the Lengths paſſed through be continually duplicate to that of their Times, and that the line of the *Impetus* laſt acquired, be equal to the line of the whole time of the Motion.

Let A B (in the 8th. figure) be a Length tranſmitted with Uniform Motion in the time A C; and let it be required to find another Length which ſhall be tranſmitted in the ſame time with Motion Uniformly accelerated, ſo, that the line of the *Impetus* laſt acquired be equal to the ſtraight line A C.

Let the parallelogram A B D C be completed; and let B D be divided in the middle at E; and between B E and B D let B F be a mean proportional; and let A F be drawn and produced till it meet with C D produced in G; and laſtly, let the parallelogram A C G H be completed. I ſay A H is the Length required.

For as duplicate proportion is to ſingle proportion, ſo let A H be to A I, that is, let A I be the half of A H; and let I K be drawn parallel to the ſtraight line A C, and cutting the Diagonal A D in K, and the ſtraight line A G in L. Seeing therefore A I is the half of A H, I L will alſo be the half of B D, that is, equal to B E, and I K equal to B F; for B D, (that is, G H), B F, and B E (that is, I L) being continual proportionals, A H, A B, and A I will alſo be continual proportionals. But as A B is to A I, that is, as A H is to A B, ſo is B D to I K, and ſo alſo is G H, that is, B D to B F; and therefore B F and I K are equal. Now the proportion of A H to A I is duplicate to the proportion of A B to A I, that is, to that

of

of B D to I K, or of G H to I K. Wherefore the point K will be in a Parabola, whoſe diameter is A H & baſe G H, which G H is equal to A C. The Body therefore proceeding from Reſt in A with motion Uniformly accelerated in the time A C, when it has paſſed through the Length A H, will acquire the *Impetus* G H equal to the time A C; that is, ſuch *Impetus*, as that with it the Body will paſs through the Length A C in the time A C. Wherefore any Length being given, &c. which was propounded to be done.

14 Any Length being given which in a given Time is tranſmitted with Uniform Motion, To find out what Length ſhall be tranſmitted in the ſame Time with Motion ſo accelerated, that the Lengths tranſmitted be continually in triplicate proportion to that of their Times, and the line of the *Impetus* laſt of all acquired be equal to the Line of Time given.

Let the given Length A B (in the 9th figure) be tranſmitted with Uniform motion in the Time A C; aud let it be required to find what Length ſhall be tranſmitted in the ſame time with motion ſo accelerated, that the Lengths tranſmitted be continually in triplicate proportion to that of their Times, and the *Impetus* laſt acquired be equal to the Time given.

Let the parallelogram A B D C be completed; and let B D be ſo divided in E, that B E be a third part of the whole B D; and let B F be a mean proportional between B D and B E; and let A F be drawn and produced till it meet the ſtraight line C D in G; and laſtly, let the parallelogram A C G H be completed. I ſay A H is the Length required.

For as triplicate proportion is to ſingle proportion, ſo let A H be to another line A I, that is, make A I a third part of the whole A H; and let I K be drawn parallel to the ſtraight line A C, cutting the Diagonal A D in K, and the ſtraight line A G in L; then, as A B is to A I, ſo let A I be to another A N; and from the point N let N Q be drawn parallel to A C, cutting A G, A D, and F K produced, in P, M and O; and laſt of all let F O and L M be drawn, which will be equal and parallel to the ſtraight lines B N and I N. By this conſtruction, the Lengths tranſmitted A H, A B, A I and A N will be continual proportionals; and in like manner, the Times G H, B F, I L and N P, that is, N Q, N O, N M and N P will be continual proportionals, and in the ſame proportion with A H,

A H, A B, A I, and A N. Wherefore the proportion of A H to A N is the ſame with that of B D, that is, of N Q to N P; and the proportion of N Q to N P triplicate to that of N Q to N O, that is, triplicate to that of B D to I K; Wherefore alſo the Length A H is to the Length A N in triplicate proportion to that of the Time B D to the Time I K; and therefore the crooked line of the firſt three ſided figure of two means, whoſe Diameter is A H, and baſe G H equal to A C, ſhall paſs through the point O; and conſequently A H ſhall be tranſmitted in the time A C, and ſhall have its laſt acquired *Impetus* G H equal to A C, and the proportions of the Lengths acquired in any of the times triplicate to the proportions of the times themſelves. Wherefore A H is the Length required to be found out,

By the ſame method, if a Length be given which is tranſmitted with Uniform Motion in any given Time, another Length may be found out, which ſhall be tranſmitted in the ſame Time with motion ſo accelerated, that the Lengths tranſmitted ſhall be to the Times in which they are tranſmitted, in proportion quadruplicate, quintuplicate, and ſo on infinitely. For if B D be divided in E, ſo, that B D be to B E as 4 to 1; and there be taken between B D and B E a mean proportional F B; and as A H is to A B, ſo A B be made to a third, and again ſo that third to a fourth, and that fourth to a fifth A N, ſo that the proportion of A H to A N be quadruplicate to that of A H to A B, and the parallelogram N B F O be completed; the crooked line of the firſt three-ſided Figure of three Means will paſs through the point O; and conſequently the Body moved will acquire the *Impetus* G H equal to A C in the time A C. And ſo of the reſt.

15 Alſo, if the proportion of the Lengths tranſmitted, be to that of their Times, as any number to any number, the ſame method ſerves for the finding out of the Length tranſmitted with ſuch *Impetus*, and in ſuch Time.

For let A C (in the 10 figure) be the time, in which a Body is tranſmitted with Uniform Motion from A to B; and the parallelograin A B D C being completed, let it be required to find out a Length in which that Body may be moved in the ſame time A C, frō A w^th^ motion ſo accelerated, that the proportion of the Lengths trainſmitted, to that of the Times be continually as 3 to 2.

Let

Let B D be ſo divided in E, that B D be to B E as 3 to 2; and between B D and B E let B F be a mean proportionall; and let A F be drawn and produced till it meet with C D produced in G; and making A M a mean proportional between A H and A B, let it be as A M to A B, ſo A B to A I; and ſo the proportion of A H to A I will be to that of A H to A B, as 3 to 2. (for of the proportions of which that of A H to A M is one, that of A H to A B is two, and that of A H to A I is three;) & conſequently as 3 to 2 to that of G H to B F, & (F K being drawn parallel to B I, and cutting A D in K) ſo likewiſe to that of G H or B D to I K; Wherefore the proportion of the Length A H to A I is to the proportion of the Time B D to I K, as 3 to 2; and therefore, if in the time A C, the Body be moved with accelerated motion, as was propounded, till it acquire the *Impetus* H G equal to A C, the Length tranſmitted in the ſame Time will be A H.

16 But if the proportion of the Lengths to that of the Times had been as 4 to 3, there ſhould then have been taken two mean proportionals between A H and A B, and their proportion ſhould have been continued one term further, ſo that A H to A B might have three of the ſame proportions, of which A H to A I has four; and all things elſe ſhould have been done as is already ſhewn. Now the way how to interpoſe any number of Means between two Lines given, is not yet found out. Nevertheleſs, this may ſtand for a general Rule; *If there be a Time given, and a Length be tranſmitted in that Time with Uniform Motion; as for example, if the Time be* A C, *and the Length* A B; *the ſtraight line* A G, *which determines the Length* C G *or* A H *tranſmitted in the ſame Time* A C *with any accelerated motion, ſhall ſo cut* B D *in* F, *that* B F *ſhall be a mean proportional between* B D *and* B E, B E *being ſo taken in* B D, *that the proportion of Length to Length be every where to the proportion of Time to Time, as the whole* B D *is to its part* B E.

17 If in a given Time, two Lengths be tranſmitted, One with uniform motion, the other with motion accelerated in any proportion of the Lengths to the Times; and again in part of the ſame Time, parts of the ſame Lengths be tranſmitted with the ſame motions, the whole Length will exceed the other Length in the ſame proportion in which one part exceeds the other part.

For example, let A B (in the 8th. figure) be a Length tranſmitted

ted in the time A C with uniform Motion; and let A H be another Length tranſmitted in the ſame time with Motion uniformly accelerated, ſo that the *Impetus* laſt acquired be G H equal to A C; and in A H let any part A I be taken, and tranſmitted in part of the time A C with uniform Motion; and let another part A B be taken, and tranſmitted in the ſame part of the time A C with Motion uniformly accelerated. I ſay, that as A H is to A B, ſo will A B be to A I.

Let B D be drawn parallel and equal to H G, and divided in the midſt at E, and between B D and B E, let a mean proportional be taken as B F; & the ſtraight line A G (by the demonſtration of the 13th Art.) ſhal paſs through F. And dividing A H in the midſt at I, A B ſhall be a mean proportional between A H and A I. Again (becauſe A I and A B are deſcribed by the ſame Motions) if I K be drawn parallel and equal to B F or A M, and divided in the midſt at N, and between I K and I N be taken the mean proportional I L, the ſtraight line A F will (by the demonſtration of the ſame 13th Art.) paſs through L. And dividing A B in the midſt at O, the line A I will be a mean proportional between A B and A O. Where A B is divided in I and O in like manner as A H is divided in B and I; and as A H to A B ſo is A B to A I. Which was to be proved.

Coroll. Alſo as A H to A B, ſo is H B to B I; and ſo alſo B I to I O.

And as this (where one of the Motions is uniformly accelerated) is proved out of the demonſtration of the 13th Article; ſo (when the accelerations are in double proportion to the times) the ſame may be proved by the demonſtration of the 14th Art. and by the ſame method in all other accelerations, whoſe proportions to the times are explicable in numbers.

18 If two ſides which contain an Angle in any Parallelogram, be moved in the ſame time to the ſides oppoſite to them, one of them with Uniform Motion, the other with Motion Uniformly accelerated; that ſide which is moved Uniformly will effect as much with its concourſe through the whole Length tranſmitted, as it would do if the other Motion were alſo Uniform, and the Length tranſmitted by it in the ſame time were a mean proportional between the whole and the half.

Let the ſide A B of the Parallelogram A B D C (in the 11th Figure) be underſtood to be moved with Uniform Motion till it be coincident with C D, and let the time of the Motion be A C

or B D. Alſo in the ſame time let the ſide A C be underſtood to be moved with Motion uniformly accelerated, till it be coincident with B D; then dividing A B in the middle in E, let A F be made a mean proportional between A B and A E; and drawing F G parallel to A C, let the ſide A C be underſtood to be moved in the ſame time AC with uniform Motion till it be coincident with F G. I ſay the whole AB confers as much to the velocity of the Body placed in A when the Motion of AC is uniformly accelerated till it come to B D, as the part A F confers to the ſame when the ſide AC is moved Uniformly and in the ſame time to F G.

For ſeeing A F is a mean proportional between the whole A B & it is half A E, B D wil (by the 13th *Art.*) be the laſt *Impetus* acquired by AC with motion uniformly acceletated till it come to the ſame B D; and conſequently the ſtraight line F B will be the exceſs by wch the Length tranſmitted by AC with motion uniformly accelerated, will exceed the Length tranſmitted by the ſame AC in the ſame time with Uniform Motion, and with *Impetus* every where equal to B D. Wherefore if the whole A B be moved Uniformly to C D in the ſame time in which A C is moved Uniformly to F G, the part F B (ſeeing it concurs not at all with the Motion of the ſide A C which is ſupposed to be moved onely to F G) will cõfer nothing to its motion. Again, ſuppoſing the ſide A C to be moved to BD with Motion Uniform'y accelerated, the ſide AB with its uniform Motion to CD will leſs put forwards the Body when it is accelerated in all the paralleIs, then when it is not at all accelerated; & by how much the greater the acceleration is, by ſo much the leſs it will put it forwards (as is ſhewn in the 12th *Artic.*) When therefore AC is in FG with accelerated Motion, the Body will not be in the ſide C D at the point G, but at the point D; ſo that G D wil be the exceſs by which the Length tranſmitted with accelerated Motion to BD, exceeds the Length tranſmitted with Uniform Motion to FG; ſo that the Body by its acceleration avoids the action of the part A F, & comes to the ſide C D in the time A C, and makes the Length C D, which is equal to the Length A B. Wherefore Uniform Motion from A B to C D in the time A C works no more in the whole Length A B upon the Body uniformly accelerated from A C to B D, then if A C were moved in the ſame time with uniform Motion to FG; the difference conſiſting onely in this,

F G,

F G, that by which the accelerated Motion exceeds the Uniform Motion, is altogether in F B, or G D; but when the ſame A B works upon the Body accelerated, that by which the accelerated Motion exceeds the Uniform Motion, is diſperſed through the whole Length A B or C D, yet ſo that if it were collected and put together, it would be equal to the ſame F B or G D. Wherefore, If two ſides which contain an angle &c; which was to be demonſtrated.

19 If two tranſmitted Lengths have to their Times any other proportion explicable by number, & the ſide A B be ſo divided in E, that A B be to A E in the ſame proportion which the Lengths tranſmitted have to the Times in which they are tranſmitted, and between A B and A E there be taken a mean proportional A F, it may be ſhewn by the ſame method, that the ſide which is moved with Uniform Motion, works as much with its concourſe through the whole Length A B, as it would do if the other Motion were alſo Uniform, and the Length tranſmitted in the ſame Time A C were that mean proportional A F.

And thus much concerning Motion by concourſe.

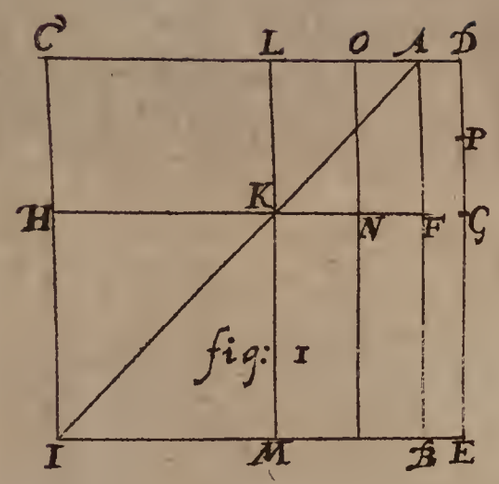


fig: 1

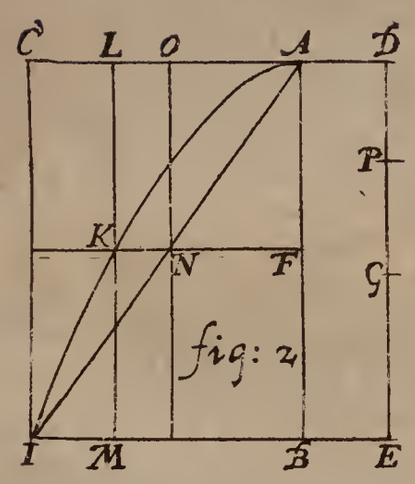


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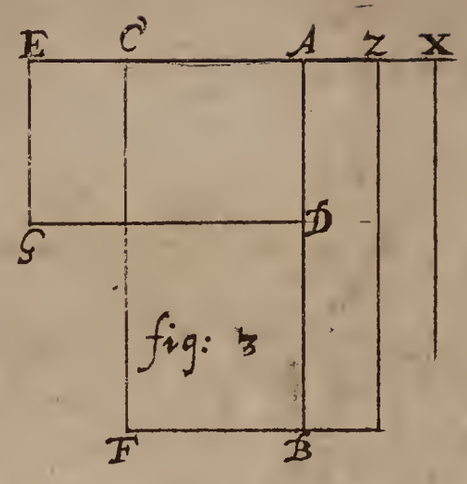


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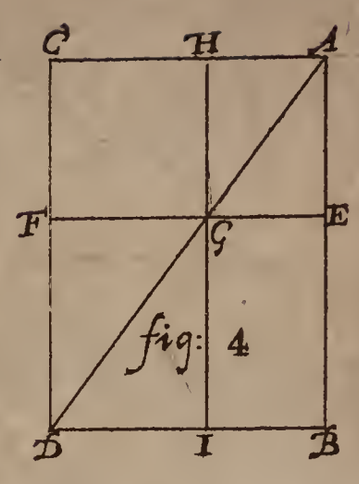


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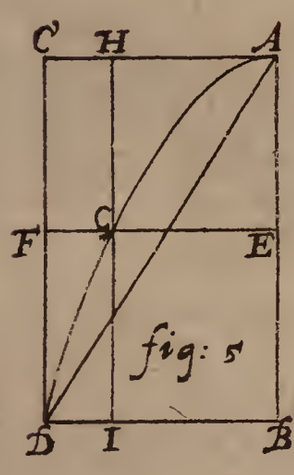


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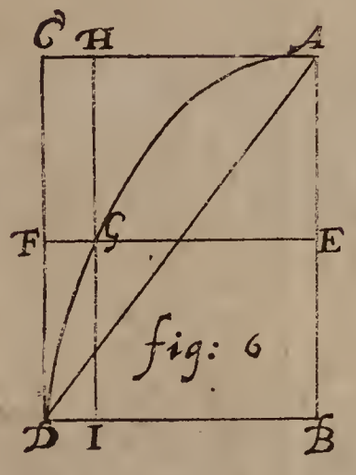


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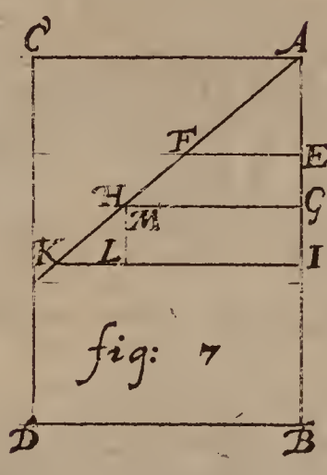


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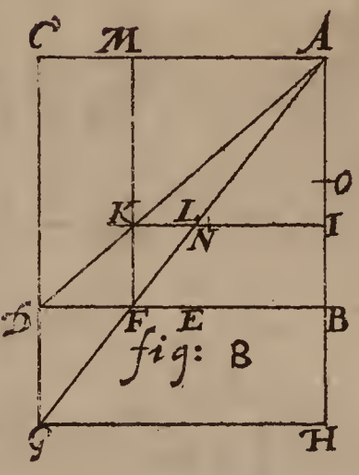


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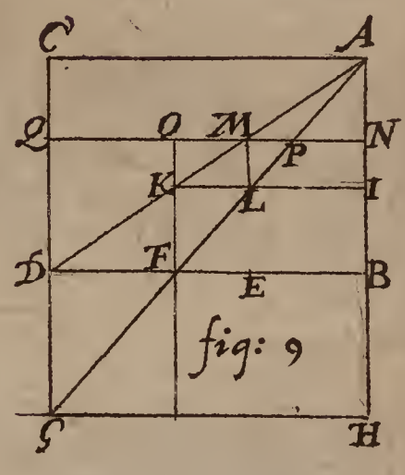


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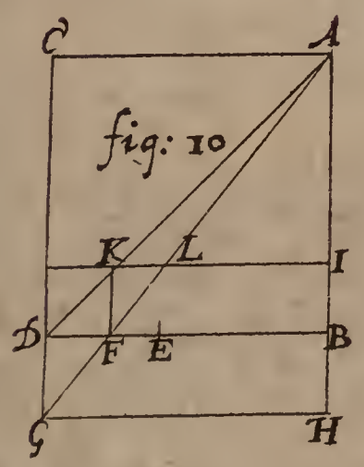


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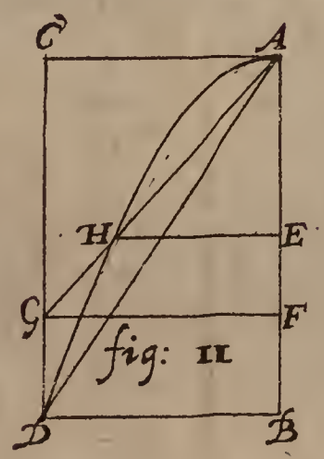


fig: 11

CHAP. XVII.

Of Figures Deficient.

1 *Definitions of a Deficient Figure; of a Complete Figure; of the Complement of a Deficient Figure; and of Proportions which are Proportional and Commensurable to one another.* 2 *The proportion of a Deficient Figure to its Complement.* 3 *The proportions of Deficient Figures to the Parallelograms in which they are described, set forth in a Table.* 4 *The Description and Production of the same Figures.* 5 *The drawing of Tangents to them.* 6 *In what proportion the same Figures exceed a straight lined Triangle of the same Altitude and Base.* 7 *A Table of Solid Deficient Figures described in a Cylinder.* 8 *In what proportion the same Figures exceed a Cone of the same Altitude and Base.* 9 *How a plain Deficient Figure may be described in a Parallelogram, so, that it be to a Triangle of the same Base and Altitude, as another Deficient Figure (plain or solid) twice taken, is to the same Deficient Figure together with the Complete Figure in which it is described.* 10 *The transferring of certain properties of Deficient Figures described in a Parallelogram to the proportions of the Spaces transmitted with several degrees of Velocity.* 11 *Of Deficient Figures deseribed in a Circle.* 12 *The proposition demonstrated in the 2d. Article, confirmed from the Elements of Philosophy.* 13 *An unusual way of reasoning concerning the Equality between the superficies of a portion of a Sphere, and a Circle.* 14. *How from the description of Deficient Figures in a Parallelogram, any number of mean Proportionals may be found out between two given straight lines.*

1 I Call those *Deficient Figures*, which may be understood to be generated by the Uniform Motion of some quantity, which decreases continually, till at last it have no magnitude at all.

And I call that a *Complete Figure*, answering to a Deficient Figure, which is generated with the same motion, and in the same time, by a Quantity which retaines alwayes its whole magnitude.

The *Complement* of a Deficient Figure is that, which being added to the Deficient Figure, makes it Complete.

Four Proportions are said to be *Proportionall*, when the first of them

them is to the ſecond, as the third is to the fourth. For example, if the firſt proportion be duplicate to the ſecond; and again the third be duplicate to the fourth, thoſe Proportions are ſaid to be *Proportionall.*

And *Commenſurable* Proportions are thoſe, which are to one another as number to number. As when to a proportion given, one proportion is duplicate, another triplicate, the duplicate proportion will be to the triplicate proportion as 2 to 3; but to the given proportion it will be as 2 to 1; and therefore I call thoſe three proportions *Commenſurable.*

2 A Deficient Figure, which is made by a Quantity continually decreaſing to nothing by proportions every where proportionall and commenſurable, is to its Complement, as the proportion of the whole altitude, to an altitude diminiſhed in any time, is to the proportion of the whole Quantity which deſcribes the Figure, to the ſame Quantity diminiſhed in the ſame time.

Let the quantity A B (in the 1 figure) by its motion through the altitude A C, deſcribe the Complete Figure A D; and againe, let the ſame quantity, by decreaſing continually to nothing in C, deſcribe the Deficient Figure A B E F C, whoſe Complement will be the Figure B D C F E. Now let A B be ſuppoſed to be moved till it lie in G K, ſo that the altitude diminiſhed be G C, and A B diminiſhed be G E; and let the proportion of the whole altitude A C to the diminiſhed altitude G C, be (for example) triplicate to the proportion of the whole quantity A B or G K, to the diminiſhed quantity G E. And in like manner, let H I be taken equal to G E, & let it be diminiſhed to H F; and let the proportion of G C to H C be triplicate to that of H I to H F; & let the ſame be done in as many parts of the ſtraight line A C as is poſſible; and a line be drawn through the points B, E, F and C. I ſay the Deficient Figure A B E F C, is to its Complement B D C E F as 3 to 1, or as the proportion of A C to G C is to the proportion of A B, that is, of G K to G E.

For (by the ſecond Article of the 15. Chap.) the proportion of the complement B E F C D to the deficient figure A B E F C, is all the proportions of D B to O E, and of D B to Q F, and of all the lines parallel to D B terminated in the line B E F C, to all the parallels to A B terminated in the ſame points of the line B E F C.

And

And seeing the proportions of D B to O E, and of D B to Q F &c. are every where triplicate of the proportions of A B to G E, and of A B to H F &c. the proportions of H F to A B, and of G E to A B &c. (by the 16 Article of the 13 Chap.) are triplicate of the proportions of Q F to D B, and of O E to D B &c. and therefore the deficient figure A B E F C which is the aggregate of all the lines H F, G E, A B, &c. is triple to the complement B E F C D made of all the lines Q F, O E, D B, &c. which was to be proved.

It follows from hence, That the same complement B E F C D is $\frac{1}{4}$ of the whole Parallelogram. And by the same method may be calculated in all other Deficient Figures generated as above declared, the proportion of the Parallelogram to either of its parts; as that when the parallels encrease fom a point in the same proportion, the Parallelogram will be divided into two equal Triangles; when one encrease is double to the other, it will be divided into a Semiparabola and its Complement, or into 2 and 1.

The same construction standing, the same conclusion may otherwise be demonstrated, thus.

Let the straight line C B be drawn cutting G K in L, & through L let M N be drawn parallel to the straight line A C; wherefore the Parallelograms G M and L D will be equal. Then let L K be divided into three equal parts, so that it may be to one of those parts in the same proportion which the proportion of A C to G C or of G K to G L hath to the proportion of G K to G E. Therefore L K will be to one of those three parts as the Arithmetical proportion between G K and G L is to the Arithmetical proportion between G K and the same G K want the third part of L K; and K E will be somwhat greater then a third of L K. Seeing now the altitude A G or M L is by reason of the continual decrease, to be supposed less then any quantity that can be given; L K (which is intercepted between the Diagonal B C and the side B D) will be also less then any quantity that can be given; and consequently, if G be put so neer to A in *g*, as that the difference between C *g* and C A be less then any quantity that can be assigned, the difference also between C *l* (removing L to *l*) and C B, will be less then any quantity that can be assigned; and the line *g l* being drawn & produced to the line B D in *k* cutting the crooked line

in

in *e*, the proportion of G *k* to G *l* will ſtill be triplicate to the proportion of G *k* to G *e*, and the difference between *k* and *e* the third part of *k l* will be leſs then any quantity that can be given; and therefore the Parallelogram *e* D will differ from a third part of the Parallelogram A *e* by a leſs difference then any quantity that can be aſſigned. Again, let H I be drawn parallel and equal to *g e*, cutting C B in P, the crooked line in F, and B D in I, and the proportion of C *g*, to C H will be triplicate to the proportion of H F to H P, and I F will be greater then the third part of P I. But again, ſetting H in *h* ſo neer to *g*, as that the difference between C *h* and C *g* may be but as a point, the point P will alſo in *p* be ſo neer to *l*, as that the difference between C *p* and C *l* will be but as a point; and drawing *h p* till it meet with *g k* in *i*, cutting the crooked line in *f*, and having drawn *e o* parallel to B D, cutting D C in *o*, the Parallelogram *f o* will differ leſs from the third part of the Parallelogram *g f*, then by any quantity that can be given. And ſo it will be in all other Spaces generated in the ſame manner. Wherefore the differences of the Arithmetical and Geometrical Means, which are but as ſo many points B, *e*, *f*, &c. (ſeeing the whole Figure is made up of ſo many indiviſible Spaces), will conſtitute a certain line, ſuch as is the line B E F C, which will divide the complete Figure A D into two parts; whereof one, namely A B E F C, which I call a Deficient Figure, is triple to the other, namely B D C E F, which I call the Complement thereof. And whereas the proportion of the altitudes to one another, is in this caſe everywhere triplicate to that of the decreaſing quantities to one another; in the ſame manner if the proportion of the altitudes had been every where quadruplicate to that of the decreaſing quantities it might have been demonſtrated, that the Deficient Figure had been quadruple to its Complement; and ſo in any other proportion. Wherefore, a Deficient Figure, which is made, &c. Which was to be demonſtrated.

The ſame rule holdeth alſo in the diminution of the Baſes of Cylinders, as is demonſtrated Chap. 15. Art. 2.

2 By this Propoſition, the magnitudes of all Deficient Figures (when the proportions by which their baſes decreaſe continually, are proportionall to thoſe by which their altitudes decreaſe) may be compared with the magnitudes of their Complements;

and

and consequently, with the magnitudes of their Complete Figures. And they will be found to be as I have set them down in the following Tables; in which I compare a Parallelogram with three-sided Figures; and first with a straight lined triangle, made by the base of the Parallelogram continually decreasing in such manner, that the altitudes be alwayes in proportion to one another as the bases are, and so the triangle will be equal to its Complement; or the proportions of the altitudes and bases wil be as 1 to 1, and then the triangle will be half the Parallelogram. Secondly, with that three-sided Figure which is made by the continual decreasing of the bases in subduplicate proportion to that of the altitudes; and so the Deficient Figure will be double to its Complement, and to the Parallelogram as 2 to 3. Then, with that, where the proportion of the altitudes is triplicate to that of the bases; and then the Deficient Figure will be triple to its Complement, and to the Parallelogram as 3 to 4. Also the proportion of the altitudes to that of the bases may be as 3 to 2; and then the Deficient Figure will be to its Complement as 3 to 2, & to the Parallelogram as 3 to 5; and so forwards according as more mean proportionals are taken, or as the proportions are more multiplyed, as may be seen in the following Table. For example, if the bases decrease so, that the proportion of the altitudes to that of the bases be alwayes as 5 to 2, and it be demanded what proportion the Figure made has to the Parallelogram, which is supposed to be Unity; then, seeing that where the proportion is taken five times, there must be four Means; look in the Table amongst the three-sided figures of four Means, and seeing the proportion was as 5 to 2, look in the uppermost row for the number 2, and descending in the 2d Columne till you meet with that three-sided Figure, you will finde $\frac{5}{7}$; which shews that the Deficient Figure is to the Parallelogram as $\frac{5}{7}$ to 1, or as 5 to 7.

	1	2	3	4	5	6	7
Parallelogram	1						
Straight-sided Triangle	1/2						
Three-sided figure of 1 Mean	2/3						
Three-sided figure of 2 Means	3/4	3/5					
Three-sided figure of 3 Means	4/5	4/6	4/7				
Three-sided figure of 4 Means	5/6	5/7	5/8	5/9			
Three-sided figure of 5 Means	6/7	6/8	6/9	6/10	6/11		
Three-sided figure of 6 Means	7/8	7/9	7/10	7/11	7/12	7/13	
Three-sided figure of 7 Means	8/9	8/10	8/11	8/12	8/13	8/14	8/15

4 Now for the better understanding of the nature of these three-sided figures, I will shew how they may be described by points; and first, those which are in the first column of the Table. Any Parallelogram being described, as A B C D (in the 2d. figure,) let the Diagonal B D be drawn; and the straight-lined triangle B C D will be half the Parallelogram; Then let any number of lines, as E F, be drawn parallel to the Side B C, and cutting the Diagonal B D in G; & let it be every where, as E F to E G, so E G to another E H; and through all the points H let the line B H H D be drawn; and the Figure B H H D C will be that which I call a Three-sided Figure of one Mean, because in three proportionals, as E F, E G and E H, there is but one Mean, namely, E G; and this three-sided figure will be $\frac{2}{3}$ of the Parallelogram, and is called a *Parabola*. Again, let it be as E G to E H, so E H to another E I, and let the line B I I D be drawn, making the three-sided figure B I I D C; & this will be $\frac{3}{4}$ of the Parallelogram, and is by many called a *Cubique Parabola*. In like manner, if the proportions be further continued in E F, there will be made the rest of the three-sided

ſided figures of the firſt Column; which I thus demonſtrate. Let there be drawn ſtraight lines, as H K and G L parallel to the baſe D C. Seeing therefore the proportion of E F to E H is duplicate of that of E F to E G, or of B C to B L, that is, of C D to L G, or of K M (producing K H to A D in M) to K H, the proportion of B C to B K will be duplicate to that of K M to K H; but as B C is to B K, ſo is D C, or K M to K N, and therefore the proportion of K M to K N is duplicate to that of K M to K H; and ſo it will be whereſoever the parallel K M be placed. Wherefore the Figure B H H D C is double to its Complement B H H D A, and conſequently $\frac{2}{3}$ of the whole Parallelogram. In the ſame manner if through I, be drawn O P I Q parallel and equal to C D, it may be demonſtrated that the proportion of O Q to O P, that is, of B C to B O, is triplicate to that of O Q to O I, and therefore that the Figure B I I D C is triple to its Complement B I I D A, and conſequently $\frac{3}{4}$ of the whole Parallelogram, &c.

Secondly, ſuch three-ſided figures as are in any of the tranſverſe rowes, may be thus deſcribed. Let A B C D (in the 3d. *Figure*) be a Parallelogram, whoſe Diagonal is B D. I would deſcribe in it ſuch figures, as in the preceding Table I call Three-ſided Figures of three Means. Parallel to D C, I draw E F as often as is neceſſary, cutting B D in G; and between E F and E G I take three proportionals E H, E I and E K. If now there be drawn lines through all the points H, I & K; that through all the points H will make the figure B H D C, which is the firſt of thoſe three-ſided figures; and that through all the points I, will make the figure B I D C, which is the ſecond; and that which is drawn through all the points K, will make the figure B K D C the third of thoſe three-ſided figures. The firſt of theſe (ſeeing the proportion of E F to E C is quadruplicate of that of E F to E H) will be to its Complement as 4 to 1, and to the Parallelogram as 4 to 5. The ſecond (ſeeing the proportion of E F to E G is to that of E F to E I as 4 to 2) will be double to its Complement, and $\frac{4}{6}$ or $\frac{2}{3}$ of the Parallelogram. The third (ſeeing the proportion of E F to E G is to that of E F to E K as 4 to 3) will be to its Complement as 4 to 3, and to the Parallelogram as 4 to 7.

Any of theſe figures being deſcribed, may be produced at pleaſure, thus; Let A B C D (in the 4th figure) be a Parallelogram, and in it let the figure B K D C be deſcribed, namely, the third three-ſided figure of three Means. Let B D be produced indefinitely to E, and let E F be made parallel to the baſe D C, cutting

A D produced in G, and B C produced in F; and in G E let the point H be ſo taken, that the proportion of F E to F G may be quadruplicate to that of F E to F H (which may be done by making F H the greateſt of three proportionals between F E and F G); the crooked line B K D produced, will paſs through the point H. For if the ſtraight line B H be drawn, cutting C D in I, and H L be drawn parallel to G D, and meeting C D produced in L; it will be as F E to F G, ſo C L to C I; that is, in quadruplicate proportion to that of F E to F H, or of C D to C I. Wherefore if the line B K D be produced according to its generation, it will fall upon the point H.

5 A ſtraight line may be drawn ſo, as to touch the crooked line of the ſaid figure in any point, in this manner. Let it be required to draw a Tangent to the line B K D H (in the 4th figure) in the point D. Let the points B and D be connected, and drawing D A equal and parallel to B C, let B and A be connected; and becauſe this figure is by conſtruction the third of three Means, let there be taken in A B three points, ſo, that by them the ſame A B be divided into four equal parts; of which take three, namely, A M, ſo that A B may be to A M, as the fignre B K D C is to its Complement. I ſay the ſtraight line M D, will touch the figure in the point given D. For let there be drawn any where between A B and D C a parallel, as R Q, cutting the ſtraight line B D, the crooked line B D, the ſtraight line M D, and the ſtraight line A D in the points P, K, O and Q. R K will therefore (by conſtruction) be the leaſt of three Means in Geometrical proportion between R Q and R P. Wherefore (by the *Coroll.* of the 28th Article of the 13th Chapter) R K will be leſs then R O; and therefore M D will fall without the figure. Now if M D be produced to N, F N will be the leaſt of three Means in Arithmetical proportion between F E and F G; and F H will be the greateſt of three Means in Geometrical proportion between the ſame F E and F G. Wherefore (by the ſame *Coroll.* of the 28 *Artic.* of the 13th Chap.) F H will be leſs then F N; and therefore D N will fall without the figure, and the ſtraight line M N will touch the ſame figure onely in the point D.

6 The proportion of a Deficient Figure to its Complement being known, it may alſo be known what proportion a ſtraight-

lined

lined triangle has to the excefs of the Deficient Figure above the ſame triangle; and theſe proportions I have ſet down in the following Table; where if you ſeek (for example) how much the fourth three-ſided figure of five Means exceeds a triangle of the ſame altitude and baſe, you will find in the concourſe of the fourth column with the three-ſided figures of five Means, $\frac{2}{10}$; by which is ſignified, that that three-ſided figure exceeds the triangle by two tenths, or by one fifth part of the ſame triangle.

The Excefs of

	1	2	3	4	5	6	7
The Triangle	1						
a Three-ſided figure of one Mean	1/3						
a Three-ſided figure of 2 Means	2/4	1/5					
a Three-ſided figure of 3 Means	3/5	2/6	1/7				
a Three-ſided figure of 4 Means	4/6	3/7	2/8	1/9			
a Three-ſided figure of 5 Means	5/7	4/8	3/9	2/10	1/11		
a Three-ſided figure of 6 Means	6/8	5/9	4/10	3/11	2/12	1/13	
a Three-ſided figure of 7 Means	7/9	6/10	5/11	4/12	3/13	2/14	1/15

7 In the next Table are ſet down the proportion of a Cone, and the Solids of the ſaid three-ſided figures, namely, the proportions between them and a Cylinder. As for example, in the concourſe of the ſecond Column with the three-ſided figures of four Means, you have $\frac{5}{9}$; which gives you to underſtand, that the Solid of the ſecond three-ſided figure of four Means is to the Cylinder as $\frac{5}{9}$ to 1, or as 5 to 9,

The-

	1	2	3	4	5	6	7
The Cylinder	1						
A Cone	1/3						
The Solids of: a Three-sided figure of one Mean	2/4						
a Three-sided figure of 2 Means	3/5	3/7					
a Three-sided figure of 3 Means	4/6	4/8	4/10				
a Three-sided figure of 4 Means	5/7	5/9	5/11	5/13			
a Three-sided figure of 5 Means	6/8	6/10	6/12	6/14	6/16		
a Three-sided figure of 6 Means	7/9	7/11	7/13	7/15	7/17	7/19	
a Three-sided figure of 7 Means	8/10	8/12	8/14	8/16	8/18	8/20	8/22

8 Laſtly, the Exceſs of the Solids of the ſaid three-ſided figures, above a Cone of the ſame altitude and baſe, are ſet down in the Table which follows

	1	2	3	4	5	6	7
The Cone	1						
The Exceſſes of the Solids of theſe three-ſided figures above a Cone: Of the Solid of a three-ſided figure of one Mean	6/12						
Of the Solid of a three-ſided figure of 2 Means	12/15	6/21					
Of the Solid of a three-ſided figure of 3 Means	18/18	12/24	6/30				
Of the Solid of a three-ſided figure of 4 Means	24/21	18/27	12/33	6/39			
Of the Solid of a three-ſided figure of 5 Means	30/24	24/30	18/36	12/42	6/48		
Of the Solid of a three-ſided figure of 6 Means	36/27	30/33	24/39	18/45	12/51	6/57	
Of the Solid of a three-ſided figure of 7 Means	42/30	36/36	30/42	24/48	18/54	12/60	6/66

9 If any of these Deficient Figures, of which I have now spoken, as A B C D (in the 5th figure) be inscribed within the Complete figure B E, having A D C E for its Complement; and there be made upon C B produced, the triangle A B I; and the Parallelogram A B I K be completed; and there be drawn parallel to the straight line C I, any number of lines as M F, cutting every one of them the crooked line of the Deficient Figure in D, and the straight lines A C, A B and A I in H, G and L; and as G F is to G D, so G L be made to another G N; and through all the points N there be drawn the line A N I, there will be a Deficient Figure A N I B, whose Complement will be A N I K. I say the figure A N I B is to the triangle A B I, as the Deficient Figure A B C D twice taken, is to the same Deficient Figure together with the Complete figure B E.

For as the proportion of A B to A G, that is, of G M to G L, is to the proportion of G M to G N; so is the magnitude of the figure A N I B, to that of its Complement A N I K (by the 2d. *Art.* of this Chapter.)

But (by the same Article), As the proportion of A B to A G, that is, of G M to G L, is to the proportion of G F to G D, that is, (by construction) of G L to G N; so is the figure A B C D to its Complement A D C E.

And by Composition, As the proportion of G M to G L, together with that of G L to G N, is to the proportion of G M to G L; so is the complete figure B E, to the Deficient Figure A B C D.

And by Conversion, As the proportion of G M to G L, is to both the proportions of G M to G L and of G L to G N, that is, to the proportion of G M to G N (which is the proportion compounded of both); so is the Deficient Figure A B C D, to the complete Figure B E.

But it was, As the proportion of G M to G L, to that of G M to G N; so the figure A N I B to its Complement A N I K. And therefore, A B C D. B E :: A N I B. A N I K are proportionals. And by Composition, A B C D + B E. A B C D :: B K. A N I B are proportionals.

And

And by doubling the Conſequents { ABCD+BE. 2 ABCD :: BK. 2 ANIB are proportionals.

And by taking the halfes of the third & the fourth { ABCD+BE. 2 ABCD :: ABI. ANIB are alſo proportionals; which was to be proved.

10 From what has been ſaid of Deficient Figures deſcribed in a Parallelogram, may be found out what proportions Spaces tranſmitted with accelerated Motion in determined times, have to the times themſelves, according as the moved Body is accelerated in the ſeveral times with one or more degrees of Velocity.

For, let the Parallelogram A B C D (in the 6th figure) and in it the three-ſided figure D E B C be deſcribed; and let F G be drawn any where parallel to the baſe, cutting the Diagonal B D in H, and the crooked line B E D in E; & let the proportion of B C to B F be (for example) triplicate to that of F G to F E; whereupon the figure D E B C will be triple to its Complement B E D A; and in like manner, I F being drawn parallel to B C, the three-ſided figure E K B F will be triple to its Complement B K E I. Wherefore, the parts of the Deficient Figure cut off from the Vertex by ſtraight lines parallel to the baſe, namely D E B C and E K B F, will be to one another as the Parallelograms A C and I F; that is, in proportion compounded of the proportions of the altitudes and baſes. Seeing therefore the proportion of the altitude B C to the altitude B F is triplicate to the proportion of the baſe D C to the baſe F E, the figure D E B C to the figure E K B F will be quadruplicate to the proportion of the ſame D C to F E. And by the ſame method, may be found out, what proportion any of the ſaid three-ſided figures, has to any part of the ſame cut off from the Vertex by a ſtraight line parallel to the baſe.

Now as the ſaid figures are underſtood to be deſcribed by the continual decreaſing of the baſe, as of C D (for example) till it end in a point, as in B; ſo alſo they may be underſtood to be deſcribed by the continual encreaſing of a point, as of B, till it acquire any magnitude, as that of C D.

Suppoſe now the figure B E D C to be deſcribed by the encreaſing of the point B to the magnitude C D. Seeing therefore the proportion of B C to B F is triplicate to that of C D to F E, the pro-

proportion of F E to C D will by Converſion (as I ſhall preſently demonſtrate) be triplicate to that B F to B C. Wherefore, if the ſtraight line B C be taken for the meaſure of the time in which the point B is moved, the Figure E K B F will repreſent the Sum of all the encreaſing Velocities in the time B F; and the figure D E B C will in like manner repreſent the Summe of all the encreaſing Velocities in the time B C. Seeing therefore the proportion of the figure E K B F to the figure D E B C, is compounded of the proportions of altitude to altitude, and baſe to baſe; and ſeeing the proportion of F E to C D is triplicate to that of B F to B C; the proportion of the figure E K B F to the figure D E B C, will be quadruplicate to that of BF to BC; that is, the proportion of the Sum of the Velocities in the time BF, to the Sum of the Velocities in the time BC wil be quadruplicate to the proportion of BF to BC. Wherfore if a Body be moved from B with Velocity ſo encreaſing, that the Velocity acquired in the time B F, be to the Velocity acquired in the time B C in triplicate proportion to that of the times themſelves B F to B C, and the Body be carried to F in the time B F; the ſame Body in the time B C will be carried through a line equal to the fifth proportional from B F in the continual proportion of B F to B C. And by the ſame manner of working, we may determine, what Spaces are tranſmitted by Velocities encreaſing according to any other proportions.

It remains, that I demonſtrate the proportion of F E to C D, to be triplicate to that of B F to B C. Seeing therefore the proportion of C D, that is of F G to F E is ſubtriplicate to that of B C to B F; the proportion of F G to F E will alſo be ſubtriplicate to that of F G to F H. Wherefore the proportion of F G to F H is triplicate to that of F G, that is, of C D to F E. But in four continual proportionals, of which the leaſt is the firſt, the proportion of the firſt to the fourth (by the *16 Art.* of the 13 Chap.) is ſubtriplicate to the proportion of the third to the ſame fourh. Wherefore the proportion of F H to G F is ſubtriplicate to that of F E to C D; and therefore the proportion of F E to C D is triplicate to that of F H to F G, that is, of B F to B C, which was to be proved.

It may from hence be collected, that when the Velocity of a

Body, is encreaſed in the ſame proportion with that of the times, the degrees of Velocity above one another proceed as numbers do in immediate ſucceſſion from Unity, namely, as 1,2,3,4, &c. And when the Velocity is encreaſed in proportion duplicate to that of the times, the degrees proceed as numbers from Unity skipping One, as 1, 3, 5, 7, &c. Laſtly, when the proportions of the Velocities are triplicate to thoſe of the times, the progreſſion of the degrees is as that of numbers from Unity skipping Two in every place, as 1, 4, 7, 10, &c. and ſo of other proportions. For Geometrical proportionals, when they are taken in every point, are the ſame with Arithmetical proportionals.

11 Moreover, it is to be noted, that as in quantities which are made by any magnitudes decreaſing, the proportions of the figures to one another, are as the proportions of the altitudes to thoſe of the baſes; ſo alſo it is in thoſe which are made with motion decreaſing, which motion is nothing elſe but that power by which the deſcribed figures are greater or leſs. And therefore in the deſcription of *Archimedes* his *Spiral*, which is done by the continual diminution of the Semidiameter of a Circle in the ſame proportion in which the Circumference is diminiſhed, the Space which is contained within the Semidiameter and the Spiral Line, is a third part of the whole Circle. For the Semidiameters of Circles, in as much as Circles are underſtood to be made up of the aggregate of them, are ſo many Sectors; and therefore in the deſcription of a Spiral, the Sector which deſcribes it, is diminiſhed in duplicate proportions to the diminutions of the Circumference of the Circle in which it is inſcribed; ſo that the Complement of the Spiral (that is, that ſpace in the Circle which is without the Spiral Line,) is double to the ſpace within the Spiral Line. In the ſame manner, if there be taken a mean proportional every where between the Semidiameter of the Circle which contains the Spiral, and that part of the Semidiameter which is within the ſame, there will be made another figure, which will be half the Circle. And to conclude this Rule ſerves for all ſuch Spaces as may be deſcribed by a Line or Superficies decreaſing either in magnitude or power; ſo that if the proportions in which they decreaſe, be commenſurable to the proportions of the times in which they decreaſe,

creaſe, the magnitudes of the figures they deſcribe will be known.

12 The truth of that propoſition which I demonſtrated in the ſecond *Article* (which is the foundation of all that has been ſaid concerning Deficient Figures) may be derived from the Elements of Philoſophy, as having its original in this; *That all equality and inequality between two effects, (that is, all Proportion) proceeds from, and is determined by the equal and unequal cauſes of thoſe effects, or from the proportion which the cauſes concurring to one effect, have to the cauſes which concurre to the producing of the other effect*; and that therefore the proportions of Quantities are the ſame with the proportions of their cauſes. Seeing therefore two Deficient Figures (of which one is the Complement of the other) are made, one by motion decreaſing in a certain time and proportion, the other by the loſs of Motion in the ſame time, the cauſes which make and determine the quantities of both the figures, ſo, that they can be no other then they are, differ onely in this, that the proportions by which the quantity which generates the figure proceeds in deſcribing of the ſame, (that is, the proportions of the remainders of all the times and altitudes) may be other proportions then thoſe by which the ſame generating quantity decreaſes in making the Complement of that Figure, (that is, the proportions of the quantity which generates the Figure continually diminiſhed.) Wherefore, as the proportions of the quantity in which Motion is loſt, is to that of the decreaſing quantities by which the Deficient Figure is generated, ſo will the Defect or Complement be to the Figure it ſelf which is generated.

13 There are alſo other quantities which are determinable from the knowledge of their cauſes, namely, from the compariſon of the Motions by which they are made, and that more eaſily then from the common Elements of Geometry. For example, That the Superficies of any portion of a Sphere, is equal to that Circle, whoſe Radius is a ſtraight Line drawn from the Pole of the portion to the Circumference of its baſe, I may demonſtrate in this manner. Let B A C (in the 7 Figure) be a portion of a Sphere, whoſe Axis is A E, & whoſe baſe is B C; & let A B be the ſtraight line drawn from the Pole A to the baſe in B; and let A D, equal to A B, touch the great Circle B A C in the Pole A. It is to be proved

that the Circle made by the Radius A D, is equal to the Superficies of the portion B A C. Let the plain A E B D be understood to make a revolution about the Axis A E; & it is manifest that by the straight line A D a Circle will be described; and by the arch A B the Superficies of a portion of a Sphere, and lastly, by the Subtense A B the Superficies of a right Cone. Now seeing both the straight line A B, and the arch A B make one and the same revolution, and both of them have the same extreme points A and B, the cause why the the Spherical Superficies which is made by the arch, is greater then the Conical Superficies which is made by the Subtense, is, that A B the arch, is greater then A B the Subtense; and the cause why it is greater consists in this, that although they be both drawn from A to B, yet the Subtense is drawn straight, but the arch angularly, namely according to that angle which the arch makes with the Subtense, which angle is equal to the angle D A B (for an angle of contingence adds nothing to an angle of a Segment, as has been shewn in the 14 Chapter at the 16th Article.) Wherefore the magnitude of the angle D A B is the cause why the Superficies of the portion described by the arch A B, is greater then the Superficies of the right Cone described by the Subtense A B.

Again, the cause why the Circle described by the Tangent A D is greater then the Superficies of the right Cone described by the Subtense A B (notwithstanding that the Tangent and the Subtense are equal, and both moved round in the same time) is this, that A D stands at right angles to the Axis, but A B obliquely; which obliquity consists in the same angle D A B. Seeing therefore the quantity of the angle D A B is that which makes the excess both of the Superficies of the Portion, and of the Circle made by the Radius A D, above the superficies of the Right Cone described by the subtense A B; it follows, that both the Superficies of the Portion, and that of the Circle, do equally exceed the Superficies of the Cone. Wherefore, the Circle made by A D, or A B, and the Spherical Superficies made by the arch A B, are equal to one another; which was to be proved.

14 If these Deficient Figures which I have described in a Parallelogram, were capable of exact description, then any number of

of mean proportionals might be found out between two ſtraight lines given. For example, in the Parallelogram A B C D, (in the 8th. Figure) let the three-ſided figure of two Means be deſcribed, (which many call a *Cubical Parabola*); and let R and S be two given ſtraight lines; between which, if it be required to find two mean proportionals, it may be done thus. Let it be as R to S, ſo B C to B F; and let F E be drawn parallel to B A, and cut the crooked line in E; then through E let G H be drawn parallel and equal to the ſtraight line A D, and cut the Diagonal B D in I; for thus we have G I the greateſt of two Means between G H and G E, as appears by the deſcription of the figure in the 4th Article. Wherefore if it be as G H to G I, ſo R to another line T, that T will be the greateſt of two Means between R and S. And therefore if it be again as R to T, ſo T to another line X, that will be done which was required.

In the ſame manner, four mean proportionals may be found out, by the deſcription of a three-ſided figure of four Means; and ſo, any other number of Means, &c.

AD CAP. XVII

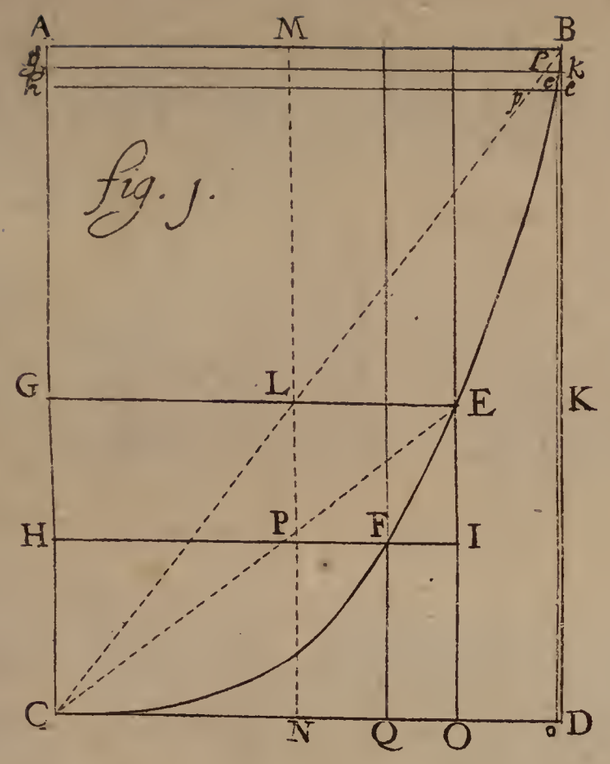


fig. 1.

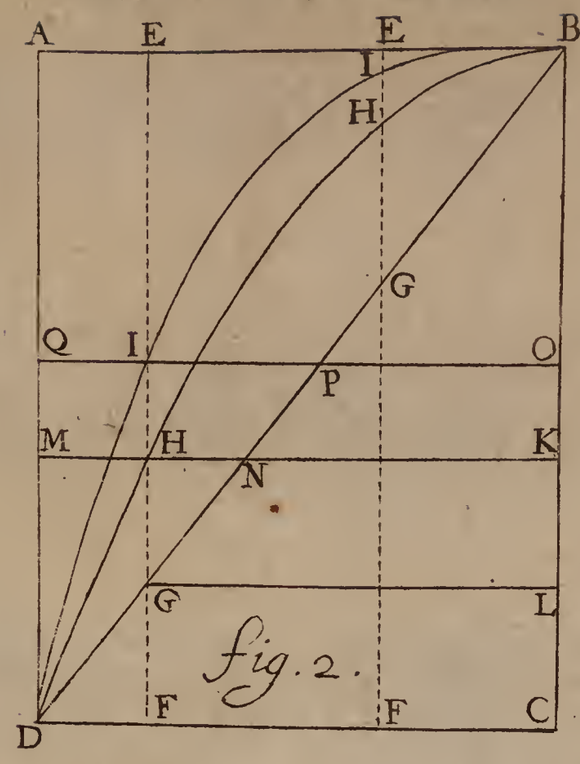


fig. 2.

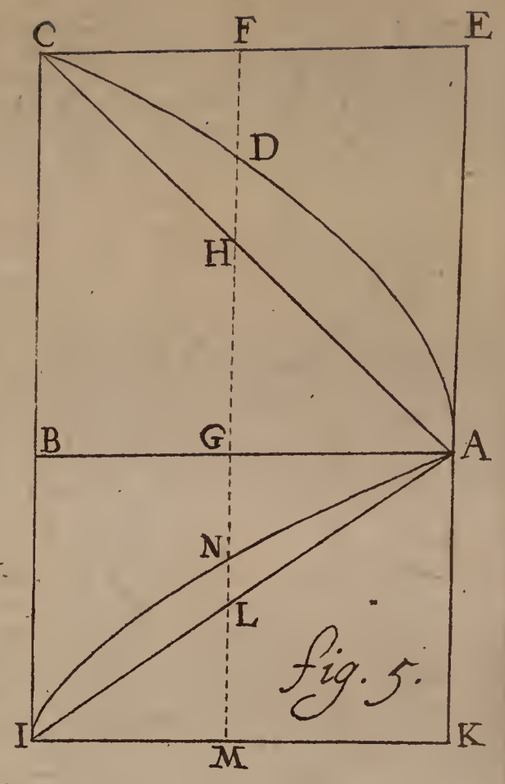


fig. 5.

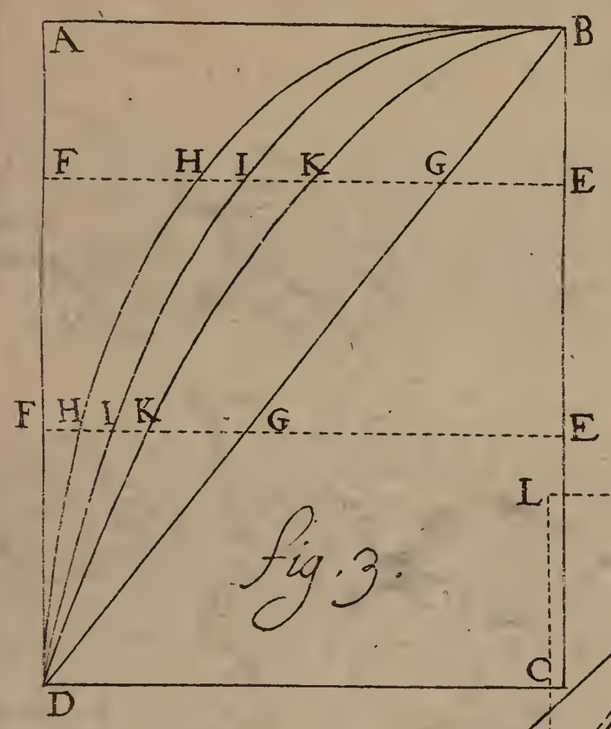


fig. 3.

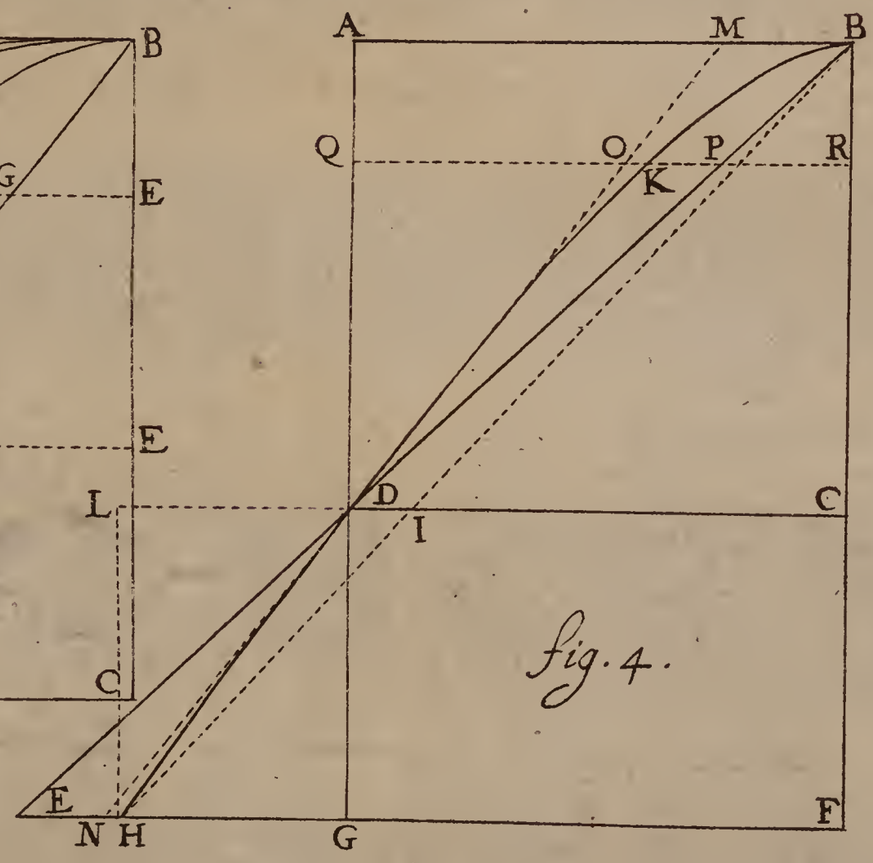


fig. 4.

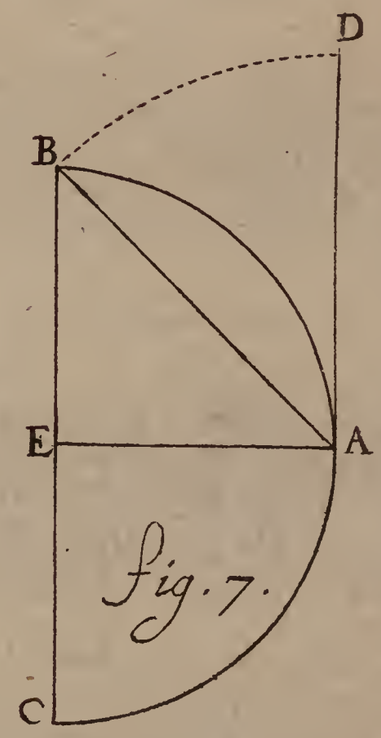


fig. 7.

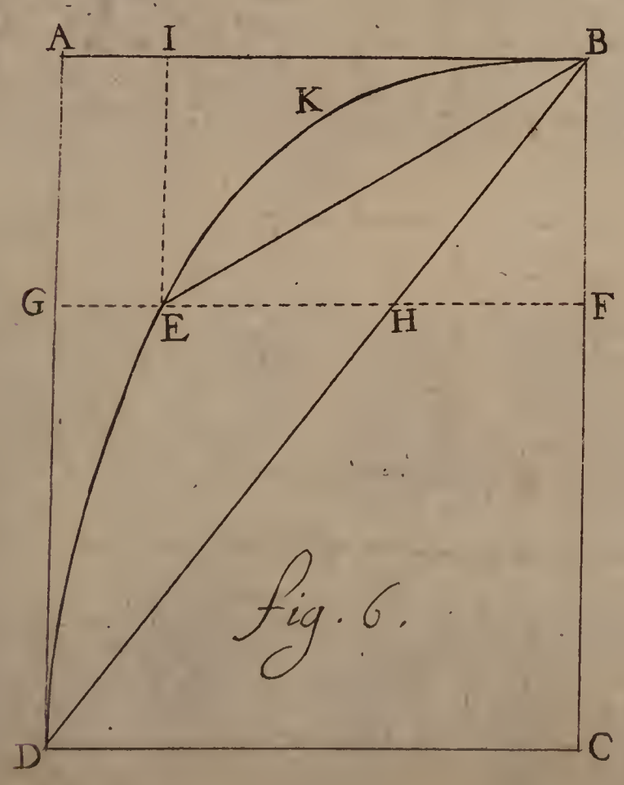
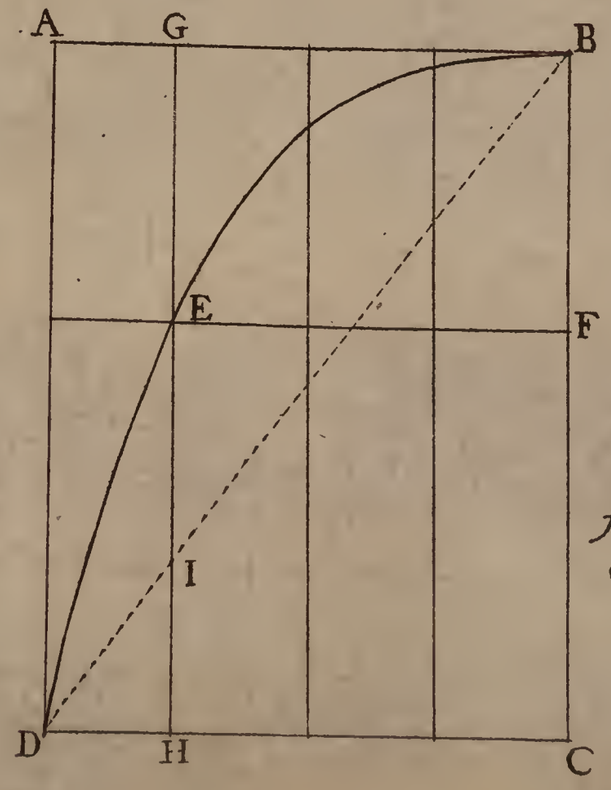


fig. 6.



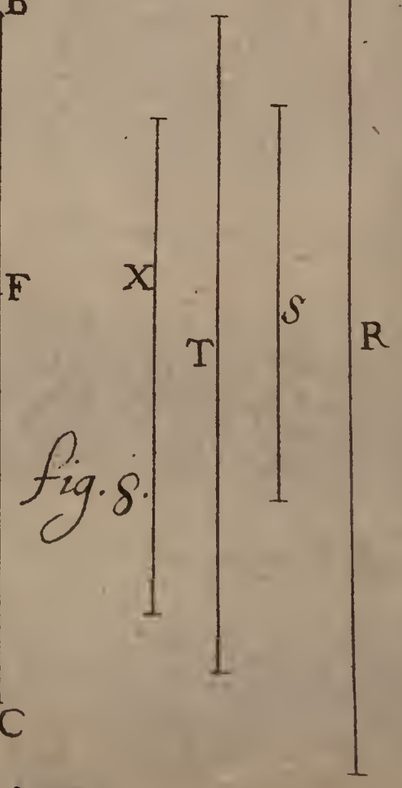


fig. 8.

CHAP. XVIII.

Of the Equation *of* Straight *Lines with the* Crooked *Lines of* Parabolas *and other Figures made in imitation of* Parabolas.

1 *To find a ſtraight Line equal to the crooked Line of a Semiparabola.* 2 *To find a ſtraight Line equal to the Crooked Line of the firſt Semiparabola-ſter, or to the Crooked Line of any other of the Deficient Figures of the Table of the 3d. Article of the precedent Chapter.*

1 A *Parabola* being given, to find a Straight Line equal to the Crooked Line of the Semiparabola.

Let the Parabolical Line given be A B C (in the firſt Figure), and the Diameter found be A D, and the baſe drawn D C, and the Parallelogram A D C E being completed, draw the ſtraight Line A C. Then dividing A D into two equal parts in F, draw F H equal and parallel to D C, cutting A C in K, and the parabolical line in O; and between F H and F O take a mean proportional F P, and draw A O, A P and P C. I ſay that the two Lines A P and P C taken together as one Line, is equal to the parabolical line A B O C.

For the line A B O C being a parabolical line, is generated by the concourſe of two Motions, one Uniform from A to E, the other in the ſame time uniformly accelerated from reſt in A to D. And becauſe the motion from A to E is uniform, A E may repreſent the times of both thoſe motions from the beginning to the end. Let therefore A E be the time; and conſequently the lines

or

ordinately applyed in the Semiparabola, will deſigne the parts of time wherein the Body that deſcribeth the line A B O C is in every point of the ſame; ſo that as at the end of the time A E or D C it is in C, ſo at the end of the time F O it will be in O. And becauſe the Velocity in A D is encreaſed uniformly, that is, in the ſame proportion with the time, the ſame lines ordinately applyed in the Semiparabola will deſigne alſo the continual augmentations of the *Impetus*, till it be at the greateſt, deſigned by the baſe D C. Therefore ſuppoſing Uniform motion in the line A F, in the time F K the Body in A by the concourſe of the two uniform motions in A F and F K will be moved uniformly in the line A K; and K O wil be the encreaſe of the *Impetus* or Swiftneſs gained in the time F K; and the line A O will be uniformly deſcribed by the concourſe of the two uniform motions in A F and F O in the time F O. From O draw O L parallel to E C, cutting A C in L; & draw L N parallel to D C, cutting E C in N, and the parabolical line in M; and produce it on the other ſide to A D in I; and I N, I M and I L will be (by the conſtruction of a Parabola) in continual proportion, & equal to the three lines F H, F P and F O; and a ſtraight line parallel to E C paſſing through M will fall on P; and therefore O P will be the encreaſe of *Impetus* gained in the time F O or I L. Laſtly, produce P M to C D in Q; and Q C, or M N, or P H will be the encreaſe of *Impetus* proportional to the time F P, or I M, or D Q. Suppoſe now uniform motion from H to C in the time P H. Seeing therefore in the time F P with uniform motion and the *Impetus* encreaſed in proportion to the times, is deſcribed the ſtraight line A P; and in the reſt of the time and *Impetus*, namely P H, is deſcribed the line C P uniformly; it followeth that the whole line A P C is deſcribed with the whole *Impetus*, and in the ſame time wherewith is deſcribed the paraboliall line A B C; and therefore the line A P C, made of the two ſtraight lines A P and P C, is equal to the parabolical line A B C; which was to be proved.

2. To find a Straight line equal to the Crooked line of the firſt Semiparabolaſter.

Let A B C be the Crooked line of the firſt Semiparabolaſter; A D the Diameter; D C the Baſe; and let the Parallelogram com-

completed be A D C E, whoſe Diagonal is A C. Divide the Diameter into two equal parts in F, and draw F H equal and parallel to D C, cutting A C in K, the Crooked line in O, and E C in H. Then draw O L parallel to E C, cutting A C in L; and draw L N parallel to the baſe D C, cutting the Crooked line in M, and the ſtraight line E C in N; and produce it on the other ſide to A D in I. Laſtly, through the point M draw P M Q parallel and equal to H C, cutting F H in P; and joyn C P, A P and A O. I ſay the two Straight lines A P and P C are equal to the Crooked line A B O C.

For the line A B O C being the Crooked line of the firſt Semiparabolaſter, is generated by the concourſe of two Motions, one uniform from A to E, the other in the ſame time accelerated from reſt in A to D, ſo as that the *Impetus* encreaſeth in proportion perpetually triplicate to that of the encreaſe of the time, or (which is all one) the lengths tranſmitted are in proportion triplicate to that of the times of their tranſmiſſion; for as the *Impetus* or Quickneſſes encreaſe, ſo the Lengths tranſmitted encreaſe alſo. And becauſe the motion from A to E is uniform, the line A E may ſerve to repreſent the time, and conſequently the lines ordinately drawn in the Semiparabolaſter, will deſigne the parts of time wherein the Body beginning from reſt in A, deſcribeth by its motion the Crooked line A B O C. And becauſe D C which repreſents the greateſt acquired *Impetus* is equal to A E, the ſame ordinate lines will repreſent the ſeveral augmentations of the *Impetus* encreaſing from reſt in A. Therefore ſuppoſing uniform Motion from A to F in the time F K, there will be deſcribed by the concourſe of the two uniform Motions A F and F K the line A K uniformly, and K O will be the encreaſe of *Impetus* in the time F K; And by the concourſe of the two uniform Motions in A F and F O, will be deſcribed the line A O uniformly. Through the point L draw the ſtraight line L M N parallel to D C, cutting the ſtraight line A D in I, the crooked line A B C in M, and the ſtraight line E C in N; and through the point M the ſtraight line P M Q parallel and equal to H C, cutting D C in Q, and F H in P. By the concourſe therefore of the two uniform Motions in A F and F P in the time F P will be uniformly deſcribed the ſtraight line A P; and L M or O P will be the encreaſe of *Impetus* to be added for the time F O. And becauſe the proportion of I N

to I L is triplicate to the proportion of I N to I M, the proportion of F H to F O will alſo be triplicate to the proportion of F H to F P; and the proportional *Impetus* gained in the time F P is P H. So that F H being equal to P C which deſigned the whole *Impetus* acquired by the acceleration, there is no more encreaſe of *Impetus* to be computed. Now in the time P H ſuppoſe an uniform motion from H to C; and by the two uniform motions in C H and H P will be deſcribed uniformly the Straight line P C. Seeing therefore the two Straight lines A P and P C are deſcribed in the time A E with the ſame encreaſe of *Impetus* wherewith the Crooked line A B C is deſcribed in the ſame time A E, that is, ſeeing the Line A P C and the Line A B C are tranſmitted by the ſame Body in the ſame Time, & with equal Velocities, the Lines themſelves are equal; which was to be demonſtrated.

By the ſame method, if any of the Semiparabolaſters in the Table of the 3d Article of the precedent Chapter be exhibited, may be found a Straight line equal to the Crooked line thereof, namely, by dividing the Diameter into two equal parts, and proceeding as before. Yet no man hitherto hath compared any Crooked with any Straight Line, though many Geometricians of every Age have endeavoured it. But the cauſe why they have not done it may be this, that there being in *Euclide* no Definition of Equality, nor any mark by which to judge of it beſides Congruity (which is the 8th. *Axiome* of the firſt Book of his Elements) a thing of no uſe at all in the comparing of Straight and Crooked; and others after *Euclide* (except *Archimedes* and *Apollonius*, and in our time *Bonaventura*) thinking the induſtry of the Ancients had reached to all that was to be done in Geometry, thought alſo, that all that could be propounded, was either to be deduced from what they had written, or elſe that it was not at all to be done. It was therefore diſputed by ſome of thoſe Ancients themſelves, whether there might be any Equality at all between Crooked and Straight Lines; Which queſtion *Archimedes* (who aſſumed that ſome Straight line was equal to the Circumference of a Circle) ſeems to have deſpiſed, as he had reaſon. And there is a late Writer that granteth that between a Straight and a Crooked Line there is Equality; but now, now ſayes he, ſince the fall of *Adam*, without the ſpecial aſſiſtance of Divine Grace, it is not to be found.

Cap: XVIII.

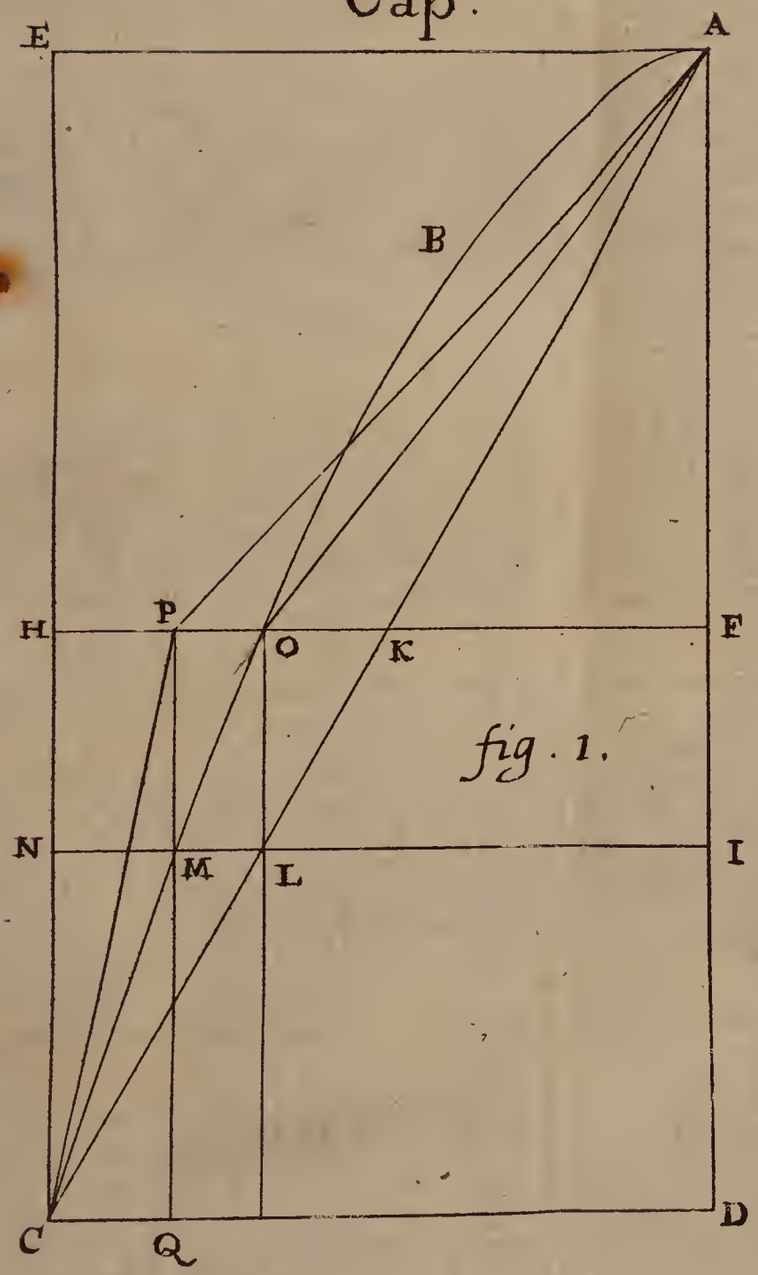


fig. 1.

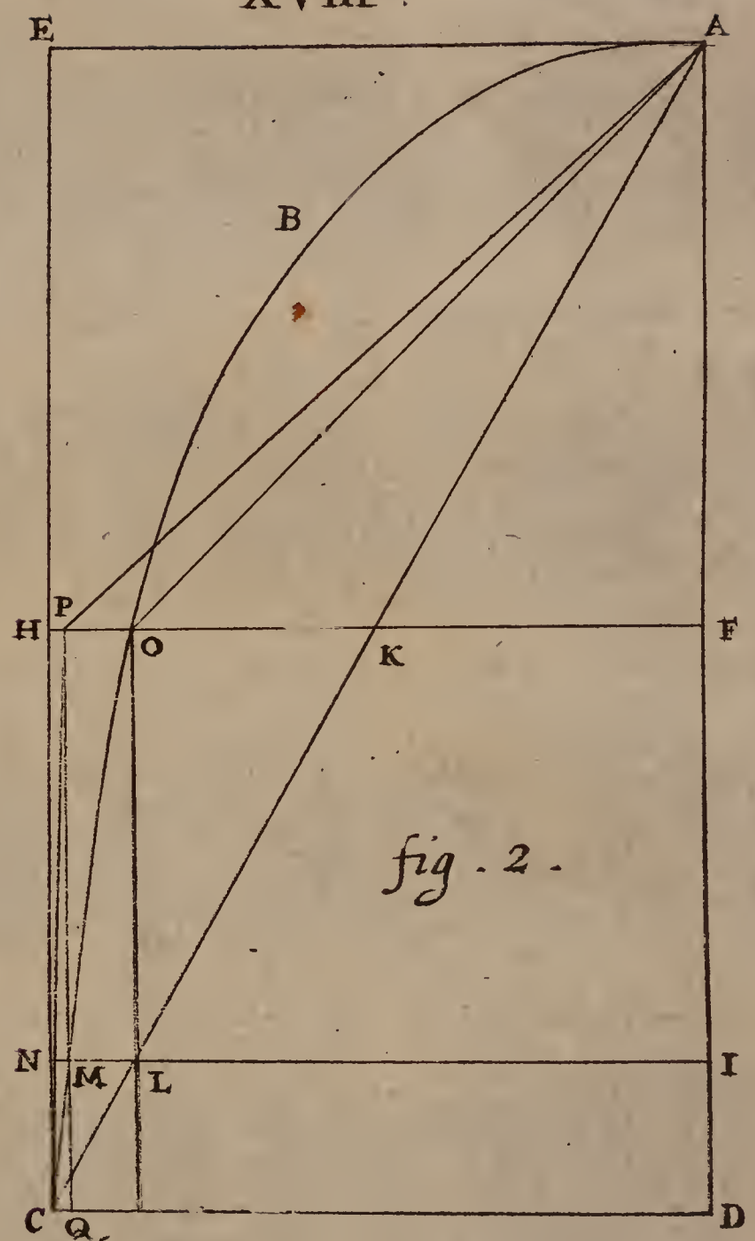


fig. 2.

CHAP. XIX.

Of Angles *of* Incidence *and* Reflection, *equal by ſuppoſition.*

1 *If two ſtraight lines falling uponanother ſtraight line be parallel, the lines reflected from them ſhall alſo be parallel.* 2 *If two ſtraight lines drawn from one point, fall upon another ſtraight line, the lines reflected from them, if they be drawn out the other way, will meet in an angle equal to the angle made by the lines of Incidence.* 3 *If two ſtraight parallel lines drawn (not oppoſitely but) from the ſame parts, fall upon the Circumference of a Circle, the lines reflected from them, if produced they meet within the Circle, will make an angle double, to that which is made by two ſtraight lines drawn from the Center to the points of Incidence.* 4 *If two ſtraight lines drawn from the ſame point without a Circle fall upon the Circumference, and the lines reflected from them, being produced meet within the Circle, they will make an angle equal to twice that angle which is made by two ſtraight lines drawn from the Center to the points of Incidence, together with the angle which the incident lines themſelves make.* 5 *If two ſtraight lines drawn from one point fall upon the concave Circumference of a Circle, and the angle they make be leſs then twice the angle at the Center, the lines reflected from them, and meeting within the Circle, will make an angle which being added to the angle of the incident lines, will be equal to twice the angle at the Center.* 6 *If through any one point two unequal Chords be drawn cutting one another, and the Center of the Circle be not placed between them, and the lines reflected from them concurre whereſoever, there cannot through the point through which the two former lines were drawn, be drawn any other ſtraight line, whoſe reflected line ſhall paſs through the common point of the two former lines reflected.* 7 *In equal Chords the ſame is not true.* 8 *Two points being given in the Circumference of a Circle, to draw two ſtraight lines to them, ſo as that their reflected lines may contain any angle given.* 9 *If a ſtraight line falling upon the Circumference of a Circle be produced till it reach the Semidiameter, and that part of it which is intercepted between the Circumference and the Semidiameter, be equal to that part of the Semidiameter which is between the point of concourſe & the center, the reflected line will be parallel to the Semidiameter.* 10 *If from a point within a*

Circle, two ſtraight lines be drawn to the Circumference, and their reflected lines meet in the Circumference of the ſame Circle, the angle made by the reflected lines, will be a third part of the angle made by the incident lines.

WHether a Body, falling upon the ſuperficies of another Body and being reflected from it, do make equal angles at that ſuperficies, it belongs not to this place to diſpute, being a knowledge which depends upon the natural cauſes of Reflection; of which hitherto nothing has been ſaid, but ſhall be ſpoken of hereafter.

In this place therefore let it be ſuppoſed, that the angle of Incidence is equal to the angle of Reflection, that our preſent ſearch may be applyed not to the finding out of the cauſes, but ſome conſequences of the ſame.

I call an *Angle of Incidence*, that which is made between a ſtraight line and another line (ſtraight or crooked) upon which it falls, and which I call the *Line Reflecting*; and an *Angle of Reflection* equal to it, that which is made at the ſame point between the ſtraight line which is reflected, and the line reflecting.

1 If two ſtraight lines which fall upon another ſtraight line be be parallel, their reflected lines ſhall be alſo parallel.

Let the two ſtraight lines A B and C D (in the 1 figure) which fall upon the ſtraight line E F, at the points B and D, be parallel; and let the lines reflected from them be B G and D H. I ſay B G and D H are alſo parallel.

For the angles A B E and C D E are equal by reaſon of the parallelleliſme of A B and C D; and the angles G B F and H D F are equal to them by ſuppoſition; for the lines B G and D H are reflected from the lines A B and C D. Wherefore B G and D H are parallel.

2 If two ſtraight lines drawn from the ſame point, fall upon another ſtraight line, the lines reflected from them, if they be drawn out the other way, will meet in an angle equal to the angle of the Incident lines.

From the point A C (in the 2d. figure) let the two ſtraight lines A B and A D be drawn; and let them fall upon the ſtraight line E K at the points B and D; and let the lines B I and D G be reflected from them. I ſay, I B and G D do converge, and that if they be

be produced on the other ſide of the line E K they ſhall meet, as in F; and that the angle B F D ſhal be equal to the angle B A D.

For the angle of Reflection I B K is equal to the angle of Incidence A B E; and to the angle I B K, its vertical angle E B F is equal; and therefore the angle A B E is equal to the angle E B F. Again, the angle A D E is equal to the angle of Reflection G D K, that is, to its vertical angle E D F; and therefore the two angles A B D and A D B of the triangle A B D, are one by one equal to the two angles F B D and F D B of the triangle F B D; Wherfore alſo the third angle B A D is equal to the third angle B F D; which was to be proved.

Corollary 1. If the ſtraight line A F be drawn, it will be perpendicular to the ſtraight line E K. For both the angles at E will be equal, by reaſon of the equality of the two angles A B E and F B E, and of the two ſides A B and F B.

Corollary 2. If upon any point between B and D there fall a ſtraight line, as A C, whoſe reflected line is C H, this alſo produced beyond C, will fall upon F; which is evident by the demonſtration above.

3 If from two points taken without a Circle, two ſtraight parallel lines drawn (not oppoſitely but) from the ſame parts, fall upon the Circumference; the lines reflected from them, if produced they meet within the Circle, will make an angle double to that which is made by two ſtraight lines drawn from the Center to the points of Incidence.

Let the two ſtraight parallels A B and D C (in the 3d figure) fall upon the Circumference B C at the points B and C; and let the Center of the Circle be E; and let A B reflected be B F, and D C reflected be C G; and let the lines F B and G C produced meet within the Circle in H; and let E B and E C be connected. I ſay the angle F H G is double to the angle B E C.

For ſeeing A B and D C are parallels, and E B cuts A B in B, the ſame E B produced will cut D C ſomewhere; let it cut it in D, & let D C be produced howſoever to I, and let the interſection of D C & B F be at K. The angle therefore I C H (being external to the triangle C K H,) will be equal to the two oppoſite angles C K H and C H K. Again, I C E (being external to the triangle C D E)

is

is equal to the two angles at D and E. Wherefore the angle I C H, being double to the angle I C E, is equal to the angles at D and E twice taken; and therefore the two angles C K H and C H K are equal to the two angles at D and E twice taken. But the angle C K H is equal to the angles D and A B D, that is, D twice taken, (for A B and D C being parallels, the altern angles D, and A B D are equal). Wherefore C H K, that is the angle, F H G is also equal to the angle at E twice taken; which was to be proved.

Corollary. If from two points taken within a circle, two ſtraight parallels fall upon the circumference, the lines reflected from them ſhall meet in an angle, double to that which is made by two ſtraight lines drawn from the center to the points of Incidence. For the parallels L B and I C falling upon the points B and C, are reflected in the lines B H and C H, and make the angle at H double to the angle at E, as was but now demonſtrated.

4 If two ſtraight lines drawn from the ſame point without a circle, fall upon the circumference, and the lines reflected from them being produced meet within the circle, they will make an angle equal to twice that angle which is made by two ſtraight lines drawn from the center to the points of Incidence together with the angle which the incident lines themſelves make.

Let the two ſtraight lines A B and A C (in the 4th figure) be drawn from the point A to the circumference of the circle, whoſe center is D; and let the lines reflected from them be B E and C G, and being produced make within the circle the angle H; alſo let the two ſtraight lines D B and D C be drawn from the center D to the points of Incidence B and C. I ſay the angle H is equal to twice the angle at D together with the angle at A.

For let A C be produced howſoever to I. Therefore the angle C H (which is external to the triangle C K H) will be equal to the two angles G K H and C H K. Again, the angle I C D (which is external to the triangle C L D) will be equal to the two angles C L D and C D L. But the angle I C H is double to the angle I C D, and is therefore equal to the angles C L D and C D L twice taken. Wherefore the angles C K H and C H K are equal to the angles C L D and C D L twice taken. But the angle C L D (being exter-

nal

nal to the triangle AL B) is equal to the two angles LAB & LBA; & consequently CLD twice taken is equal to LAB &L A B twice taken. Wherefore C K H & C H K are equal to the angle C D L together with L A B and L B A twice taken. Alſo the angle CKH is equal to the angle L A B once, and A B K, that is, L B A twice taken. Wherefore the Angle C HK is equal to the remaining angle C D L (that is, to the angle at D) twice taken, and the angle L A B (that is, the angle at A) once taken; which was to be proved.

Corollary. If two ſtraight converging lines, as I C and M B fall upon the concave circumference of a circle, their reflected lines, as C H and B H, will meet in the angle H, equal to twice the angle D, together with the angle at A made by the incident lines produced. Or, if the Incident lines be H B and I C, whoſe reflected lines C H and B M meet in the point N, the angle C N B will be equal to twice the angle D, together with the angle C K H made by the lines of Incidence. For the angle C N B is equal to the angle H (that is, to twice the angle D) together with the two angles A and N B H (that is K B A). But the angles K B A and A are equal to the angle C K H. Wherefore the angle C N B is equal to twice the angle D, together with the angle C K H made by the lines of Incidence I C and H B produced to K.

5 If two ſtraight lines drawn from one point, fall upon the concave circumference of a circle, and the angle they make be leſſe then twice the angle at the center; the lines reflected from them, and meeting within the circle, will make an angle, which being added to the angle of the incident lines, will be equal to twice the angle at the center.

Let the two Lines A B and A C (in the 5th figure) drawn from the point A, fall upon the concave circumference of the circle whoſe center is D; & let their reflected Lines B E and C E meet in the point E; alſo let the angle A be leſs then twice the angle D. I ſay the angles A and E together taken are equal to twice the angle D.

For let the ſtraight Lines A B and E C cut the ſtraight Lines D C and D B in the points G and H; and the angle B H C will be equal to the two angles E B H and E; alſo the ſame angle B H C will be equal to the two angles D and D C H; and in like manner the angle B G C will be equal to the two angles A C D & A, & the

ſame

ſame angle B G C will be alſo equal to the two angles D B G and D. Wherefore the four angles E B H, E, A C D and A are equal to the four angles D, D C H, D B G and D. If therefore equals be taken away on both ſides, namely, on one ſide A C D and E B H, and on the other ſide D C H and D B G (for the angle E B H is equal to the angle D B G, and the angle A C D equal to the angle D C H) the remainders on both ſides will be equal, namely, on one ſide the angles A and E, and on the other the angle D twice taken. Wherefore the angles A and E are equal to twice the angle D.

Corollary. If the angle A be greater then twice the angle D, their reflected Lines will diverge. For, by the *Corollary* of the third Propoſition, if the angle A be equal to twice the angle D, the reflected Lines B E and C E will be parallel; and if it be leſſe, they will concurre, as has now been demonſtrated; and therefore if it be greater, the reflected Lines B E and C E will diverge, and conſequently, if they be produced the other way, they will concurre, and make an angle equal to the exceſſe of the angle A above twice the angle D; as is evident by the fourth Article.

6 If through any one point, two unequal chords be drawn, cutting one another, either within the circle, or (if they be produced) without it, and the center of the circle be not placed between them, and the Lines reflected from them concurre whereſoever; there cannot through the point through which the former Lines were drawn, be drawn another ſtraight Line, whoſe reflected Line ſhall paſſe through the point where the two former reflected Lines concurre.

Let any two unequal chords, as B K and C H (in the 6th Figure) be drawn through the point A in the circle B C; and let their reflected Lines B D and C E meet in F; and let the center not be between A B and A C; and from the point A let any other ſtraight Line as A G be drawn to the circumference between B and C. I ſay G N, which paſſes through the point F, where the reflected Lines B D and C E meet, will not be the reflected Line of A G.

For let the arch B L be taken equal to the arch B G, and the ſtraight Line B M equal to the ſtraight Line B A; and L M being drawn, let it be produced to the circumference in O. Seeing therefore

fore B A and B M are equal, and the arch B L equal to the arch B G, and the angle M B L equal to the angle A B G; A G and M L will alſo be equal, and (producing G A to the circumference in I) the whole lines L O and G I will in like manner be equal. But L O is greater then G F N (as ſhall preſently be demonſtrated) and therefore alſo G I is greater then G N. Wherefore the angles N G C and I G B are not equal. Wherefore the Line G F N is not reflected from the Line of Incidence A G; and conſequently no other ſtraight Line (beſides A B and A C) which is drawn through the point A, and falls upon the circumference B C, can be reflected to the point F; which was to be demonſtrated.

It remains that I prove L O to be greater then G N; which I ſhall do in this manner. L O and G N cut one another in P; and P L is greater then P G. Seeing now L P. P G :: P N. P O are proportionals, therefore the two Extremes L P and P O together taken, (that is L O), are greater then P G and P N together taken, (that is, G N;) which remained to be proved.

7 But if two equal chords be drawn through one point within a circle, and the Lines reflected from them meet in another point, then another ſtraight Line may be drawn between them through the former point, whoſe reflected Line ſhall paſs through the later point.

Let the two equal chords B C and E D (in the 7th figure) cut one another in the point A within the circle B C D; and let their reflected Lines C H and D I meet in the point F. Then dividing the arch C D equally in G, let the two chords G K and G L be drawn through the points A and F. I ſay G L will be the Line reflected from the chord K G. For the four chords B C, C H, E D and D I, are by ſuppoſition all equal to one another; and therefore the arch B C H is equal to the arch E D I; as alſo the angle B C H to the angle E D I; & the angle A M C to its vertical angle F M D; and the ſtraight Line D M to the ſtraight Line C M; and in like manner, the ſtraight Line A C to the ſtraight Line F D; and the chords C G and G D being drawn, will alſo be equal; as alſo the angles F D G and A C G, in the equal Segments G D I and G C B. Wherefore the ſtraight Lines F G and A G are equal; and therefore the angle F G D is equal to the angle A G C, that is, the an-

gle of Incidence equal to the angle of Reflection. Wherefore the line G L is reflected from the incident Line K G; which was to be proved.

Corollary. By the very ſight of the figure, it is manifeſt, that if G be not the middle point between C and D, the reflected Line G L will not paſs through the point F.

8 Two points in the circumference of a circle being given, to draw two ſtraight Lines to them, ſo as that their reflected Lines may be parallel, or contain any angle given.

In the circumference of the circle whoſe center is A (in the 8th. figure) let the two points B and C be given; and let it be required to draw to them from two points taken without the circle, two incident Lines, ſo, that their reflected Lines may (firſt) be parallel.

Let A B and A C be drawn; as alſo any incident Line D C, with its reflected Line C F; and let the angle E C D be made double to the angle A; and let H B be drawn parallel to E C, and produced till it meet with D C produced in I. Laſtly, producing A B indefinitely to K, let G B be drawn, ſo, that the angle G B K may be equal to the angle H B K, and then G B will be the reflected Line of the incident Line H B. I ſay D C and H B are two incident Lines, whoſe reflected Lines C F and B G are parallel.

For ſeeing the angle E C D is double to the angle B A C, the angle H I C is alſo (by reaſon of the parallels E C and H I) double to the ſame B A C; Therefore alſo F C and G B (namely the lines reflected from the incident lines D C and H B are parallel. Wherefore the firſt thing required, is done.

Secondly, let it be required to draw to the points B & C two ſtraight lines of Incidence, ſo, that the lines reflected from them may contain the given angle Z.

To the angle E C D made at the point C, let there be added on one ſide the angle D C L equal to half Z, and on the other ſide the angle E C M equal to the angle D C L; and let the ſtraight Line B N be drawn parallel to the ſtraight line C M; and let the angle K B O be made equal to the angle N B K; which being done, B O will be the Line of Reflection from the Line of Incidence N B. Laſtly, from the incident Line L C, let the reflected Line C O be drawn, cutting B O at O, and making the angle C O B. I ſay the angle C O B is equal to the angle Z.

Let

Let N B be produced till it meet with the ſtraight line L C produced in P. Seeing therefore the angle L C M is by conſtruction equal to twice the angle BAC together with the angle Z; the angle N P L (which is equal to L C M by reaſon of the parallels N P and M C) will alſo be equal to twice the ſame angle B A C together with the angle Z. And ſeeing the two ſtraight lines O C and O B fall from the point O upon the points C and B; and their reflected lines L C and N B meet in the point P; the angle N P L will be equal to twice the angle B A C together with the angle COP. But I have already proved the angle NPL to be equal to twice the angle B A C together with the angle Z. Therefore the angle C O P is equal to the angle Z; Wherefore, Two points in the circumference of a Circle being given, I have drawn, &c. which was to be done.

But if it be required to draw the incident Lines from a point within the circle, ſo, that the Lines reflected from them may contain an angle equal to the angle Z, the ſame method is to be uſed, ſaving that in this caſe the angle Z is not to be added to twice the angle B A C, but to be taken from it.

9. If a ſtraight line falling upon the circumference of a circle, be produced till it reach the Semidiameter, and that part of it which is intercepted between the circumference and the Semidiameter, be equal to that part of the Semidiameter which is between the point of concourſe and the center, the reflected Line will be parallel to the Semidiameter.

Let any Line A B (in the 9th figure) be the Semidiameter of the circle whoſe center is A; and upon the circumference B D let the ſtraight Line C D fall, and be produced till it cut A B in E, ſo, that E D and E A may be equal; & from the incident Line C D let the Line D F be reflected. I ſay A B and D F will be parallel.

Let A G be drawn through the point D. Seeing therefore E D and E A are equal, the angles E D A and E A D will alſo be equal. But the angles F D G and E D A are equal (for each of them is half the angle E D H or F D C.) Wherefore the angles F D G and E A D are equal, and conſequently D F and A B are parallel; which was to be proved.

Corollaby. If E A be greater then E D, then D F and A B being produced will concurre; but if E A be leſs then E D, then B A and D H being produced will concurre.

10. If from a point within a circle, two ſtraight Lines be drawn to the Circumference, and their reflected Lines meet in the Circumference of the ſame circle, the angle made by the Lines of Reflection, will be a third part of the angle made by the Lines of Incidence.

From the point B. (in the 10th figure) taken within the circle whoſe center is A, let the two ſtraight lines B C and B D be drawn to the circumference; and let their reflected Lines C E and D E meet in the circumference of the ſame circle at the point E. I ſay the angle C E D will be a third part of the angle C B D.

Let A C and A D be drawn. Seeing therefore the angles C E D and C B D together taken, are equal to twice the angle C A D (as has been demonſtrated in the 5th article); and the angle C A D twice taken is quadruple to the angle C E D; the angles C E D and C B D together taken, will alſo be equal to the angle C E D four times taken; and therefore if the angle C E D be taken away on both ſides, there will remain the angle C B D on one ſide, equal to the angle C E D thrice taken on the other ſide; which was to be demonſtrated.

Coroll. Therefore a point being given within a Circle, there may be drawn two Lines from it to the Circumference, ſo as their reflected Lines may meet in the Circumference. For it is but triſecting the Angle C B D; which how it may be done, ſhall be ſhewn in the following Chapter.

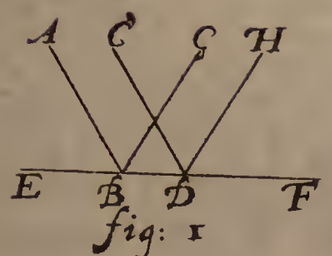


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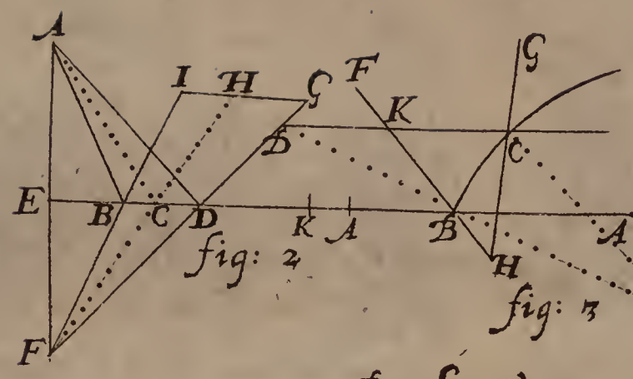


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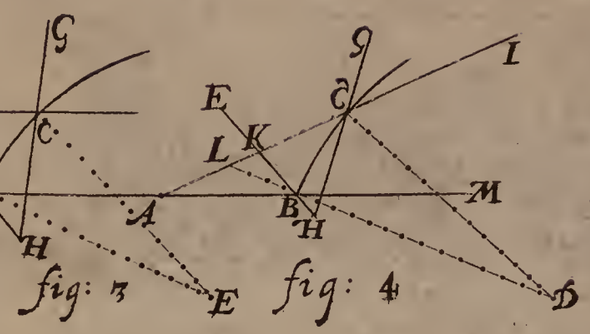


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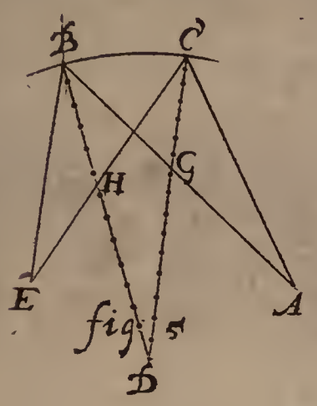


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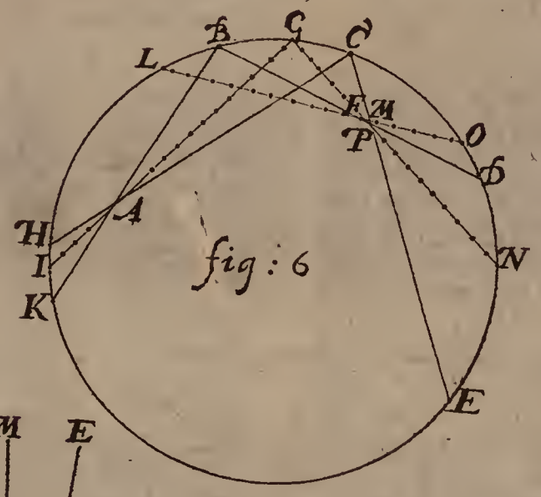


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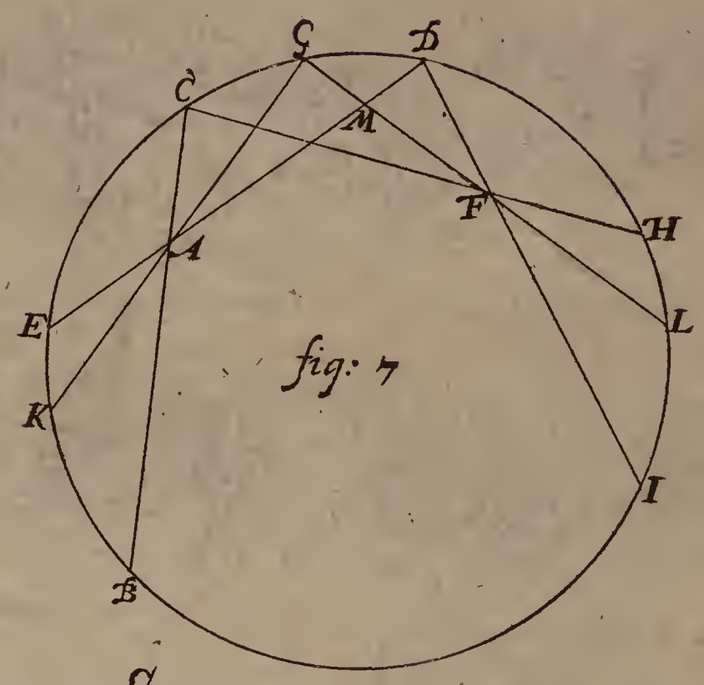


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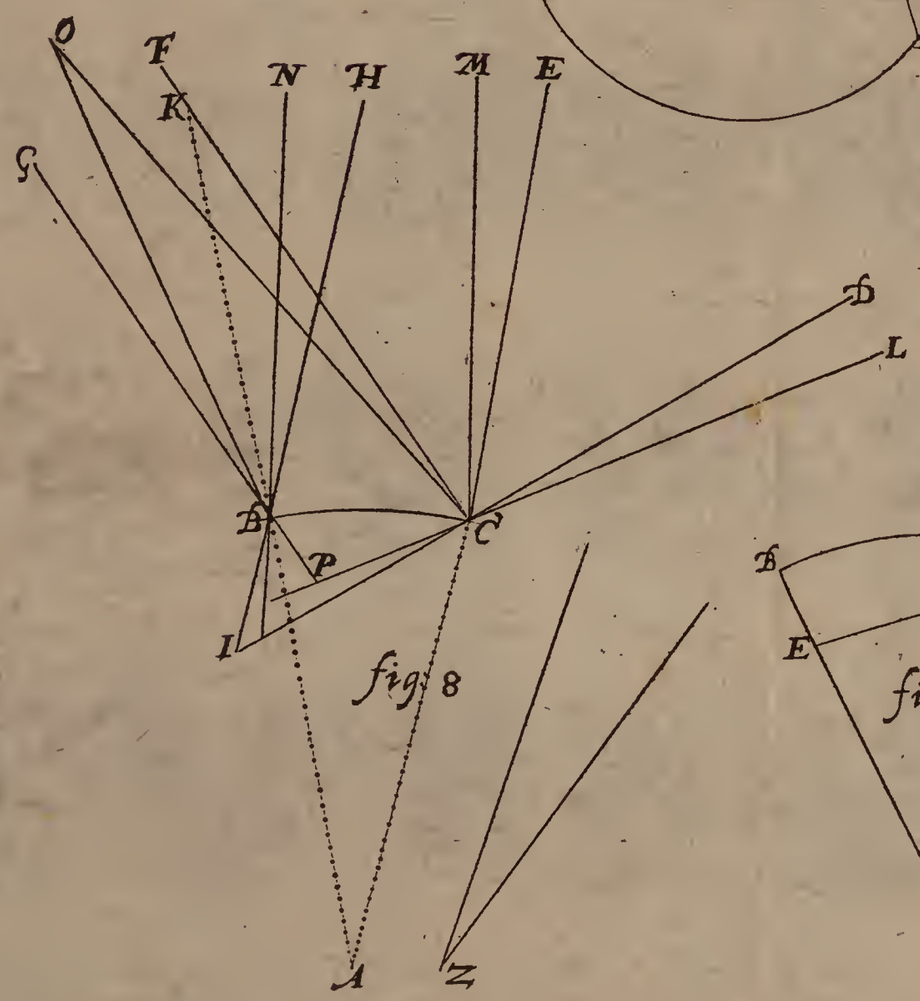


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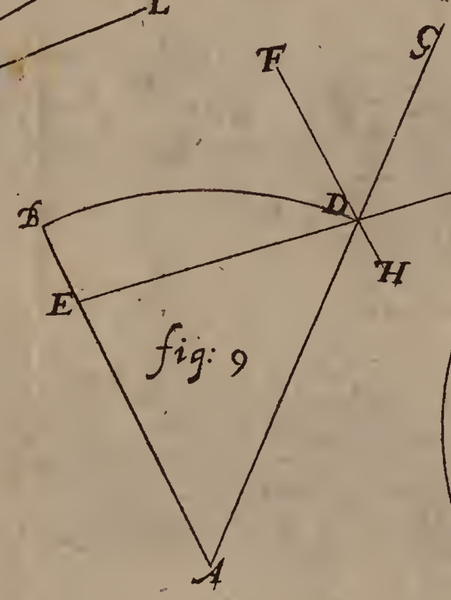


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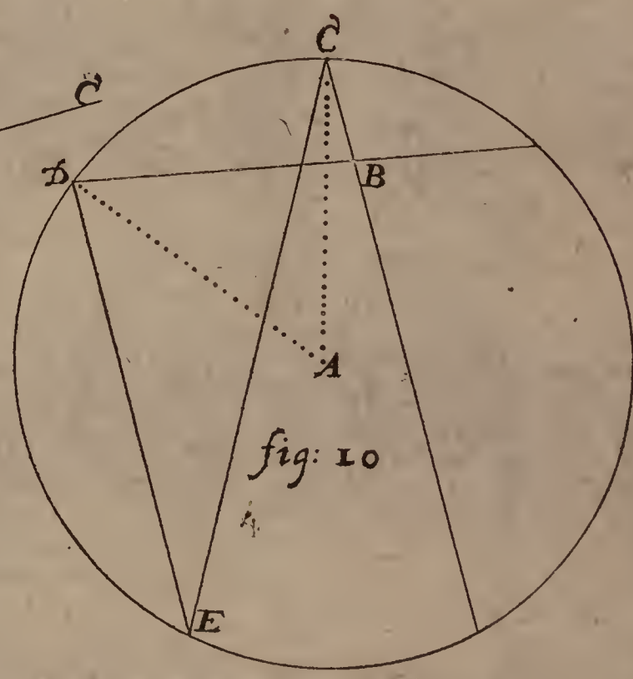


fig: 10

CHAP. XX.

Of the Dimenſion *of a* Circle, *and the* Diviſion *of* Angles *or* Arches.

1 *The Dimenſion of a Circle neer determined in Numbers by* Archimedes *and others.* 2 *The firſt attempt for the finding out of the Dimenſion of a Circle by Lines.* 3 *The ſecond attempt for the finding out of the Dimenſion of a Circle from the conſideration of the nature of Crookedneſs.* 4 *The third attempt; and ſome things propounded to be further ſearched into.* 5 *The Equation of the Spiral of* Archimedes *with a ſtraight Line.* 6 *Of the Analyſis of Geometricians by the Powers of Lines.*

1 IN the comparing of an Arch of a Circle with a Straight Line, many and great Geometricians, even from the moſt ancient times, have exerciſed their wits; and more had done the ſame, if they had not ſeen their pains, though undertaken for the common good, if not brought to perfection, vilified by thoſe that envy the prayſes of other men. Amongſt thoſe Ancient Writers whoſe Works are come to our hands, *Archimedes* was the firſt that brought the Length of the Perimeter of a Circle within the limits of Numbers very litle differing from the truth; demonſtrating the ſame to be leſs then three Diameters and a ſeventh part, but greater then three Diameters and ten ſeventy one parts of the Diameter. So that ſuppoſing the Radius to conſiſt of 10000000 equal parts, the Arch of a Quadrant will be between 15714285 and 15704225 of the ſame parts. In our times *Ludovicus Van Cullen* & *Willebrordus Snellius* with joint endeavour have come yet neerer to the truth; and pronounced from true Principles, that the Arch of a Quadrant (putting, as before 10000000 for Radius) differs not one whole Unity from the number 15707963; which, if they had ex-

exhibited their arithmetical operations(and no man had discovered any errour in that long work of theirs) had been demonstrated by them. This is the furthest progress that has been made by the way of Numbers ; and they that have proceeded thus far deserve the praise of Industry. Nevertheless,if we consider the benefit (which is the scope at which all Speculation should aime) the improvement they have made has been little, or none. For any ordinary man may much sooner, & more accurately find a Straight Line equal to the Perimeter of a Circle, and consequently square the Circle, by winding a small thred about a given Cylinder, then any Geometrician shall do the same by dividing the Radius into 1coooooo equal parts. But though the length of the Circumference were exactly set out,either by Numbers, or mechanically,or onely by chance, yet this would contribute no help at all towards the Section of Angles, unless happily these two Problemes, *To divide a given Angle according to any proportion assigned*, and *To finde a Straight Line equal to the Arch of a Circle*, were reciprocal, and followed one another. Seeing therefore the benefit proceeding from the knowledge of the Length of the Arch of a Quadrant, consists in this,that we may thereby divide an Angle according to any proportion, either accurately, or at least accurately enough for common use; and seeing this cannot be done by Arithmetick,I thought fit to attempt the same by Geometry;and in this Chapter to make trial whether it might not be performed by the drawing of Straight and Circular Lines.

2 Let the Square A B C D(in the first figure) be described;and with the *Radii* A B, B C and D C the three Arches B D,C A and A C ; of which let the two B D and C A cut one another in E, and the two B D and A C in F. The Diagonals therefore B D and A C being drawn will cut one another in the center of the Square G, and the two Arches B D and C A into two equal parts in H and Y ; and the Arch B H D will be trisected in I and E. Through the Center G let the two Straight Lines K G L and M G N be drawn parallel and equal to the sides of the Square A B and A D,cutting the four sides of the same Square in the points K, L, M and N; which being done, K L will pass through F, and M N through E Then let O P be drawn parallel and equal to the side B C,cutting the Arch B F D in F, and the sides A B and D C

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in O and P. Therefore O F will be the Sine of the arch B F, which is an arch of 30 degrees; and the ſame O F will be equal to half the Radius. Laſtly, dividing the arch B F in the middle in Q, let R Q the Sine of the arch B Q be drawn and produced to S. ſo that Q S be equal to R Q, and conſequently R S be equal to the chord of the arch B F; and let F S be drawn and produced to T in the ſide B C. I ſay, the Straight Line B T is equal to the Arch B F; and conſequently that B V the triple of B T is equal to the Arch of the Quadrant B F E D.

Let T F be produced till it meet the ſide B A produced in X; and dividing O F in the middle in Z, let Q Z be drawn and produced till it meet with the ſide B A produced. Seeing therefore the Straight Lines R S and O F are parallel, and divided in the midſt in Q and Z, Q Z produced will fall upon X, and X Z Q produced to the ſide B C will cut B T in the midſt in α.

Upon the Straight line F Z the fourth part of the Radius A B let the equilateral triangle *a* Z F be conſtituted; & upon the center *a*, with the Radius *a* Z let the arch Z F be drawn; which arch Z F will therefore be equal to the arch Q F the half of the arch B F. Again, let the ſtraight line Z O be cut in the midſt in *b*, and the ſtraight line *b* O in the midſt in *c*; and let the biſection be continued in this manner till the laſt part O *c* be the leaſt that can poſſibly be taken; and upon it, and all the reſt of the parts equal to it into which the ſtraight line O F may be cut, let ſo many equilateral triangles be underſtood to be conſtituted; of which let the laſt be *d* O *c*. If therefore upon the center *d*, with the Radius *d* O be drawn the arch O *c*, and upon the reſt of the equal parts of the ſtraight line O F be drawn in like manner ſo many equal arches, all thoſe arches together taken will be equal to the whole arch B F, & the half of them, namely, thoſe that are comprehended between O & Z, or between Z & F will be equal to the arch B Q or Q F and in ſumme, what part ſoever the ſtraight line O *c* be of the ſtraight line O F, the ſame part will the arch O *c* be of the arch O F, though both the arch and the chord be infinitely biſected. Now ſeeing the arch O *c* is more crooked then that part of the arch B F which is equal to it; and ſeeing alſo that the more the ſtraight line X *c* is produced the more it diverges from the ſtraight line X O, if

the

the points O and *c* be underſtood to be moved forwards with ſtraight motion in X O and X *c*, the arch O *c* will thereby be extended by little and little, til at the laſt it come ſome-where to have the ſame crookedneſs with that part of the arch B F which is equal to it. In like manner, if the ſtraight line X *b* be drawn, and the point *b* be underſtood to be moved forwards at the ſame time, the arch *c b* will alſo by little and little be extended, till its crookedneſs come to be equal to the crookedneſs of that part of the arch B F which is equal to it. And the ſame will happen in all thoſe ſmal equal arches which are deſcribed upon ſo many equal parts of the ſtraight line O F. It is alſo manifeſt, that by ſtraight motion in X O and X Z all thoſe ſmall arches will lie in the arch B F in the points B, Q and F. And though the ſame ſmall equall arches ſhould not be coincident with the equall parts of the arch B F in all the other points thereof, yet certainly they will conſtitute two crooked lines, not onely equall to the two arches B Q and Q F and equally crooked, but alſo having their cavity towards the ſame parts; which how it ſhould be, unleſſe all thoſe ſmall arches ſhould be coincident with the arch B F in all its points, is not imaginable. They are therefore coincident, and all the ſtraight lines drawne from X & paſſing through the points of diviſion of the ſtraight line O F, will alſo divide the arch B F into the ſame proportions into which O F is divided.

Now ſeeing X *b* cuts off from the point B the fourth part of the arch B F, let that fourth part be B *e*; and let the Sine thereof *f e* be produced to F T in *g*, for ſo *f e* will be the fourth part of the ſtraight line *f g*, becauſe as O *b* is to O F, ſo is *f e* to *f g*. But B T is greater then *f g*; and therefore the ſame B T is greater then four Sines of the fourth part of the arch B F. And in like manner, if the arch B F be ſubdivided into any number of equal parts whatſoever, it may be proved that the ſtraight line B T is greater then the Sine of one of thoſe ſmall arches ſo many times taken as there be parts made of the whole arch B F. Wherefore the Straight line B T is not leſſe then the Arch B F. But neither can it be greater, becauſe if any ſtraight line whatſoever, leſſe then B T, be drawn below B T parallel to it and terminated in the ſtraight lines X B and X T, it would cut the arch B F; and

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ſo the Sine of ſome one of the parts of the arch B F taken ſo often as that ſmall arch is found in the whole arch B F, would be greater then ſo many of the ſame arches; which is abſurd. Wherefore the Straight line B T is equal to the Arch B F; & the Straight line B V equal to the Arch of the Quadrant B F D; and B V four times taken, equal to the Perimeter of the Circle deſcribed with the Radius A B. Alſo the Arch B F and the Straight line BT are every where divided into the ſame proportions; and conſequently any given Angle, whether greater or leſs then B A F may be divided into any proportion given.

But the ſtraight line B V (though its magnitude fall within the terms aſſigned by *Archimedes*) is found, if computed by the Canon of Sines, to be ſomwhat greater then that wch is exhibited by the *Ludolphine* numbers. Nevert. eleſs, if in the place of BT, another ſtraight line, though never ſo little leſs, be ſubſtituted, the diviſion of Angles is immediatly loſt, as may by any man be demonſtrated by this very Scheme.

Howſoever, if any man think this my Straight line BV to be too great, yet, ſeeing the Arch and all the Parallels are every where ſo exactly divided, and B V comes ſo neer to the truth, I deſire he would ſeach out the reaſon, Why (granting B V to be preciſely true) the Arches cut off ſhonld not be equal.

But ſome man may yet ask the reaſon why the ſtraight lines drawn from X through the equal parts of the arch B F ſhould cut off in the Tangent B V ſo many ſtraight lines equal to them, ſeeing the connected ſtraight line XV paſſes not through the point D, but cuts the ſtraight line A D produced in *l*; and conſequently require ſome determination of this Probleme. Concerning which, I will ſay what I think to be the reaſon, namely; that whileſt the magnitude of the Arch doth not exceed the magnitude of the Radius, that is, the magnitude of the Tangent B C, both the Arch and the Tangent are cut alike by the ſtraight lines drawn from X; otherwiſe not. For AV being connected, cutting the arch B H D in I, if X C being drawn ſhould cut the ſame arch in the ſame point I, it would be as true that the Arch B I is equal to the Radius B C, as it is true that the Arch B F is equal to the ſtraight line B T, and drawing X K it would cut the arch B I in the midſt in *i*; Alſo

drawing A *i* and producing it to the Tangent B C in *k*, the ſtraight line B *k* will be the Tangent of the arch B *i*, (which arch is equal to half the Radius) and the ſame ſtraight line B *k* will be equal to the ſtr aight line *k* I. I ſay all this is true, if the preceding demonſtration be true; and conſequently the proportional ſection of the Arch and its Tangent proceeds hitherto. But it is manifeſt by the Golden Rule, that taking B *h* double to B T, the line X *h* ſhall not cut off the arch B E which is double to the arch B F, but a much greater. For the magnitude of the ſtraight lines X M, X B and M E being known (in numbers) the magnitude of the ſtraight line cut off in the Tangent by the ſtraight line X E produced to the Tangent may alſo be known; and it will be found to be leſs then B *h*; Wherfore the ſtraight line X *h* being drawn will cut off a part of the arch of the Quadrant greater then the arch B E. But I ſhall ſpeak more fully in the next Article concerning the magnitude of the arch B I.

And let this be the firſt attempt for the finding out of the dimenſion of a Circle by the Section of the arch B F.

3. I ſhall now attempt the ſame by arguments drawn from the nature of the Crookedneſs of the Circle it ſelf; but I ſhall firſt ſet down ſome Premiſſes neceſſary for this ſpeculation; and

Firſt, If a Straight line be bowed into an Arch of a Circle equal to it, as when a ſtretched thred which toucheth a Right Cylinder, is ſo bowed in every point, that it be every where coincident with the Perimeter of the baſe of the Cylinder, the Flexion of that line will be equal, in all its points; and conſequently the Crookedneſs of the Arch of a Circle is every where Uniform; which needs no other demonſtration then this, That the Perimeter of a Circle is an Uniform line.

Secondly, and conſequently, If two unequal Arches of the ſame Circle be made by the bowing of two ſtraight lines equal to them, the Flexion of the longer line (whileſt it is bowed into the greater Arch) is greater then the Flexion of the ſhorter line (whileſt it is bowed into the leſſer Arch) according to the proportion of the Arches themſelves; and conſequently, the Crookedneſs of the greater Arch is to the Crookedneſs of the leſſer Arch; as the greater Arch is to the leſſer Arch.

Thirdly, If two unequal Circles and a ſtraight line touch one ano-

another in the ſame point, the Crookedneſs of any Arch taken in the leſſer Circle, will be greater then the Crookedneſs of an Arch equal to it taken in the greater Circle, in reciprocal proportion to that of the *Radii* with which the Circles are deſcribed; or, which is all one, any ſtraight line being drawn from the point of Contact till it cut both the circumferences, as the part of that ſtraight line cut off by the circumference of the greater Circle to that part which is cut off by the circumference of the leſſer Circle.

For let A B and A C (in the ſecond figure) be two Circles, touching one another and the ſtraight line A D in the point A; and let their Centers be E and F; and let it be ſuppoſed, that as A E is to A F, ſo is the Arch A B to the Arch A H. I ſay the Crookedneſs of the Arch A C is to the Crookedneſs of the Arch A H, as A E is to A F. For let the ſtraight line A D be ſuppoſed to be equal to the Arch A B, and the ſtraight line A G to the Arch A C; and let A D (for example) be double to A G. Therefore by reaſon of the likeneſs of the Arches A B and A C, the ſtraight line A B will be double to the ſtraight line A C, and the Radius A E double to the Radius A F, and the Arch A B double to the Arch A H. And becauſe the ſtraight line A D is ſo bowed to be coincident with the Arch A B equal to it, as the ſtraight line A G is bowed to be coincident with the Arch A C equal alſo to it, the Flexion of the ſtraight line A G into the Crooked line A C will be equal to the Flexion of the ſtraight Line A D into the Crooked line A B. But the Flexion of the ſtraight line A D into the Crooked line A B is double to the the Flexion of the ſtraight line A G into the Crooked line A H; and therefore the Flexion of the ſtraight line A G into the Crooked line A C is double to the Flexion of the ſame ſtraight line A G into the Crooked line A H. Wherefore, as the Arch A B is to the Arch A C or A H; or as the Radius A E is to the Radius A F; or as the Chord A B is to the Chord A C; ſo reciprocally is the Flexion or Uniform Crookedneſs of the Arch A C, to the Flexion or Uniform Crookedneſs of the Arch A H, namely, here double. And this may by the ſame method be demonſtrated in Circles whoſe Perimeters are to one another triple, quadruple, or in whatſoever given proportion. The Crookedneſs therefore of two equal Arches

taken in ſeveral Circles are in proportion reciprocall to that of their *Radii*, or like Arches, or like Chords; which was to be demonſtrated.

Let the Square A B C D be again deſcribed (in the third Figure,) and in it the Quadrants A B D, B C A and D A C; and dividing each ſide of the Square A B C D in the midſt in E, F, G and H, let E G and F H be connected, which will cut one another in the center of the Square at I; and divide the arch of the Quadrant A B D into three equal parts in K and L. Alſo the Diagonals A C and B D being drawn will cut one another in I, and divide the arches B K D and C L A into two equal parts in M and N. Then with the Radius B F let the arch F E be drawn, cutting the Diagonal B D in O; and dividing the arch B M in the midſt in P, let the ſtraight line E *a* equal to the chord B P be ſet off from the point E in the arch E F, and let the arch *a b* be taken equal to the arch *O a*, and let B *a* and B *b* be drawn and produced to the arch A N in *c* and *d*; and laſtly, let the ſtraight line A *d* be drawn. I ſay the Straight line A *d* is equal to the Arch A N or B M.

I have proved in the preceding article, that the arch E O is twice as crooked as the arch B P, that is to ſay, that the arch E O is ſo much more crooked then the arch B P, as the arch B P is more crooked then the ſtraight line E *a*. The crookedneſs therefore of the chord E *a*, of the arch B P, and of the arch E O are as 0, 1, 2. Alſo the difference between the arches E O and E O, the difference between the arches E O and E *a*, and the difference between the arches E O and E *b* are as 0, 1, 2. So alſo the difference between the arches A N and A N, the difference between the arches A N and A *c*, and the difference between the arches A N and A *d* are as 0, 1, 2; and the ſtraight line A *c* is double to the chord B P or E *a*; and the ſtraight line A *d* double to the chord E *b*.

Again, let the ſtraight line B F be divided in the midſt in Q, and the arch B P in the midſt in R; and deſcribing the Quadrant B Q S (whoſe arch Q S is a fourth part of the arch of the Quadrant B M D, as the arch B R is a fourth part of the arch B M which is the arch of the Semiquadrant A B M) let the chord S *e* equal to the chord B R be ſet off from the point S in the arch S Q; and let B *e* be drawn and produced to the arch A N in *f*; which being done, the ſtraight line A *f* will be quadruple to the chord

chord B R or S *e*. And seeing the crookedneſs of the arch S *e* or of the arch A *c* is double to the crookedneſs of the arch B R, the exceſs of the crookedneſs of the arch A *f* above the crookedneſs of the arch A *c* will be ſubduple to the exceſs of the crookedneſs of the arch A *c* above the crookedneſs of the arch A N; and therefore the arch N *c* will be double to the arch *c f*. Wherefore the arch *c d* is divided in the midſt in *f*, and the arch N *f* is $\frac{3}{4}$ of the arch N *d*. And in like manner if the arch B R be biſected in V, and the ſtraight Line B Q in X, and the quadrant B X Y be deſcribed, and the ſtraight Line Y *g* equal to the chord B V be ſet off from the point Y in the arch Y X, it may be demonſtrated that the ſtraight Line B *g* being drawn and produced to the arch A N will cut the arch *f d* into two equal parts, and that a ſtraight Line drawn from A to the point of that Section, will be equal to eight chords of the arch B V, and ſo on perpetually; and conſequently, that the ſtraight Line A *d* is equal to ſo many equal chords of equal parts of the arch B M, as may be made by infinite biſections. Wherefore the Straight Line A *d* is equal to the Arch B M or A N, that is, to half the Arch of the Quadrant A B D or B C A.

Corollary. An Arch being given not greater then the arch of a Quadrant (for being made greater it comes again towards the Radius B A produced, from which it receded before) if a ſtraight Line double to the chord of half the given arch be adapted from the beginning of the arch, and by how much the arch that is ſubtended by it is greater then the given arch, by ſo much a greater arch be ſubtended by another ſtraight Line, this Straight Line ſhall be equal to the firſt given Arch.

Suppoſing the Straight Line B V (in the firſt Figure) be equal to the arch of the Quadrant B H D, and A V be connected cutting the arch B H D in I, it may be asked what proportion the arch B I has to the arch I D. Let therefore the arch A Y be divided in the midſt in *o*, and in the ſtraight line A D let A *p* be taken equal, and A *q* double to the drawn chord A *o*. Then upon the center A, with the Radius A *q* let an arch of a circle be drawne cutting the arch A Y in *r*, and let the arch Y *r* be doubled at *t*; which being done, the drawne ſtraight line A *t* (by what has been laſt demonſtrated) will be equall to the arch A Y.

A-

Again, upon the Center A with the Radius A *t* let the arch *t u* be drawne cutting A D in *u*; and the ſtraight line A *u* will be equall to the arch A Y. From the point *u* let the ſtraight line *u s* be drawn equal and parallell to the ſtraight line A B, cutting M N in *x*, and biſected by M N in the ſame point *x*. Therefore the ſtraight line A *x* being drawn and produced till it meet with B C produced in V, it will cut off B V double to B *s*, that is, equal to the arch B H D. Now let the point where the ſtraight line A V cuts the arch B H D, be I; and let the arch D I be divided in the midſt in *y*; and in the ſtraight line D C, let D *z* be taken equal, and D *ſ* double to the drawn chord D *y*; and upon the center D with the Radius D *ſ* let an arch of a circle be drawn cutting the arch B H D in the point *n*; and let the arch *n m* be taken equal to the arch I *n*; which being done, the ſtraight line D *m* will (by the laſt foregoing *Corollary*) be equal to the arch D I. If now the ſtraight lines D *m* and C V be equal, the arch B I will be equal to the Radius A B or B C; and conſequently X C being drawn will paſs through the point I. Moreover, if the ſemicircle B H D *c* being completed, the ſtraight lines *c* I and B I be drawn making a right angle (in the Semicircle) at I, and the arch B I be divided in the midſt at *i*, it will follow that A *i* being connected will be parallel to the ſtraight line *c* I, and being produced to B C in *k*, will cut off the ſtraight line B *k* equal to the ſtraight line *k* I, and equal alſo to the ſtraight line A *γ* cut off in A D by the ſtraight line *c* I. All which is manifeſt, ſuppoſing the arch B I and the Radius B C to be equal.

But that the arch B I and the Radius B C are preciſely equal, cannot (how true ſoever it he) be demonſtrated, unleſs that be firſt proved w^ch is contained in the firſt article, namely, that the ſtraight lines drawn from X through the equal parts of O F, (produced to a certain length) cut off ſo many parts alſo in the Tangent B C ſeverally equal to the ſeveral arches cut off; which they do moſt exactly as far as B C in the Tangent, and B I in the arch B E; in ſo much that no inequality between the arch B I and the Radius B C can be diſcovered either by the hand or by ratiocination. It is therefore to be further enquired, whether the ſtraight line A V cut the arch of the Quadrant in I in the ſame proportion as the point C divides the

the ſtraight line B V which is equal to the arch of the Quadrant. But however this be, it has been demonſtrated that the ſtraight line B V is equal to the arch B H D.

4 I ſhall now attempt the ſame dimenſion of a Circle another way, aſſuming the two following *Lemma's*.

Lemma 1. If to the Arch of a Quadrant, and the Radius, there be taken in continual proportion a third Line Z; then the Arch of the Semiquadrant, Half the chord of the Quadrant, and Z will alſo be in continual proportion.

For ſeeing the Radius is a mean proportional between the Chord of a Quadrant and its Semichord, and the ſame Radius a mean proportional between the Arch of the Quadrant and Z, the Square of the Radius will be equal as well to the Rectangle made of the Chord and Semichord of the Quadrant, as to the Rectangle made of the Arch of the Quadrant and Z; and theſe two Rectangles will be equal to one another. Wherfore, as the Arch of a Quadrant is to its Chord, ſo reciprocally is half the Chord of the Quadrant to Z. But as the Arch of the Quadrant is to its Chord, ſo is half the Arch of the Quadrant to half the Chord of the Quadrant. Wherefore, as half the arch of the Quadrant is to half the Chord of the Quadrant (or to the Sine of 45 degrees) ſo is half the Chord of the Quadrant to Z; which was to be proved.

Lemma 2. The Radius, the Arch of the Semiquadrant, the Sine of 45 degrees, and the Semiradius are proportional.

For ſeeing the Sine of 45 degrees is a mean proportional between the Radius and the Semiradius; and the ſame Sine of 45 degrees is alſo a mean proportional (by the precedent *Lemma*) between the Arch of 45 degrees and Z; the Square of the Sine of 45 degrees will be equal as well to the Rectangle made of the Radius and Semiradius, as to the Rectangle made of the Arch of 45 degrees and Z. Wherefore, as the Radius is to the Arch of 45 degrees, ſo reciprocally is Z to the Semiradius; which was to be demonſtrated.

Let now A B C D (in the fourth Figure) be a Square; and with the *Radii* A B, B C and D A let the three Quadrants A B D, B C A and D A C be deſcribed; and let the ſtraight lines E F and G H drawn parallel to the Sides B C & A B, divide the Square A B C D into foure equal Squares. They will therefore cut the arch of the

Qua-

Quadrant A B D into three equal parts in I and K, and the arch of the Quadrant B C A into three equal parts in K and L. Alſo let the Diagonals A C and B D be drawn, cutting the arches B I D and A L C in M and N. Then upon the center H with the Radius H F equal to half the Chord of the arch B M D, or to the Sine of 45 degrees, let the arch F O be drawn cutting the arch C K in O; and let A O be drawn and produced till it meet with B C produced in P; alſo let it cut the arch B M D in Q, and the ſtraight line D C in R. If now the ſtraight line H Q be equal to the ſtraight line D R, and being produced to D C in S cut off D S equal to half the ſtraight line B P; I ſay then the Straight Line B P will be equal to the Arch B M D.

For ſeeing P B A and A D R are like triangles, it will be as P B to the Radius B A or A D, ſo A D to D R; and therefore as well P B, A D and D R, as P B, A D (or A Q) and Q H are in continuall proportion, and producing H O to D C in T, D T will be equal to the Sine of 45 degrees, as ſhall by and by be demonſtrated. Now D S, D T and D R are in continual porportion by the firſt *Lemma*; and by the ſecond *Lemma* D C. D S:: D R. D F are proportionals. And thus it will be, whether B P be equal or not equal to the arch of the Quadrant B M D. But if they be equal, it will then be, as that part of the arch B M D which is equal to the Radius, is to the remainder of the ſame arch B M D; ſo A Q to H Q, or ſo B C to C P. And then will B P and the arch B M D be equal. But it is not demonſtrated that the Straight Lines H Q and D R are equal; though if from the point B there be drawn (by the conſtruction of the firſt figure) a Straight Line equal to the arch B M D, then D R to H Q, and alſo the half of the Straight Line B P to D S, will always be ſo equal, that no inequality can be diſcovered between them. I will therefore leave this to be further ſearched into. For though it be almoſt out of doubt, that the Straight Line B P and the arch B M D are equal, yet that may not be received without demonſtration; and means of Demonſtration the Circular Line admitteth none that is not grounded upon the nature of Flexion, or of Angles. But by that way I have already exhibited a Straight Line equal to the Arch of a Quadrant in the Firſt and Second aggreſſion.

In

It remains that I prove D T to be equal to the Sine of 45 degrees.

In B A produced let AV he taken equal to the Sine of 45 degrees; and drawing and producing V H, it will cut the arch of the Quadrant C N A in the midst in N, and the same arch again in O, and the Straight line D C in T, so, that D T will be equal to the Sine of 45 degrees, or to the straight line AV; also the Straight line V H will be equal to the straight line H I or the Sine of 60 degrees.

For the square of AV is equal to two squares of the Semiradius; and consequently the square of VH is equal to three Squares of the Semiradius. But H I is a mean proportional between the Semiradius and three *Semiradii*; and therefore the square of H I is equal to three Squares of the Semiradius. Wherefore H I is eqval to H V. But because A D is cut in the midst in H, therefore V H and H T are equal; and therefore also D T is equal to the Sine of 45 degrees. In the Radius B A let B X be taken equal to the Sine of 45 degrees; for so V X will be equal to the Radius; and it will be as V A to A H the Semiradius, so V X the Radius to X N the Sine of 45 degrees. Wherefore V H produced passes through N. Lastly, upon the center V with the Radius V A let the arch of a circle be drawn cutting VH in Y; which being done, VY will be equal to H O (for H O is by construction equal to the Sine of 45 degrees) and YH will be equal to OT; & therefore VT passes through O. All which was to be demonstrated.

I will here add certain Problemes, of which if any Analyst can make the construction, he will thereby be able to judge clearly of what I have now said concerning the dimension of a Circle. Now these Problemes are nothing else (at least to sense) but certain symptomes accompanying the construction of the first and third figure of this Chapter.

Describing therefore again the Square A B C D (in the fifth figure) and the three Quadrants A B D, B C A and D A C, let the Diagonals A C & B D be drawn, cutting the arches B H D & C I A in the middle in H and I; & the straight lines E F and G L, dividing the square A B C D into four equal squares, and trisecting the arches B H D and C I A, namely, B H D in K and M, and C I A

in M and O. Then dividing the arch B K in the midſt in P, let Q P the Sine of the arch B P be drawn and produced to R, ſo that Q R be double to Q P; and connecting K R, let it be produced one way to B C in S, and the other way to B A produced in T. Alſo let B V be made triple to B S, and conſequently (by the ſecond article of this Chapter) equall to the arch B D. This conſtruction is the ſame with that of the firſt figure, which I thought fit to renew diſcharged of all lines but ſuch as are neceſſary for my preſent purpoſe.

In the fitſt place therefore, if A V be drawn, cutting the arch B H D in X, and the ſide D C in Z, I deſire ſome Analyſt would (if he can) give a reaſon, Why the ſtraight lines T E and T C ſhould cut the arch B D the one in Y, the other in X, ſo as to make the arch B Y equal to the arch Y X; or if they be not equal, that he would determine their difference.

Secondly, if in the ſide D A, the ſtraight line D *a* be taken equal to D Z, and V *a* be drawn; Why V *a* and V B ſhould be equal; or if they be not equal, What is the difference.

Thirdly, drawing Z *b* parallel and equal to the ſide C B, cutting the arch B H D in *c*, and drawing the ſtraight line A *c*, and producing it to B V in *d*; Why A *d* ſhould be equal and parallel to the ſtraight line *a* V, and conſequently equal alſo to the arch B D.

Fourthly, drawing *e* K the Sine of the arch B K, & taking (in *e* A produced) *e f* equal to the Diagonal A C, and connecting *f* C; Why *f* C ſhould paſs through *a* (which point being given, the length of the arch B H D is alſo given) and *c*; and why *f e* and *f c* ſhould be equal; or if not, why unequal.

Fifthly, drawing *f* Z, I deſire he would ſhew, Why it is equal to B V, or to the arch B D; or if they be not equal, What is their difference.

Sixtly, granting *f* Z to be equal to the arch B D, I deſire he would determine whether it fall all without the arch B C A, or cut the ſame, or touch it, and in what point.

Seventhly, the Semicircle B D *g* being completed; Why *g* I being drawn and produced, ſhould paſs through X (by which point X the length of the arch B D is determined). And the ſame *g I* being yet further produced to D C in *h*; Why A *d* (which is equal

to

to the arch B D) ſhould paſs through that point *h*.

Eighthly, upon the Center of the ſquare A B C D, which let be *k*, the arch of the quadrant E *i* L being drawn, cutting *e* K produced in *i*; Why the drawn ſtraight line *i* X ſhould be parallel to the ſide C D.

Ninthly, in the ſides B A and B C taking B *l* and B *m* ſeverally equal to half B V, or to the arch B H, and drawing *m n* parallel and equal to the ſide B A, cutting the arch B D in *o*; Why the ſtraight line wich connects V *l* ſhould paſs through the point *o*,

Tenthly, I would know of him, Why the ſtraight line which connects *a* H ſhould be equal to B *m*; or if not, how much it differs from it.

The Analyſt that can ſolve theſe Problemes without knowing firſt the length of the arch B D, or uſing any other known Method then that which proceeds by perpetual biſection of an angle, or is drawn from the conſideration of the nature of Flexion, ſhall do more then ordinary Geometry is able to perform. But if the Dimenſion of a Circle cannot be found by any other Method; then I have either found it, or it is not at all to be found.

From the known Length of the Arch of a Quadrant, and from the proportional Diviſion of the Arch and of the Tangent BC, may be deduced the Section of an Angle into any given proportion; as alſo the Squaring of the Circle, the Squaring of a given Sector, and many the like propoſitions, which it is not neceſſary here to demonſtrate. I will therefore onely exhibit a Straight line equal to the Spiral of *Archimedes*, and ſo diſmiſs this ſpeculation.

5 The length of the Perimeter of a Circle being found, that Straight line is alſo found, which touches a Spiral at the end of its firſt converſion. For upon the center A (in the ſixth figure) let the circle B C D E be deſcribed; and in it let *Archimedes* his Spiral A F G H B be drawn, beginning at A and ending at B. Through the center A let the ſtraight line C E be drawn, cutting the Diameter B D at right angles; and let it be produced to I, ſo, that A I be equal to the Perimeter B C D E B. Therefore I B being drawn will touch the Spiral A F G H B in B; which is demonſtrated by *Archimedes* in his book *de Spiralibus*.

And for a Straight Line equal to the given Spiral A F G H B, it may be found thus.

Let the ſtraight line AI (which is equal to the Perimeter BCDE) be biſected in K; and taking KL equal to the Radius AB, let the rectangle IL be completed. Let ML be underſtood to be the axis, and KL the baſe of a Parabola, and let MK be the crooked line thereof. Now if the point M be conceived to be ſo moved by the concourſe of two movents, the one frō IM to KL with velocity encreaſing continually in the ſame proportion with the Times, the other from ML to IK uniformly, that both thoſe motions begin together in M and end in K; *Galilæus* has demonſtrated that by ſuch motion of the point M, the crooked line of a Parabola will be deſcribed. Again, if the point A be conceived to be moved uniformly in the ſtraight line AB, and in the ſame time to be carried round upon the center A by the circular motion of all the points between A and B; *Archimedes* has demonſtrated that by ſuch motion will be deſcribed a Spiral line. And ſeeing the circles of all theſe motions are concentrick in A; and the interiour circle is alwayes leſſe then the exteriour in the proportion of the times in which AB is paſſed over with uniform motion; the velocity alſo of the circular motion of the point A, will continually encreaſe proportionally to the times. And thus far the generations of the Parabolical line MK, and of the Spiral line AFGHB, are like. But the Uniform motion in AB concurring with circular motion in the Perimeters of all the concentrick circles, deſcribes that circle, whoſe center is A, and Perimeter BCDE; and therefore that circle is (by the *Coroll.* of the firſt article of the 16 Chapter) the aggregate of all the Velocities together taken of the point A whilſt it deſcribes the Spiral AFGHB. Alſo the rectangle IKLM is the aggregate of all the Velocities together taken of the point M, whileſt it deſcribes the crooked line MK. And therefore the whole velocity, by which the Parabolicall line MK is deſcribed, is to the whole velocity with which the Spiral line AFGHB is deſcribed in the ſame time, as the rectangle IKLM, is to the Circle BCDE, that is to the triangle AIB. But becauſe AI is biſected in K & the ſtraight lines IM & AB are equal, therefore the rectangle IKLM and the triangle AIB are alſo equal. Wherefore the Spiral line AFGHB, and the Parabolical line MK, being deſcribed with equal velocity and in equal times, are equal to one another. Now in the firſt article of the 18 Chapter a ſtraight line is found

out

out equal to any Parabolical line. Wherefore alſo a Straight line is found out, equal to a given Spiral line of the firſt revolution deſcribed by *Archimedes*; which was to be done.

6 In the ſixth Chapter, which is of *Method*, that which I ſhould there have ſpoken of the *Analyticks* of Geometricians, I thought fit to deferre, becauſe I could not there have been underſtood, as not having then ſo much as named *Lines*, *Superficies*, *Solids*, *Equal and Unequal &c.* Wherefore I will in this place ſet down my thoughts concerning it.

Analyſis, is continual Reaſoning from the Definitions of the terms of a propoſition we ſuppoſe true, and again from the Definitions of the terms of thoſe Definitions, and ſo on, till we come to ſome things known, the Compoſition whereof is the demonſtration of the truth or falſity of the firſt ſuppoſition; and this Compoſition or Demonſtration is that we call *Syntheſis*. *Analytica* therefore is that art, by which our reaſon proceeds from ſomething ſuppoſed, to Principles, that is, to prime Propoſitions, or to ſuch as are known by theſe, till we have ſo many known Propoſitions as are ſufficient for the demonſtration of the truth or falſity of the thing ſuppoſed. *Synthetica* is the art it ſelf of Demonſtration. *Syntheſis* therefore and *Analyſis* differ in nothing, but in proceeding forwards or backwards; and *Logiſtica* comprehends both. So that in the *Analyſis* or *Syntheſis* of any queſtion, that is to ſay, of any Probleme, the Terms of all the Propoſitions ought to be convertible; or if they be enunciated Hypothetically, the truth of the Conſequent ought not onely to follow out of the truth of its Antecedent, but contrarily alſo the truth of the Antecedent muſt neceſſarily be inferred from the truth of the Conſequent. For otherwiſe, when by Reſolution we are arrived at Principles, we cannot by Compoſition return directly back to the thing ſought for. For thoſe Terms which are the firſt in *Analyſis*, will be the laſt in *Syntheſis*; as for example, when in *Reſolving*, we ſay, theſe two Rectangles are equal and therefore their ſides are reciprocally proportional, we muſt neceſſarily in *Compounding* ſay, the ſides of theſe Rectangles are reciprocally proportional and therefore the Rectangles themſelves are equal; Which we could not ſay, unleſs *Rectangles have their ſides reciprocally proportional, and Rectangles are equal*, were Terms convertible.

Now

Now in every *Analyſis*, that which is ſought, is the Proportion of two quantities; by which proportion (a figure being deſcribed) the quantity ſought for may be expoſed to Senſe. And this *Expoſition* is the end and Solution of the queſtion, or the conſtruction of the Probleme.

And ſeeing *Analyſis* is reaſoning from ſomething ſuppoſed, till we come to Principles, that is, to Definitions, or to Theoremes formerly known; and ſeeing the ſame reaſoning tends in the laſt place to ſome Equation; we can therefore make no end of Reſolving, till we come at laſt to the cauſes themſelves of Equality and Inequality, or to Theoremes formerly demonſtrated from thoſe cauſes; and ſo have a ſufficient number of thoſe Theoremes for the demonſtration of the thing ſought for.

And ſeeing alſo, that the end of the *Analyticks*, is either the conſtruction of ſuch a Probleme as is poſſible, or the detection of the impoſſibility thereof; whenſoever the Probleme may be ſolved, the *Analyſt* muſt not ſtay, till he come to thoſe things which contain the efficient cauſe of that whereof he is to make conſtruction. But he muſt of neceſſity ſtay when he comes to prime Propoſitions; and theſe are Definitions. Theſe Definitions therefore muſt contain the efficient cauſe of his Conſtruction; I ſay of his Conſtruction, not of the Concluſion which he demonſtrates; for the cauſe of the Concluſion is contained in the premiſed propoſitions; that is to ſay, the truth of the propoſition he proves, is drawn from the propoſitions which prove the ſame. But the cauſe of his conſtruction is in the things themſelves, and conſiſts in motion, or in the concourſe of motions. Wherefore thoſe propoſitions in which *Analyſis* ends, are Definitions, but ſuch, as ſignifie in what manner the conſtruction, or generation of the thing proceeds. For otherwiſe, when he goes back by *Syntheſis* to the proofe of his Probleme, he will come to no Demonſtration at all; there being no true Demonſtration but ſuch as is ſcientificall; and no Demonſtration is ſcientifical but that which proceeds from the knowledge of the cauſes from which the conſtruction of the Probleme is drawne. To collect therefore what has been ſaid into few words; ANALYSIS *is Ratiocination from the ſuppoſed conſtruction or generation of a thing to the efficient cauſe, or coefficient cauſes of that which is conſtructed or generated. And* SYNTHESIS *is Ratiocination from the firſt cauſes*

ſes of the Conſtruction," continued through all the middle cauſes till we come to the thing it ſelfe which is conſtructed or generated.

But becauſe there are many means by which the ſame thing may be generated, or the ſame Probleme be conſtructed, therefore neither do all Geometricians, nor doth the ſame Geometrician alwayes uſe one and the ſame Method. For if to a certain quantity given, it be required to conſtruct another quantity equal, there may be ſome that will enquire whether this may not be done by means of ſome motion. For there are quantities, whoſe equality and inequality may be argued from Motion and Time, as well as from Congruence; and there is motion, by which two quantities, whether Lines or Superficies, though one of them be crooked, the other ſtraight, may be made congruous or coincident. And this method *Archimedes* made uſe of in his Book *de Spiralibus*. Alſo the equality or inequality of two quantities may be found out and demonſtrated from the conſideration of Waight, as the ſame *Archimedes* did in his Quadrature of the Parabola. Beſides, equality and equality are found out often by the diviſion of the two quantityes into parts which are conſidered as undiviſible; as *Cavallerius Bonaventura* has done in our time, and *Archimedes* often. Laſtly, the ſame is performed by the conſideration of the *Powers* of lines, or the roots of thoſe *Powers*, and by the multiplication, diviſion, addition and ſubſtraction, as alſo by the extraction of the roots of thoſe *Powers*, or by finding where ſtraight lines of the ſame proportion terminate. For example, when any number of ſtraight lines, how many ſoever, are drawne from a ſtraight line, and paſſe all through the ſame point, looke what proportion they have, and if their parts continued from the point retaine every where the ſame proportion, they ſhall all terminate in a ſtraight line. And the ſame happens if the point be taken between two Circles. So that the places of all their points of termination make either ſtraight lines, or circumferences of Circles, and are called *Plain Places*. So alſo when ſtraight parallel lines are applyed to one ſtraight line, if the parts of the ſtraight line to which they are applyed be to one another in proportion duplicate to that of the contiguous applyed lines, they will all terminate in a Conical Section; which Section being the place of their

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termination, is called a *Solid Place*, becauſe it ſerves for the finding out of the quantity of any Equation which conſiſts of three dimenſions. There are therfore three ways of finding out the cauſe of Equality or Inequality between two given quantities; namely, Firſt by the Computation of *Motions* (for by equal Motion, & equal Time equal Spaces are deſcribed,) and Ponderation is motion. Secondly By *Indiviſibles*; becauſe all the parts together taken are equal to the whole. And thirdly by the *Powers*; for when they are equall, their roots alſo are equall; and contrarily, the *Powers* are equall, when their roots are equal. But if the queſtion be much complicated, there cannot by any of theſe wayes be conſtituted a certaine Rule, from the ſuppoſition of which of the unknown quantities the *Analyſis* may beſt begin; nor out of the variety of Equations that at firſt appeare, which we were beſt to chooſe; but the ſucceſſe will depend upon dexterity, upon formerly acquired Science, and many times upon fortune.

For no man can ever be a good *Analyſt* without being firſt a good Geometrician; nor do the rules of *Analyſis* make a Geometrician, as *Syntheſis* doth; which begins at the very Elements, and proceeds by a Logical Uſe of the ſame. For the true teaching of Geometry is by *Syntheſis*, according to *Euclides* method; and he that hath *Euclide* for his Maſter, may be a Geometrician without *Vieta* (though *Vieta* was a moſt admirable Geometrician); but he that has *Vieta* for his maſter, not ſo, without *Euclide*.

And as for that part of *Analyſis* which works by the Powers, though it be eſteemed by ſome Geometricians (not the chiefeſt) to be the beſt way of ſolving all Problemes, yet it is a thing of no great extent; it being all contained in the doctrine of rectangles, and rectangled Solids. So that although they come to an Equation which determines the quantity ſought, yet they cannot ſometimes by art exhibit that quantity in a Plain, but in ſome Conique Section; that is, as Geometricians ſay, not Geometrically, but mechanically. Now ſuch Problemes as theſe, they call *Solid*; and when they cannot exhibit the quantity ſought for with the helpe of a conique Section, they call it a *Lineary* Probleme. And therefore in the quantities of angles, and of the arches of Circles, there is no uſe at all of the *Analyticks* which proceed by the *Powers*; ſo that the

the Antients pronounced it impoſſible, to exhibit in a plaine the Diviſion of Angles, except biſection, and the biſection of the biſected parts, otherwiſe then mechanically. For *Pappus*, (before the 31 propoſition of his fourth Book) diſtinguiſhing and defining the ſeveral kinds of Problemes, ſays that *ſome are* Plain, *others* Solid, *and others* Lineary. *Thoſe therefore which may be ſolved by ſtraight lines and the circumferences of Circles* (that is, which may be deſcribed with the Rule and Compaſs, without any other Inſtrument) *are fitly called* Plain; *for the lines by which ſuch Problemes are found out, have their generation in a Plain. But thoſe which are ſolved by the uſing of ſome one or more Conique Sections in their conſtruction, are called* Solid, *becauſe their conſtruction cannot be made without uſing the ſuperficies of ſolid figures, namely of Cones. There remains the third kinde, which is called* Lineary, *becauſe other lines beſides thoſe already mentioned are made uſe of in their conſtruction, &c.* And a little after he ſayes, *Of this kinde are the* Spiral *lines, the* Quadratrices, *the* Conchoeides, *and the* Ciſſoeides. *And Geometricians think it no ſmall fault, when for the finding out of a* Plain *Probleme any man makes uſe of* Coniques, *or new* Lines. Now he ranks the *Triſection* of an angle among *Solid* Problemes, and the *Quinqueſection* among *Lineary*. But what! are the ancient Geometricians to be blamed, who made uſe of the *Quadratrix* for the finding out of a ſtraight line equal to the arch of a Circle? and *Pappus* himſelf, was he faulty when he found out the triſection of an Angle by the help of an *Hyperbole*? Or am I in the wrong, who think I have found out the conſtruction of both theſe Problemes by the Rule and Compaſs onely? Neither they, nor I. For the Ancients made uſe of this *Analyſis* which proceeds by the Powers; and with them it was a fault to do that by a more remote Power, which might be done by a neerer; as being an argument that they did not ſufficiently underſtand the nature of the thing. The virtue of this kind of *Analyſis* conſiſts in the changing and turning and toſſing of Rectangles and Analogiſmes; and the skill of *Analyſts* is meer Logick, by which they are able methodically to find out whatſoever lies hid either in the Subject or Predicate of the Concluſiō ſought for. But thisdoth not properly belong to *Algebra*, or the *Analyticks Specious, Symbolical* or *Coſſick*; which are, as I may ſay, the *Brachygraphy* of the *Analyticks*, and an art, neither of teaching nor

learning Geometry, but of regiſtring with brevity and celerity, the inventions of Geometricians. For though it be eaſie to diſcourſe by *Symbols* of very remote propoſitions; yet whether ſuch diſcourſe deſerve to be thought very profitable, when it is made without any *Ideas* of the things themſelves, I know not.

Cap: XX



Fig 1

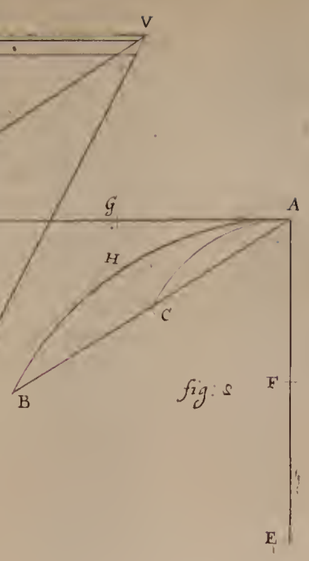


fig: 2

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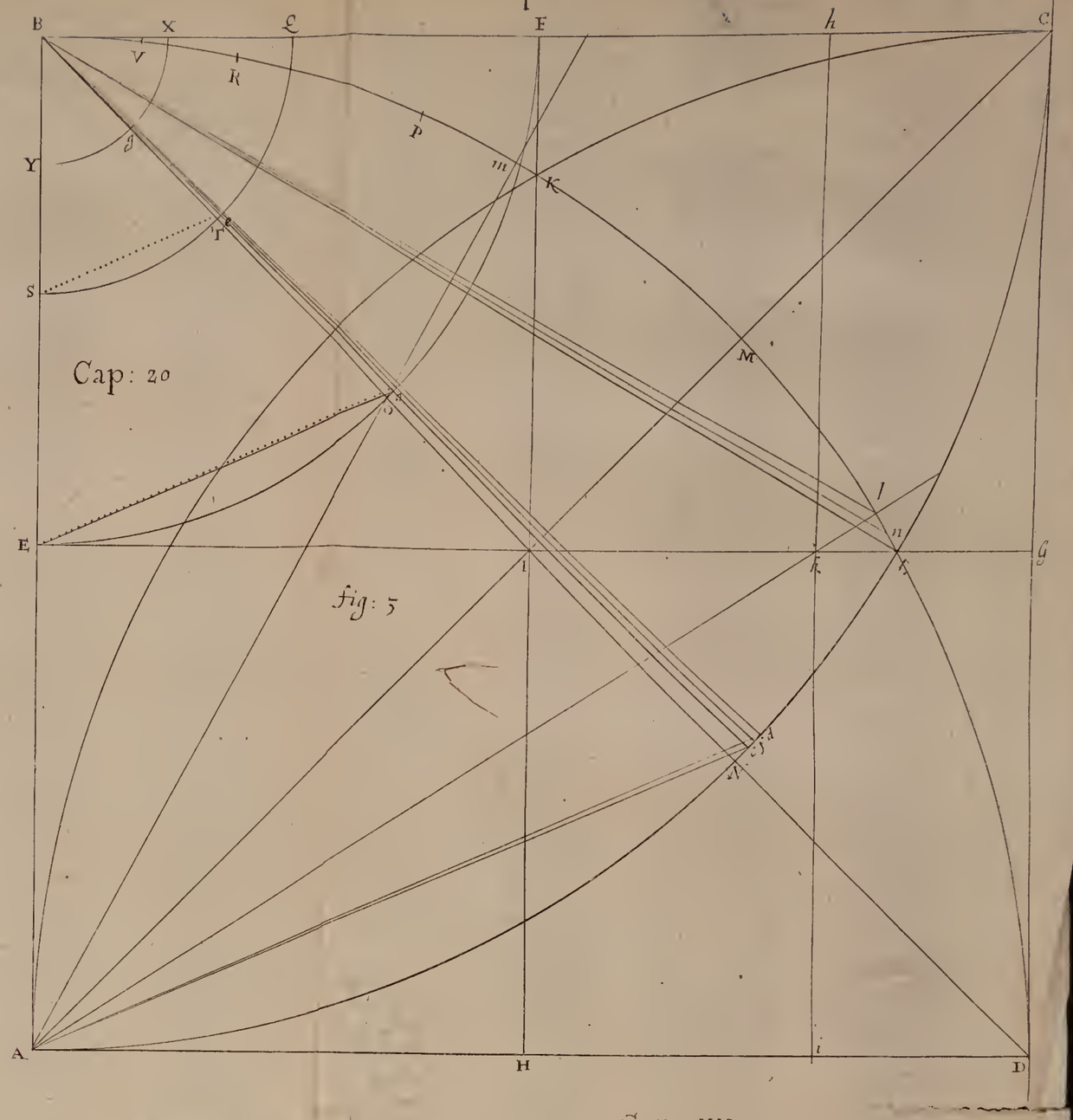


fig: 3

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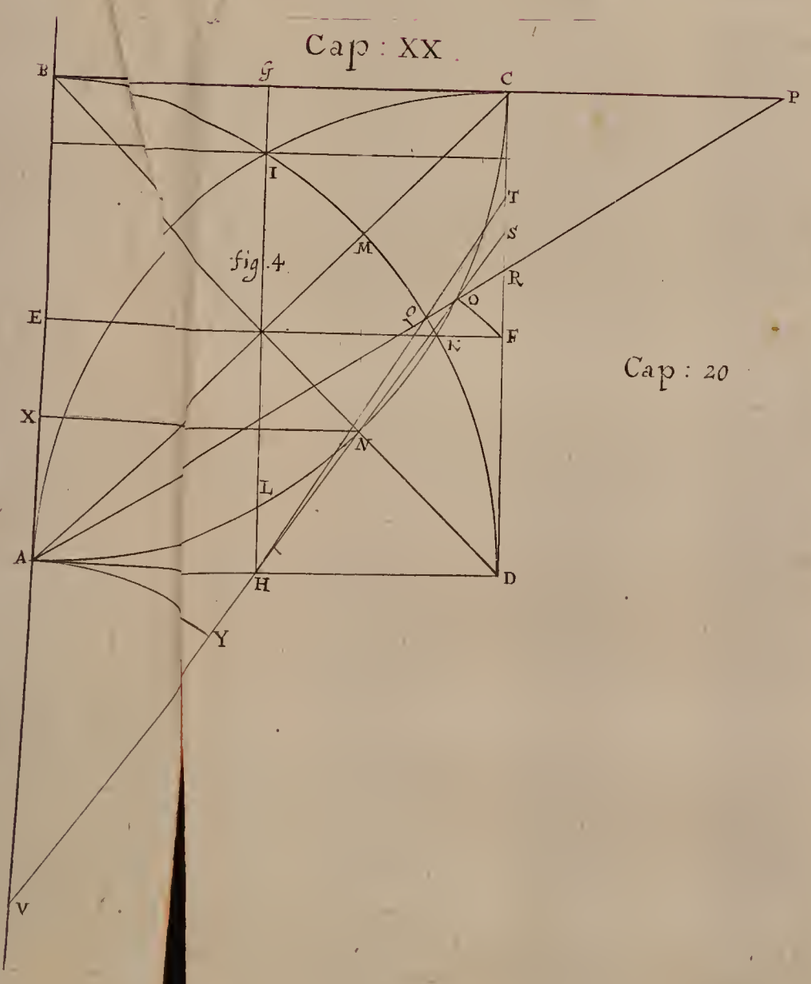


fig: 4

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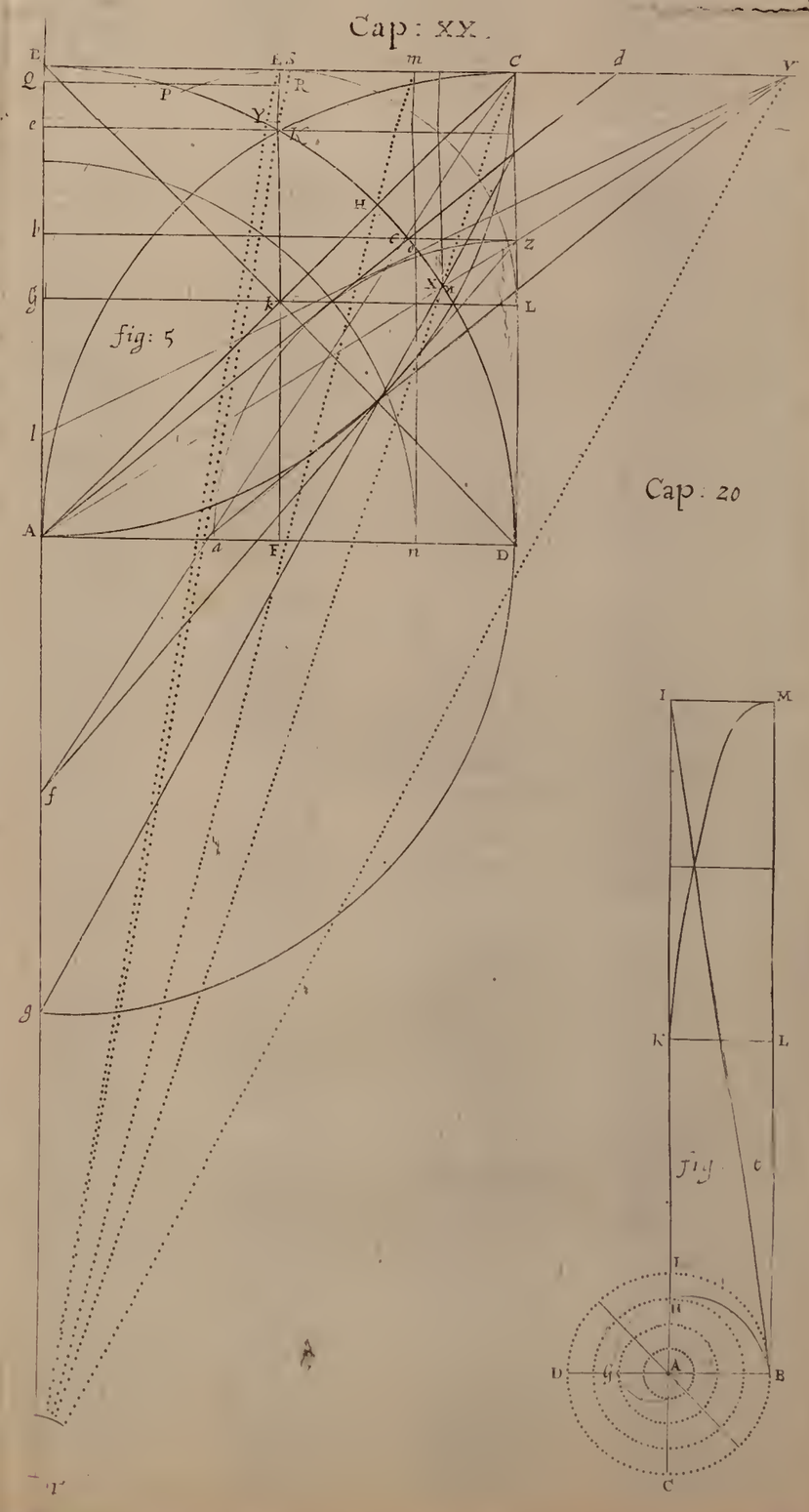


fig: 5

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fig: 6

CHAP. XXI.

Of Circular Motion.

1 *In Simple Motion, every Straight Line taken in the Body moved, is so carried, that it is always parallel to the places in which it formerly was.* 2 *If Circular Motion be made about a resting Center, and in that Circle there be an Epicyle, whose revolution is made the contrary way, in such manner, that in equal times it make equal angles, every Straight Line taken in that Epicycle will be so carried, that it will alwayes be parallel to the places in which it formerly was.* 3 *The properties of Simple Motion.* 4 *If a fluid Body be moved with simple Circular Motion, all the points taken in it will describe their Circles in times proportional to the distances from the Center.* 5 *Simple Motion dissipates Heterogeneous and congregates Homogeneous Bodies.* 6 *If a Circle made by a Movent moved with Simple Motion, be commensurable to another Circle made by a point which is carried about by the same Movent, all the points of both the Circles will at some time return to the same situation.* 7 *If a Sphere have Simple Motion, its Motion will more dissipate Heterogeneous Bodies by how much it is more remote from the Poles.* 8 *If the Simple Circular Motion of a fluid Body, be hindered by a Body which is not fluid, the fluid Body will spread it self upon the Superficies of that Body.* 9 *Circular Motion about a fixed Center, casteth off by the Tangent such things as lie upon the Circumference and stick not to it.* 10 *Such things as are moved with Simple Circular Motition, beget Simple Circular Motion.* 11 *If that which is so moved have one side hard, and the other side fluid, its Motion will not be perfectly Circular.*

1 I Have already defined *Simple Motion* to be that, in which the several points taken in a moved Body, do in several equal times describe several equal arches. And therefore in Simple Circular Motion it is necessary that every Straight Line taken in the Moved Body be alwayes carried parallel to it self; which I thus demonstrate.

First, let A B (in the first figure) be any Straight Line taken in

any Solid Body; and let A D be any arch drawn upon any Center C and Radius C A. Let the point B be understood to describe towards the same parts the arch B E, like and equall to the arch A D. Now in the same time in which the point A transmits the arch A D, the point B (which by reason of its simple motion is supposed to be carried with velocity equall to that of A) will transmit the arch B E; and at the end of the same time the whole A B will be in D E; and therefore A B and D E are equall. And seeing the arches A D and B E are like and equall, their subtending straight lines A D and B E will also be equall; and therefore the four sided figure A B D E will be a parallelogram. Wherefore A B is carried parallel to it selfe. And the same may be proved by the same method, if any other straight line be taken in the same moved Body in which the straight line A B was taken. So that all straight lines taken in a Body moved with Simple Citcular Motion will be carried parallel to themselves.

Coroll. 1 It is manifest that the same will also happen in any Body which hath Simple Motion, though not Circular. For all the points of any straight line whatsoever, will describe lines though not Circular, yet equall; so that though the crooked lines A D and B E were not arches of Circles, but of *Parabolas*, *Ellipses*, or of any other figures; yet both they, and their Subtenses, and the straight lines which joyne them, would be equal and parallel.

Coroll. 2 It is also manifest, that the *Radii* of the equall circles A D and B E, or the Axis of a Sphere, will be so carried, as to be allwayes parallel to the places in which they formerly were. For the straight line B F drawn to the center of the arch B E being equall to the Radius A C, will also be equall to the straight line F E or C D; and the angle B F E will be equall to the angle A C D. Now the intersection of the straight lines C A and B E, being at G, the angle C G E (seeing B E and A D are parallel) will be equal to the angle D A C. But the angle E B F is equal to the same angle D A C; and therefore the angles C G E and E B F are also equal. Wherefore A C and B F are parallel; which was to be demonstrated.

2 Let there be a Circle given (in the second figure), whose center

center is A, and Radius A B; and upon the center B and any Radius B C let the Epicycle C D E be described. Let the center B be understood to be carried about the center A, and the whole Epicycle with it till it be coincident with the Circle F G H, whose center is I; and let B A I be any angle given. But in the time that the center B is moved to I, let the Epicycle C D E have a contrary revolution upon its own center, namely from E by D to C according to the same proportions; that is, in such manner, that in both the Circles, equal angles be made in equal times. I say E C the Axis of the Epicycle will be alwayes carried parallel to it self. Let the angle F I G be made equal to the angle B A I; I F and A B will therefore be parallel; and how much the Axis A G has departed from its former place A C (the measure of which progression is the angle C A G, or C B D which I suppose equal to it), so much in the same time has the Axis I G (the same with B C) departed from its own former situation. Wherefore, in what time B C comes to I G by the motion from B to I upon the center A, in the same time G will come to F by the contrary motion of the Epicycle; that is, it will be turned backwards to F, & I G will lie in I F. But the angles F I G and G A C are equal; and therefore A C, that is, B C), and I G, (that is the Axis, though in different places) will be parallel. Wherefore, the Axis of the Epicycle E D C will be carried alwayes parallel to it self; which was to be proved.

Coroll. From hence it is manifest, that those two annual Motions which *Copernicus* ascribes to the Earth, are reducible to this one Circular Simple Motion, by which all the points of the moved Body are carried always with equal velocity, that is, in equal times they make equal revolutions uniformly.

This, as it is the most simple, so it is the most frequent of all Circular Motions; being the same which is used by all men when they turn any thing round with their arms, as they do in grinding or sifting. For all the points of the thing moved, describe lines which are like and equal to one another. So that if a man had a Ruler, in which many Pens points of equal length were fastned, he might with this one Motion write many lines at once.

3. Having shewed what Simple Motion is, I will here also set down some properties of the same.

First,

First, when a Body is moved with Simple Motion in a fluid Medium which hath no vacuity, it changes the ſituation of all the parts of the fluid ambient which reſiſt its motion; I ſay there are no parts ſo ſmall of the fluid ambient, how farre ſoever it be continued, but do change their ſituation, in ſuch manner, as that they leave their places continually to other ſmall parts that come into the ſame.

For (in the ſame ſecond figure) let any Body, as K L M N, be underſtood to be moved with Simple Circular Motion; and let the Circle which every point thereof deſcribes have any determined quantity, ſuppoſe that of the ſame K L M N. Wherefore the Center A, and every other point, and conſequently the moved Body it ſelf, will be carried ſometimes towards the ſide where is K, and ſometimes towards the other ſide where is M. When therefore it is carried to K, the parts of the fluid Medium on that ſide will go back; and (ſuppoſing all ſpace to be full) others on the other ſide will ſucceed. And ſo it will be when the Body is carried to the ſide M, and to N, and every way. Now when the neereſt parts of the fluid Medium go back, it is neceſſary that the parts next to thoſe neereſt parts go back alſo; and (ſuppoſing ſtill all ſpace to be full) other parts will come into their places with ſucceſſion perpetual and infinite. Wherefore all, even the leaſt parts of the fluid Medium change their places, &c. which was to be proved.

It is evident from hence, that Simple Motion, whether Circular, or not Circular, of Bodies which make perpetual returns to their former places, hath greater or leſs force to diſſipate the parts of reſiſting Bodies, as it is more or leſs ſwift, and as the lines deſcribed have greater or leſs magnitude. Now the greateſt Velocity that can be, may be underſtood to be in the leaſt circuit, and the leaſt in the greateſt; and may be ſo ſuppoſed when there is need.

4 Secondly, ſuppoſing the ſame Simple Motion in the Aire, Water, or other fluid Medium; the parts of the Medium which adhere to the Moved Body will be carried about with the ſame Motion and Velocity, ſo that in what time ſoever any point of the Movent finiſhes its Circle, in the ſame time every part of the Medium which adheres to the Movent, ſhall alſo deſcribe

ſcribe ſuch a part of its Circle, as is equal to the whole Circle of the Movent; I ſay it ſhall deſcribe a part, and not the whole Circle, becauſe all its parts receive their motion from an interiour concentrique Movent, and of Concentrique Circles the exteriour are alwayes greater then the interiour; nor can the motion imprinted by any Movent be of greater Velocity then that of the Movent it ſelf. From whence it follows, that the more remote parts of the fluid ambient, ſhall finiſh their Circles in times which have to one another the ſame proportion with their diſtances from the Movent. For every point of the fluid ambient, as long as it toucheth the Body which carries it about, is carried about with it, and would make the ſame Circle, but that it is left behind ſo much as the exteriour Circle exceeds the interiour. So that if we ſuppoſe ſome thing which is not fluid to float in that part of the fluid ambient which is neereſt to the Movent, it will together with the Movent be carried about. Now that part of the fluid ambient which is not the neereſt but almoſt the neereſt, receiving its degree of velocity from the neereſt, (which degree cannot be greater then it was in the giver) doth therefore in the ſame time make a Circular Line, not a whole Circle, yet equal to the whole Circle of the neereſt. Therefore in the ſame time that the Movent deſcribes its Circle, that which doth not touch it ſhall not deſcribe its Circle; yet it ſhall deſcribe ſuch a part of it, as is equal to the whole Circle of the Movent. And after the ſame manner, the more remote parts of the ambient will deſcribe in the ſame time ſuch parts of their Circles as ſhall be ſeverally equal to the whole Circle of the Movent; and by conſequent they ſhall finiſh their whole Circles in times proportional to their diſtances from the Movent; which was to be proved.

5 Thirdly, The ſame Simple Motion of a Body placed in a fluid Medium, congregates, or gathers into one place ſuch things as naturally float in that Medium, if they be Homogeneous; and if they be Heterogeneous, it ſeparates and diſſipates them. But if ſuch things as be Heterogeneous do not float, but ſettle, then the ſame Motion ſtirs and mingles them diſorderly together. For ſeeing Bodies which are unlike to one another, that is, Heterogeneous Bodies, are not unlike in that they are Bodies (for Bodies, as Bodies,

have

have no difference)but onely from ſome ſpecial Cauſe,that is,from ſome internal Motion,or Motions of their ſmalleſt parts(for I have ſhewn in the 9th Chapter and 9th Article, that all Mutation is ſuch Motion), it remains that Heterogeneous Bodies have their unlikeneſs or difference from one another from their internal or ſpecifical Motions. Now Bodies w^ch have ſuch difference, receive unlike & different Motions from the ſame external common Movent; and therefore they will not be moved together, that is to ſay, they will be diſſipated. And being diſſipated they will neceſſarily at ſome time or other meet with Bodies like themſelves,and be moved alike and together with them; and afterwards meeting with more Bodies like themſelves, they will unite and become greater Bodies. Wherefore Homogeneous Bodies are congregated, and Heterogenous diſſipated by Simple Motion in a Medium where they naturally float. Again, ſuch as being in a fluid Medium, do not float, but ſink, if the Motion of the fluid Medium be ſtrong enough, will be ſtirred up and carried away by that Motion, and conſequently they will be hindred from returning to that place to which they ſink naturally, and in which onely they would unite,and out of which they are promiſcuouſly carried; that is, they are diſorderly mingled.

Now this Motion by which Homogeneous Bodies are congregated, and Heterogeneous are ſcattered, is that which is commonly called *Fermentation*, from the Latine *Fervere*; as the Greeks have their Ζύμη (which ſignifies the ſame) from Ζέω *Ferveo*. For Seething makes all the parts of the Water change their places; and the parts of any thing that is thrown into it, will go ſeveral wayes according to their ſeveral natures. And yet all *Fervor* or Seething is not cauſed by Fire; for New Wine and many other things have alſo their Fermentation and *Fervor*, to which Fire contributes little, and ſome times nothing. But when in Fermentation we find Heat, it is made by the Fermentation.

6 Fourthly, in what time ſoever the Movent whoſe Center is A (in the 2d figure) moved in K L N ſhall by any number of revolutions(that is,when the Perimeters B I and K L N be commenſurable) have deſcribed a Line equal to the Circle which paſſes through the points B and I; in the ſame time all the points of the float-

floating Body whoſe Center is B, ſhall return to have the ſame ſituation in reſpect of the Movent, from which they departed. For ſeeing it is as the diſtance B A, that is, as the Radius of the Circle which paſſes through B I, is to the Perimeter it ſelf B I, ſo the Radius of the Circle K L N is to the Perimeter K L N; and ſeeing the velocities of the points B aud K are equal, the time alſo of the revolution in I B to the time of one revolution in K L N, will be as the Perimeter B I to the Perimeter K L N; and therefore ſo many revolutions in K L N as together taken are equal to the Perimeter B I, will be finiſhed in the ſame time in which the whole Perimeter B I is finiſhed; & therefore alſo the points L, N, F & H, or any of the reſt, will in the ſame time return to the ſame ſituation from which they departed; and this may be demonſtrated whatſoever be the points conſidered. Wherefore all the points ſhall in that time return to the ſame ſituation; which was to be proved.

From hence it follows, that if the Perimeters B I and L K N be not commenſurable, then all the points wil never return to have the ſame ſituation or configuration in reſpect of one another.

7 In Simple Motion, if the Body moved be of a Spherical figure, it hath leſs force towards its Poles then towards its middle, to diſſipate Heterogeneous, or to congregate Homogeneous Bodies.

Let there be a Sphere (as in the third figure) whoſe Center is A and Diameter B C; & let it be conceived to be moved with Simple Circular Motion; of which Motion let the Axis be the Straight Line D E, cutting the Diameter B C at right angles in A. Let now the Circle which is deſcribed by any point B of the Sphere, have B F for its Diameter; and taking F G equal to B C, and dividing it in the middle in H, the Center of the Sphere A, will when half a revolution is finiſhed, lie in H. And ſeeing H F aud A B are equal, a Circle deſcribed upon the Center H with the Radius H F or H G, will be equal to the Circle whoſe Center is A and Radius A B. And if the ſame Motion be continued, the point B will at the end of another half revolution return to the place from whence it began to be moved; and therefore at the end of half a revolution, the point B will be carried to F, and the whole Hemiſphere D B E into that Hemiſphere in which are the points L, K and F. Wherfore that part of the fluid Medium which is cõtiguous to the

point F, will in the same time go back the length of the Straight Line B F; and in the return of the point F to B, that is, of G to C, the fluid Medium wil go back as much in a Straight Line from the point C. And this is the effect of Simple Motion in the middle of the Sphere, where the distance from the Poles is greatest. Let now the point I be taken in the same Sphere neerer to the Pole E, and through it let the Straight Line I K be drawn parallel to the Straight Line BF, cutting the arch F L in K, & the Axis HL in M; then connecting HK, upon HF let the perpendicular KN be drawn. In the same time therefore that B comes to F, the point I will come to K, B F and I K being equal, and described with the same velocity. Now the Motion in I K to the fluid Medium upon which it works, namely to that part of the Medium which is contiguous to the point K, is oblique, whereas if it proceeded in the Straight Line H K, it would be perpendicular; and therefore the Motion which proceeds in I K has less power, then that which proceeds in H K with the same velocity. But the Motions in H K and H F do equally thrust back the Medium; and therefore the part of the Sphere at K, moves the Medium less, then the part at F; namely so much less, as K N is less then H F. Wherefore also the same Motion hath less power to disperse Heterogeneous, and to congregate Homogeneous Bodies, when it is neerer, then when it is more remote from the Poles; which was to be proved.

Corollary. It is also necessary, that in Plains which are perpendicular to the Axis, and more remote then the Pole it self from the middle of the Sphere, this Simple Motion have no effect. For the Axis D E with Simple Motion describes the Superficies of a Cylinder; and towards the Bases of the Cylinder there is in this Motion no endeavour at all.

8 If in a fluid Medium, moved about (as hath been said) with Simple Motion, there be conceived to float some other Spherical Body which is not fluid, the parts of the Medium which are stopped by that Body, will endeavour to spread themselves every way upon the Superficies of it. And this is manifest enough by experience, namely by the spreading of water poured out upon a pavement. But the reason of it may be this. Seeing the Sphere A (in the 3d figure) is moved towards B, the Medium also in which it is moved, will have the same Motion. But because in this Motion it

it falls upon a Body not liquid, as G, ſo that it cannot go on; and ſeeing the ſmall parts of the Medium can not go forwards, nor can they go directly backwards; againſt the force of the Movent; it remayns therefore that they diffuſe themſelves upon the Superficies of that Body, as towards O and P, Which was to be proved.

9 Compounded Circular Motion (in which all the parts of the moved Body do at once deſcribe Circumferences, ſome greater, others leſs, according to the proportion of their ſeveral diſtances from the common Center) carries about with it ſuch Bodies, as being not fluid, adhere to the Body ſo moved; and ſuch as do not adhere, it caſteth forwards in a Straight Line which is a Tangent to the point from which they are caſt off.

For let there be a Circle whoſe Radius is A B (in the fourth figure); and let a Body be placed in the Circumference in B, which if it be fixed there, will neceſſarily be carried about with it, as is manifeſt of it ſelf. But whileſt the motion proceeds, let us ſuppoſe that Body to be unfixed in B. I ſay the Body wil cõtinue its motion in the Tangent BC. For let both the Radius AB, and the Sphere B, be conceived to conſiſt of hard matter; and let us ſuppoſe the Radius A B to be ſtricken in the point B by ſome other Body which falls upon it in the Tangent D B. Now therefore there will be a motion made by the concourſe of two things, the one, Endeavour towards C in the Straight Line D B produced, (in which the Body B would proceed, if it were not retained by the Radius A B); the other, the Retention it ſelf. But the Retention alone cauſeth no endeavour towards the Center; and therefore the Retention being taken away, (which is done by the unfixing of B) there will remain but one Endeavour in B, namely, that in the Tangent B C. Wherefore the Motion of the Body B unfixed, will proceed in the Tangent B C; which was to be proved.

By this demonſtration it is manifeſt, that Circular Motion about an unmoved Axis, ſhakes off, and puts further from the Center of its motion ſuch things as touch, but do not ſtick faſt to its Superficies; and the more, by how much the diſtance is greater from the Poles of the Circular Motion; and ſo much the more alſo, by how much the things that are ſhaken off, are leſs driven towards the Center by the fluid ambient, for other Cauſes.

10 If in a fluid Medium a Spherical Body be moved with ſimple Circular Motion; and in the ſame Medium there float another Sphere whoſe matter is not fluid; this Sphere alſo ſhall be moved with ſimple Circular Motion.

Let B C D (in the 5th figure) be a Circle, whoſe Center is A, and in whoſe Circumference there is a Sphere ſo moved that it deſcribes with Simple Motion the Perimeter B C D. Let alſo E F G be another Sphere of Conſiſtent matter, whoſe Semidiameter is E H, and Center H; and with the Radius A H let the Circle H I be deſcribed. I ſay the Sphere E F G will (by the Motion of the Body in B C D) be moved in the Circumference H I with Simple Motion.

For ſeeing the Motion in B C D (by the 4th Article of this Chapter) makes all the points of the fluid Medium deſcribe in the ſame time Circular Lines equal to one another, the points E, H and G of the Straight Line E H G will in the ſame time deſcribe with equal *Radii* equal Circles. Let E B be drawn equal and parallel to the Straight Line A H; and let A B be connected, which will therefore be equal and parallel to E H; and therefore alſo, if upon the Center B and Radius B E the arch E K be drawn equal to the arch H I, and the ſtraight Lines A I, B K and I K be drawn, B K and A I will be equal; and they will alſo be parallel, becauſe the two arches E K and H I, that is, the two angles K B E and I A H are equal; and conſequently the Straight Lines A B and K I which connect them will alſo be equal and parallel. Wherefore K I and E H are parallel. Seeing therefore E and H are carried in the ſame time to K and I, the whole Straight Line I K will be parallel to E H, from whence it departed. And therefore, (ſeeing the Sphere E F G is ſuppoſed to be of conſiſtent matter, ſo as all its points keep alwayes the ſame ſituation) it is neceſſary that every other Straight Line taken in the ſame Sphere, be carried alwayes parallel to the places in which it formerly was. Wherefore the Sphere E F G is moved with ſimple Circular Motion; which was to be demonſtrated.

11 If in a fluid Medium, whoſe parts are ſtirred by a Body moved with Simple Motion, there float annother Body, which hath its Superficies either wholly hard, or wholly fluid; the parts of this

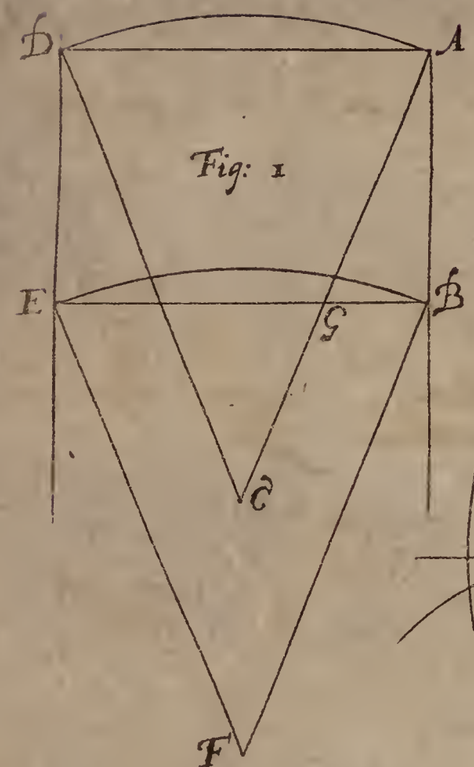
this Body ſhall approach the Center equally on all ſides; that is to ſay, the motion of the Body ſhall be Circular, and Concentrique with the motion of the Movent. But if it have one ſide hard, and the other ſide fluid, then both thoſe Motions ſhall not have the ſame center, nor ſhall the floating Body be moved in the Circumference of a perfect Circle.

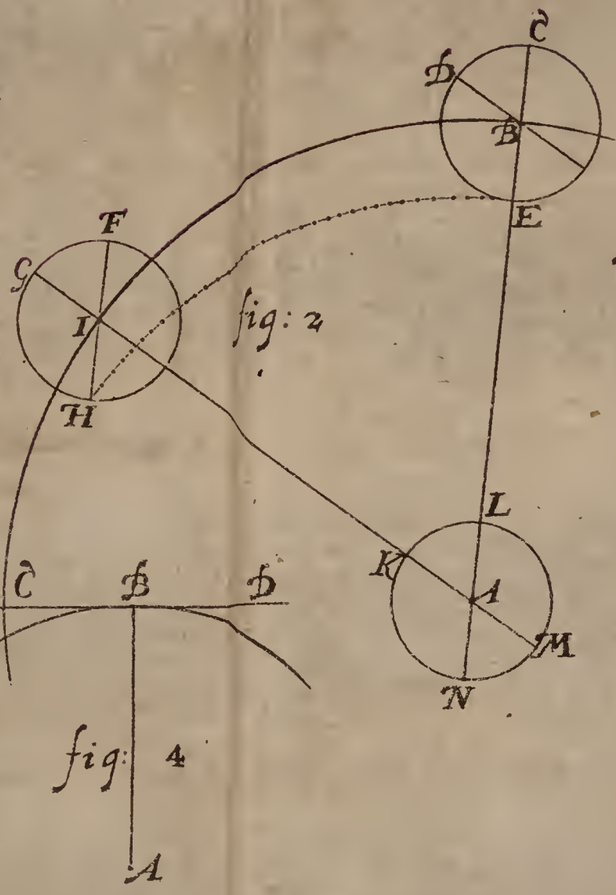
Let a Body be moved in the Circumference of the Circle K L M N (in the 2d figure) whoſe center is A. And let there be another Body at I, whoſe Superficies is either all hard, or all fluid. Alſo let the Medium in which both the Bodies are placed, be fluid. I ſay the Body at I will be moved in the Circle I B about the Center A. For this has been demonſtrated in the laſt Article.

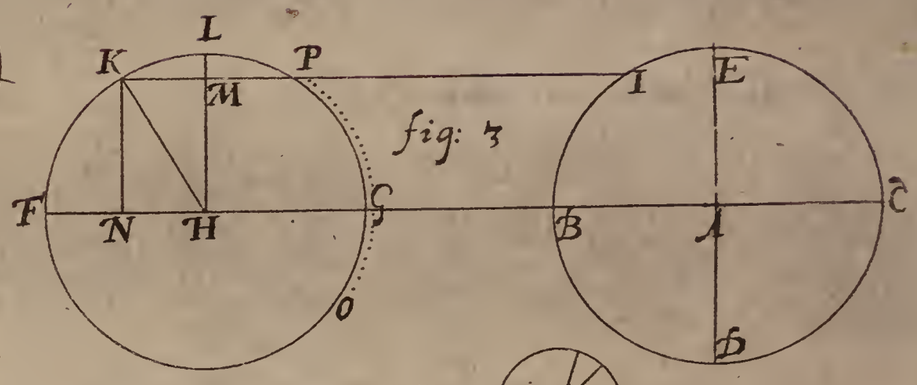
Wherefore let the Superficies of the Body at I, be fluid on one ſide, and hard on the other. And firſt, let the fluid ſide be towards the Center. Seeing therefore the Motion of the Medium is ſuch, as that its parts do continually change their places, (as hath been ſhewn in the 5th Article); if this change of place be conſidered in thoſe parts of the Medium which are contiguous to the fluid Superficies, it muſt needs be, that the ſmall parts of that Superficies enter into the places of the ſmall parts of the Medium which are contiguous to them; And the like change of place will be made with the next contiguous parts towards A. And if the fluid parts of the Body at I, have any degree at all of tenacity (for there are degrees of tenacity, as in the Aire and Water) the whole fluid ſide will be lifted up a little; but ſo much the leſs, as its parts have leſs tenacity; whereas the hard part of the Superficies which is contiguous to the fluid part, has no cauſe at all of elevation, that is to ſay, no endeavour towards A.

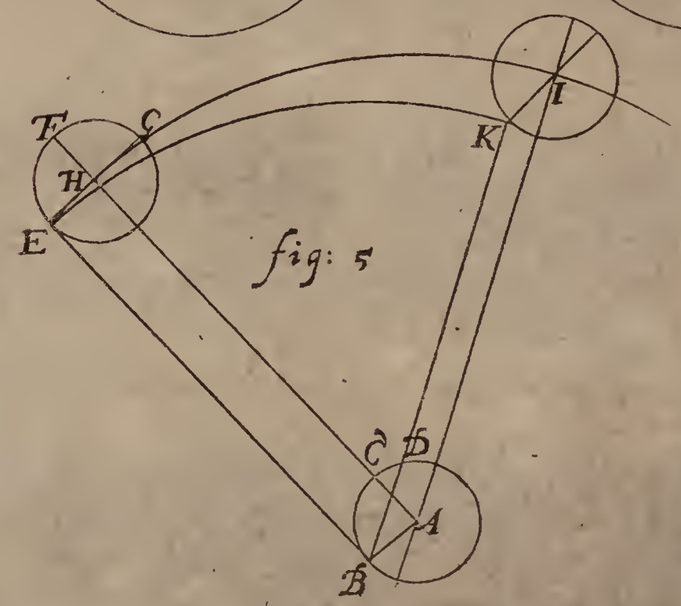
Secondly, let the hard Superficies of the Body at I, be towards A. By reaſon therefore of the ſaid change of place of the parts which are contiguous to it, the hard Superficies muſt of neceſſity (ſeeing by Suppoſition there is no empty Space) either come neerer to A, or elſe its ſmalleſt parts muſt ſupply the contiguous places of the Medium, which otherwiſe would be empty. But this cannot be by reaſon of the ſuppoſed hardneſs; and therefore the other muſt needs be, namely, that the Body come neerer to A. Wherefore the Body at I, has greater endeavour towards the center

ter A, when its hard ſide is next it, then when it is averted from it. But the Body in I, while it is moving in the circumference of the Circle I B, has ſometimes one ſide, ſometimes another turned towards the center; and therefore it is ſometimes neerer, ſometimes further off from the center A. Wherefore the Body at I, is not carried in the circumference of a perfect Circle; which was to be demonſtrated.


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Fig: 1
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F


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fig: 2
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fig: 4
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fig: 3
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I
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fig: 5
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CHAP. XXII.

Of other Variety of Motion.

1 *Endeavour and Preſſure how they differ.* 2 *Two kinds of Mediums in which Bodies are moved.* 3 *Propagation of Motion what it is* 4 *What motion Bodies have when they preſs one another.* 5 *Fluid Bodies, when they are preſſed together, penetrate one another.* 6 *When one Body preſſeth another, and doth not penetrate it, the action of the preſſing Body is perpendicular to the Superficies of the Body preſſed.* 7 *When a hard Body, preſſing another Body, penetrates the ſame, it doth not penetrate it perpendicularly, unleſs it fall perpendicularly upon it.* 8 *Motion ſometimes oppoſite to that of the Movent.* 9 *In a full Medium, Motion is propagated to any diſtance.* 10 *Dilatation and Contraction what they are.* 11 *Dilatation and Contraction ſuppoſe Mutation of the ſmalleſt parts in reſpect of their ſituation.* 12 *All Traction is Pulſion.* 13 *Such things as being preſſed, or bent, reſtore themſelves, have motion in their internal parts.* 14 *Though that which carrieth another be ſtopped, the Body carried will proceed.* 15,16 *The effects of Percuſſion not to be compared with thoſe of Waight.* 17,18 *Motion cannot begin firſt in the internal parts of a Body.* 19 *Action and Reaction proceed in the ſame Line.* 20 *Habit what it is.*

1 I Have already (in the 15th Chap. at the 2d Article) defined Endeavour to be Motion through ſome Length, though not conſidered as Length, but as a Point. Whether therefore there be reſiſtance or no reſiſtance, the Endeavour will ſtill be the ſame. For ſimply to Endeavour, is to Go. But when two Bodies having oppoſite Endeavours preſs one another, then the Endeavour of either of them is that which we call Preſſure, and is mutual when their preſſures are oppoſite.

2 Bodies moved, and alſo the Mediums in which they are moved, are of two kinds. For either they have their parts coherent in ſuch manner, as no part of the Moved Body will eaſily yeild to the Mouent, except the whole Body yeild alſo, and ſuch are the things

we

we call *Hard*; Or elſe their parts, while the whole remains unmoved, will eaſily yeild to the Movent; and theſe we call *Fluid* or *Soft* Bodies. For the words *Fluid*, *Soft*, *Tough* and *Hard* (in the ſame manner as *Great* and *Little*) are uſed onely comparatively; and are not different kinds, but different degrees of Quality.

3 To *Do*, and to *Suffer* is to Move and to be moved; and nothing is moved, but by that which toucheth it, and is alſo moved, (as has been formerly ſhewn). And how great ſoever the diſtance be, we ſay the firſt Movent moveth the laſt moved Body; but mediately; namely ſo, as that the firſt moveth the ſecond, the ſecond the third, and ſo on, till the laſt of all be touched. When therefore one Body having oppoſite Endeavour to another Body, moveth the ſame, and that moveth a third, and ſo on, I call that action *Propagation of Motion*.

4 When two fluid Bodies which are in a free and open Space, preſs one another, their parts will endeavour, or be moved towards the ſides, not onely thoſe parts which are there where the mutual contact is, but all the other parts. For in the firſt contact, the parts which are preſſed by both the endeavouring Bodies, have no place either forwards or backwards in which they can be moved; and therefore they are preſſed out towards the ſides. And this expreſſure, when the forces are equal, is in a line perpendicular to the Bodies preſſing. But whenſoever the formoſt parts of both the Bodies are preſſed, the hindermoſt alſo muſt be preſſed at the ſame time; for the motion of the hindermoſt parts cannot in an inſtant be ſtopped by the reſiſtance of the formoſt parts, but proceeds for ſome time; and therefore ſeeing they muſt have ſome place in which they may be moved, and that there is no place at all for them forwards, it is neceſſary that they be moved into the places which are towards the ſides every way. And this effect followes of neceſſity, not onely in Fluid, but in Conſiſtent and Hard Bodies, though it be not alwayes manifeſt to ſenſe. For though from the compreſſion of two ſtones we cannot with our eyes diſcerne any ſwelling outwards towards the ſides, (as we perceive in two Bodies of wax;) yet we know well enough by reaſon, that ſome tumor muſt needs be there, though it be but little.

5. But

5 But when the Space is encloſed, and both the Bodies be fluid, they will (if they be preſſed together) penetrate one anoteer, though differently according to their different endeavours. For ſuppoſe a hollow Cylinder of hard matter, well ſtopped at both ends, but filled firſt, below with ſome heavy fluid Body, as Quickſilver; and above with Water or Aire. If now the bottome of the Cylinder be turned upwards, the heavieſt fluid Body which is now at the top, having the greateſt endeavour downwards, and being by the hard ſides of the veſſel hindered from extending it ſelfe ſidewayes, muſt of neceſſity either be received by the lighter Body, that it may ſink through it, or elſe it muſt open a paſſage through it ſelfe, by which the lighter Body may aſcend. For of the two Bodies, that whoſe parts are moſt eaſily ſeparated, will the firſt be divided; which being done, it is not neceſſary that the parts of the other, ſuffer any ſeparation at all. And therefore when two Liquours which are encloſed in the ſame veſſel, change their places, there is no need that their ſmalleſt parts ſhould be mingled with one another; for a way being opened through one of them, the parts of the other need not be ſeparated.

Now if a fluid Body which is not encloſed preſs a hard Body, its endeavour will indeed be towards the internal parts of that hard Body; but (being excluded by the reſiſtance of it) the parts of the fluid Body will be moved every way according to the Superficies of the hard Body, and that equally, if the preſſure be perpendicular; for when all the parts of the Cauſe are equal, the Effects will be equal alſo. But if the preſſure be not perpendicular, then the angles of Incidence being unequal, the expanſion alſo will be unequal, namely, greater on that ſide where the angle is greater, becauſe that motion is moſt direct which proceeds by the directeſt Line.

6 If a Body, preſſing another Body do not penetrate it, it will nevertheleſs give to the part it preſſeth, an endeavour to yeild and recede in a ſtraight line perpendicular to its Superficies in that point in which it is preſſed.

Let A B C D (in the firſt figure) be a hard Body; and let another Body, falling upon it in the ſtraight line E A, with any inclination, or without inclination, preſs it in the point A. I ſay the Body ſo

preſſing,& not penetrating it, will give to the part A an endeavour to yeild or recede in a ſtraight Line perpendicular to the line AD.

For let A B be perpendicular to A D; and let B A be produced to F. If therefore A F be coincident with A E, it is of it ſelf manifeſt that the motion in E A will make A to endeavour in the line A B. Let now E A be oblique to A D; and from the point E let the ſtraight line E C be drawn, cutting A D at right augles in D; and let the rectangles A B C D and A D E F be completed. I have ſhewn (in the 8th Article of the 16th Chapter) that the Body will be carried from E to A by the concourſe of two Uniform Motions, the one in E F and its parallels, the other in E D and its parallels. But the motion in E F and its parallels (whereof D A is one) contributes nothing to the Body in A, to make it endeavour or preſs towards B; and therefore the whole endeavour which the Body hath in the inclined line E A, to paſs, or preſs the Straight line AD, it hath it all from the perpendicular motion or endeavour in F A. Wherefore the Body E after it is in A, will have onely that perpendicular endeavour which proceeds from the motion in F A, that is, in A B, which was to be proved.

7 If a hard Body falling upon, or preſſing another Body, penetrate the ſame, its endeavour after its firſt penetration will be neither in the inclined line produced, nor in the perpendicular, but ſometimes betwixt both, ſometimes without them.

Let E A G (in the ſame 1 figure) be the inclined line produced; and Firſt, let the paſſage through the Medium in which E A is, be eaſier then the paſſage through the Medium in which A G is. As ſoon therefore as the Body is within the Medium in which is A G, it will finde greater reſiſtance to its motion in D A and its parallels, then it did whileſt it was above A D; and therefore below A D it will proceed with ſlower motion in the parallels of D A, then above it. Wherefore the motion which is compounded of the two motions in E F and E D will be ſlower below AD, then above it; and therefore alſo, the Body will not proceed from A in E A produced, but below it. Seeing therefore the endeavour in A B is generated by the endeavour in F A; if to the endeavour in F A there be added the endeavour in D A, (which is not all taken away by the immerſion of the point A into the lower Medium) the Body will not proceed from A in the perpendicular A B, but be-

yond

yond it, namely, in ſome ſtraight line between A B and A G, as in the line A H.

Secondly, let the paſſage through the Medium E A, be leſs eaſie then that through A G. The motion therefore which is made by the concourſe of the motions in E F and F B, is ſlower above A D then below it; aud conſequently, the endeavour will not proceed from A in E A produced, but beyond it, as in A I. Wherefote, If a hard Body falling, &c; which was to be proved.

This Divergency of the Straight line A H from the ſtraight line A G, is that which the Writers of *Opticks* commonly call *Refraction*; which, when the paſſage is eaſier in the firſt then in the ſecond Medium, is made by diverging from the line of Inclination towards the perpendicular; and contrarily, when the paſſage is not ſo eaſie in the firſt Medium, by departing farther from the perpendicular.

8 By the 6th *Theoreme* it is manifeſt, that the force of the Movent may be ſo placed, as that the Body moved by it, may proceed in a way almoſt directly contrary to that of the Movent; as we ſee in the motion of Ships.

For let A B (in the 2d figure) repreſent a Ship, whoſe length from the prow to the poop is A B; and let the winde lie upon it in the ſtraight parallel lines C B, D E and F G, and let D E and F G be cut in E and G by a ſtraight Line drawn from B perpendicular to A B; alſo let B E and E G be equal, and the angle A B C any angle how ſmall ſoever. Then between B C and B A let the ſtraight line B I be drawn, and let the Sail be conceived to be ſpred in the ſame line B I, and the winde to fall upon it in the points L, M and B; from which points, perpendicular to B I, let B K, M Q and L P be drawn. Laſtly, let E N and G O be drawn perpendicular to B G, and cutting B K in H and K; and let H N and K O be made equal to one another, and ſeverally equal to B A. I ſay the Ship B A by the winde falling upon it in C B, D E, F G, and other lines parallel to them, will be carried forwards almoſt oppoſite to the winde, that is to ſay, in a way almoſt contrary to the way of the Movent.

For the Winde that blowes in the Line C B, will (as hath been ſhewn in the 6th Article) give to the point B an endeavour to proceed in a ſtraight line perpendicular to the ſtraight line B I, that

is, in the ſtraight line B K; and to the points M and L an endeavour to proceed in the ſtraight lines M Q and L P, which are parallel to B K. Let now the meaſure of the time be B G, which is divided in the middle in E; & let the point B be carried to H in the time B E. In the ſame time therefore by the wind blowing in D M & F L (and as many other lines as may be drawn parallel to them) the whole Ship will be applyed to the ſtraight line H N. Alſo at the end of the ſecond time E G, it will be applyed to the ſtraight line K O. Wherefore the Ship will always go forwards; and the angle it makes with the winde will be equal to the angle A B C, how ſmall ſoever that angle be; and the way it makes will in every time be equal to the ſtraight line E H. I ſay thus it would be, if the Ship might be moved with as great celerity ſidewayes from B A towards K O, as it may be moved forwards in the line B A. But this is impoſſible, by reaſon of the reſiſtance made by the great quantity of water which preſſeth the ſide, much exceeding the reſiſtance made by the much ſmaller quantity which preſſeth the prow of the Ship; ſo that the way the Ship makes ſidewayes is ſcarce ſenſible; and therefore the point B will proceed almoſt in the very line B A, making with the winde the angle A B C, how acute ſoever, that is to ſay, it will proceed almoſt in the ſtraight line B C, that is, in a way almoſt contrary to the way of the Movent; which was to be demonſtrated.

But the Sayl in B I muſt be ſo ſtretched, as that there be left in it no boſome at all; for otherwiſe the ſtraight lines L P, M Q & B K will not be perpendicular to the plain of the Sayl, but falling below P, Q and K will drive the Ship backwards. But by making uſe of a ſmall Board for a Sayl, a little Waggon with wheels for the Ship, and of a ſmooth Pavement for the Sea, I have by experience found this to be ſo true, that I could ſcarce oppoſe the board to the winde in any obliquity though never ſo ſmall, but the Waggon was carried forwards by it.

By the ſame 6th. *Theoreme*, it may be found, how much a ſtroke which falls obliquely, is weaker then a ſtroke falling perpendicularly, they being like and equal in all other reſpects.

Let a ſtroke fall upon the Wall A B obliquely, as (for example) in the ſtraight line C A (in the 3d figure). Let C E be drawn parallel

rallel to AB, & DA perpendicular to the ſame AB & equal to CA; & let both the velocity & time of the motion in CA be equal to the velocity & time of the motion in DA. I ſay the ſtroke in CA will be weaker then that in DA in the proportion of E A to D A. For producing D A howſoever to F, the endeavour of both the ſtrokes will (by the 6th Art.) proceed from A in the perpendicular A F. But the ſtroke in C A is made by the concourſe of two motions in C E and E A; of which that in C E contributes nothing to the ſtroke in A, becauſe C E and B A are parallels; and therefore the ſtroke in C A is made by the motion which is in E A onely. But the velocity or force of the perpendicular ſtroke in E A, to the velocity or force of the ſtroke in D A, is as E A to D A. Wherefore the oblique ſtroke in C A is weaker then the perpendicular ſtroke in D A, in the proportion of E A to D A or C A; Which was to be proved.

9 In a full Medium, all Endeavour proceeds as far as the Medium it ſelf reacheth; that is to ſay, if the Medium be infinite, the Endeavour will proceed infinitely.

For whatſoever Endeavoureth, is Moved, and therefore whatſoever ſtandeth in its way, it maketh it yeild, at leaſt a little, namely ſo far as the Movent it ſelf is moved forwards. But that which yeildeth is alſo moved, and conſequently maketh that to yeild which is in its way, and ſo on ſucceſſively as long as the Medium is full; that is to ſay, infinitely, if the full Medium be infinite; which was to be proved.

Now although Endeavour thus perpetually propagated, do not alwayes appear to the Senſes as Motion; yet it appears as Action, or as the efficient cauſe of ſome Mutation. For if there be placed before our Eyes ſome very little object, as (for example) a ſmall grain of ſand, which at a certain diſtance is viſible; it is manifeſt that it may be removed to ſuch a diſtance as not to be any longer ſeen, though by its action it ſtill work upon the organs of ſight, as is manifeſt from that (which was laſt proved) that all Endeavour proceeds infinitely. Let it be conceived therefore to be removed from our Eyes to any diſtance how great ſoever, and a ſufficient number of other grains of ſand of the ſame bigneſs added to it; it is evident that the aggregate of all thoſe ſands will be viſible; and though none of them can be ſeen when it is ſingle and ſevered

from

from the reſt, yet the whole heap or hill which they make wil manifeſtly appear to the ſight; which would be impoſſible if ſome action did not proceed from each ſeveral part of the whole heap.

10 Between the degrees of *Hard* and *Soft*, are thoſe things which we call *Tough*, *Tough* being that, which may be bended without being altered from what it was; and the Bending of a Line, is either the adduction or diduction of the extreme parts, that is, a morion from Straightneſs to Crookedneſs, or contrarily, whileſt the line remains ſtill the ſame it was; for by drawing out the extreme points of a line to their greateſt diſtance, the line is made ſtraight, which otherwiſe is Crooked. So alſo the Bending of a Superficies, is the diduction or adduction of its extreme lines, that is, their Dilatation and Contraction.

11 *Dilatation* and *Contraction*, as alſo all *Flexion* ſuppoſes neceſſarily that the internal parts of the Body bowed do either come neerer to the external parts, or go further from them. For though *Flexion* be conſidered onely in the length of a Body, yet when that Body is bowed, the line which is made on one ſide will be convex, and the line on the other ſide will be concave; of which the concave being the interiour line, will (unleſs ſomething be taken from it and added to the convex line) be the more crooked, that is, the greater of the two. But they are equal; and therefore in *Flexion* there is an acceſſion made from the interiour to the exteriour parts; and on the contrary, in *Tenſion*, from the exteriour to the interiour parts. And as for thoſe things which do not eaſily ſuffer ſuch tranſpoſition of their parts, they are called *Brittle*; and the great force they require to make them yeild, makes them alſo with ſudden motion to leap aſunder, and break in pieces.

12 Alſo Motion is diſtinguiſhed into *Pulſion* and *Traction*. And *Pulſion*, as I have already defined it, is when that which is moved, goes before that which moveth it. But contrarily, in *Traction* the Movent goes before that which is moved. Nevertheleſs, conſidering it with greater attention, it ſeemeth to be the ſame with *Pulſion*. For of two parts of a hard Body, when that which is foremoſt drives before it the Medium in which the motion is made, at the ſame time that which is thruſt forwards, thruſteth the next, and this again the next, and ſo on ſucceſſively. In which action, if we

ſup-

ſuppoſe that there is no place void, it muſt needs be, that by continual Pulſion, namely, when that action has gone round, the Movent will be behind that part which at the firſt ſeemed not to be thruſt forwards, but to be drawn; ſo that now the Body which was drawn, goes before the Body which gives it motion; and its motion is no longer Traction, but Pulſion.

13 Such things as are removed from their places by forcible Compreſſion or Extenſion, and as ſoon as the force is taken away, doe preſently return and reſtore themſelves to their former ſituation, have the beginning of their reſtitution within themſelves, namely, a certain motion in their internal parts, which was there, when, before the taking away of the force, they were compreſſed, or extended. For that Reſtitution is motion, and that which is at reſt cannot be moved, but by a moved and a Contiguous Movent. Nor doth the cauſe of their Reſtitution proceed from the taking away of the force by which they were compreſſed or extended; for the removing of impediments hath not the efficacy of a cauſe (as has been ſhewn at the end of the 3d Article of the 15th Chapter). The Cauſe therefore of their Reſtitution, is ſome motion either of the parts of the Ambient, or of the parts of the Body compreſſed or extended. But the parts of the Ambient have no endeavour which contributes to their Compreſſion or Extenſion, nor to the ſetting of them at liberty, or Reſtitution. It remayns therefore that from the time of their Compreſſion or Extenſion there be left ſome endeavour (or motion) by which, the impediment being removed, every part reſumes its former place; that is to ſay, the whole Reſtores it ſelf.

14 In the *Carriage* of Bodies if that Body which carries another, hit upon any obſtacle, or be by any means ſuddenly ſtopped, and that which is carried be not ſtopped, it will go on, till its motion be by ſome external impediment taken away.

For I have demonſtrated in the 8th Chapter at the 19th Article, that Motion, unleſs it be hindred by ſome external reſiſtance, will be continued eternally with the ſame celerity; and in the 7th Article of the 9th Chap. that the action of an external Agent is of no effect without contact. When therefore that which carrieth another thing, is ſtopped, that ſtop doth not preſently take away the motion of that which is carried. It will therefore proceed,

till

till its motion be by little and little extinguiſhed by ſome external reſiſtance; Which was to be proved; Though experience alone had been ſufficient to prove this.

In like manner, if that Body which carrieth another be put from reſt into ſudden motion; that which is carried will not be moved forwards together with it, but will be left behind. For the contiguous part of the Body carried, hath almoſt the ſame motion with the Body which carries it; and the remote parts will receive different Velocities according to their different diſtances from the Body that carries them; namely, the more remote the parts are, the leſs will be their degrees of Velocity. It is neceſſary therefore that the Body which is carried, be left accordingly more or leſs behind. And this alſo is manifeſt by experience, when at the ſtarting forward of the Horſe, the Rider falleth backwards.

15 In *Percuſſion* therefore, when one hard Body is in ſome ſmall part of it ſtricken by another with great force, it is not neceſſary that the whole Body ſhould yeild to the ſtroke with the ſame celerity with which the ſtricken part yeilds. For the reſt of the parts receive their motion from the motion of the part ſtricken and yeilding, which motion is leſs propagated every way towards the ſides then it is directly forwards. And hence it is, that ſometimes very hard Bodies, which being erected can hardly be made to ſtand, are more eaſily broken, then thrown down by a violent ſtroke; when nevertheleſs, if all their parts together were by any weak motion thruſt forwards they would eaſily be caſt down.

16 Though the difference between *Truſion* and *Percuſſion* conſiſt onely in this, that in *Truſion* the motion both of the Movent and Moved Body begin both together in their very contact; and in *Percuſſion* the ſtriking Body is firſt moved, and afterwards the Body ſtricken; Yet their Effects are ſo different, that it ſeems ſcarce poſſible to compare their forces with one another. I ſay, any effect of Percuſſion being propounded, as for example the ſtroke of a Beetle of any weight aſſigned, by which a Pile of any given length, is to be driven into earth of any tenacity given, it ſeems to me very hard if not impoſſible to define, with what weight, or with what ſtroke, and in what time, the ſame pile may be driven to a depth aſſigned into the ſame earth. The cauſe of which difficulty

ficulty is this, that the velocity of the Percutient is to be compared with the magnitude of the Ponderant. Now Velocity, seeing it is computed by the length of space transmitted, is to be accounted but as one Dimension; but Waight, is as a solid thing, being measured by the dimension of the whole Body. And there is no comparison to be made of a Solid Body with a Length, that is, with a Line.

17 If the internal parts of a Body be at rest, or retain the same situation with one another for any time how little soever, there cannot in those parts be generated any new motion, or endeavour, whereof the efficient cause is not without the Body of which they are parts. For if any small part which is comprehended within the Superficies of the whole Body, be supposed to be now at rest, and by and by to be moved, that part must of necessity receive its motion from some moved and contiguous Body. But (by supposition) there is no such moved and contiguous part within the Body. Wherefore, if there be any Endeavour or Motion, or change of situation, in the internal parts of that Body, it must needs arise from some efficient cause that is without the Body which contains them; Which was to be proved.

18 In hard Bodies therefore which are compressed or extended, if that which compresseth or extendeth them being taken away, they restore themselves to their former place or situation, it must needs be, that that Endeavour (or Motion) of their internal parts, by which they were able to recover their former places or situations, was not extinguished when the force by which they were compressed or extended was taken away. Therefore when the Lath of a Cross-bow bent, doth, as soon as it is at liberty, restore it self, though to him that judges by Sense, both it and all its parts seem to be at rest; yet he that judging by Reason, doth not account the taking away of impediment for an efficient cause, nor conceives that without an efficient cause any thing can pass from Rest to Motion, will conclude, that the parts were already in motion before they began to restore themselves.

19 *Action* and *Reaction* proceed in the same Line, but from opposite Terms. For seeing Reaction is nothing but Endeavour in the Patient to restore it self to that situation from which it was for-

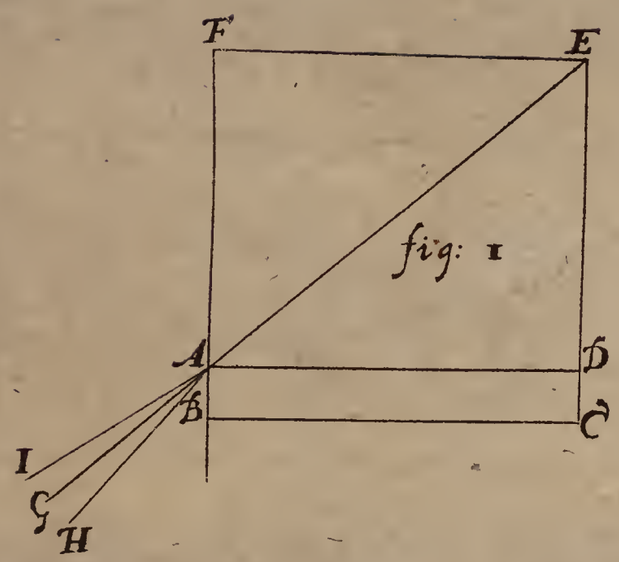
ced by the Agent; the endeavour or motion both of the Agent and Patient (or Reagent) will be propagated between the same terms, (yet so, as that in Action the Term *from which*, is in Reaction the Term *to which*). And seeing all Action proceeds in this manner, not onely between the opposite Terms of the whole line in which it is propagated, but also in all the parts of that line, the Terms *from which* and *to which*, both of the Action and Reaction, will be in the same line. Wherefore Action and Reaction proceed in the same line, &c.

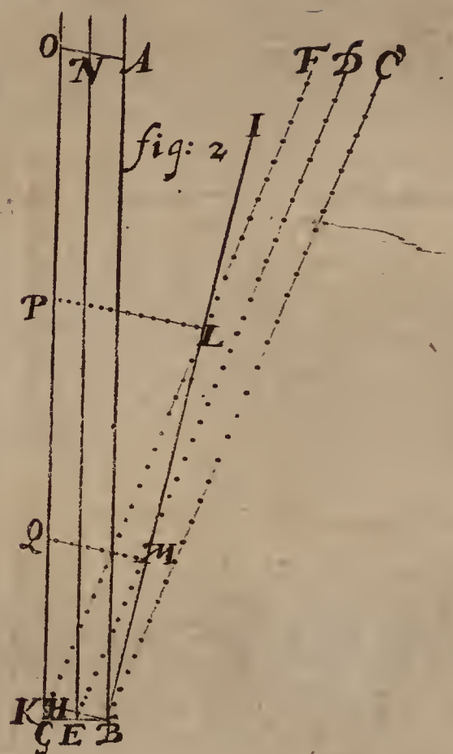
20. To what has been said of Motion, I will add what I have to say concerning Habit. *Habit* therefore is a generation of Motion, not of Motion simply, but an easie conducting of the moved Body in a certain and designed way. And seeing it is attained by the weakning of such endeavours as divert its motion, therefore such endeavours are to be weakned by little and little. But this cannot be done but by the long continuance of action, or by actions often repeated; and therefore Custome begets that Facility, which is commonly and rightly called *Habit*; and it may be defined thus; HABIT *is Motion made more easie and ready by Custome; that is to say, by perpetual endeavour, or by iterated endeavours in a way differing from that in which the Motion proceeded from the beginning, and opposing such endeavours as resist.* And to make this more perspicuous by example, We may observe, that when one that has no skill in Musique, first, puts his hand to an Instrument, he cannot after the first stroke carry to his hand to the place where he would make the second stroke, without taking it back by a new endeavour, and as it were beginning again, pass from the first to the second. Nor will he be able to go on to the third place without another new endeavour, but he will be forced to draw back his hand again, and so successively, by renewing his endeavour at every stroke, till at the last by doing this often, and by compounding many interrupted motions or endeavours into one equal endeavour, he be able to make his hand go readily on from stroke to stroke in that order and way which was at the first designed. Nor are *Habits* to be observed in living creatures only, but also in Bodies inanimate. For we find, that when the Lath of a Crossbow is strongly bent, and would if the impedi-

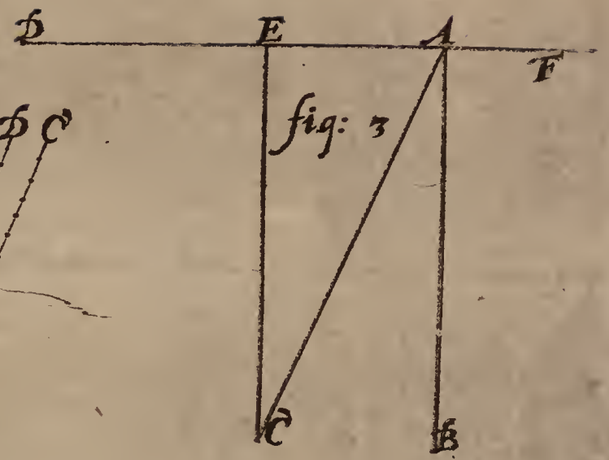
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ment were removed return again with great force, if it remain a long time bent, it will get such a *Habit*, that when it is looſed and left to its own freedome, it will not onely not reſtore it ſelf, but will require as much force for the bringing of it back to its firſt poſture, as it did for the bending of it at the firſt.

AD CAP. XXII







CHAP. XXIII.

Of the Center *of* Equiponderation *of Bodies preſsing downwards in ſtraight Parallel Lines.*

1 *Definitions and Suppoſitions.* 2 *Two Plains of Equiponderation are not parallel.* 3 *The Center of Equiponderation is in every Plain of Equiponderation.* 4 *The Moments of equal Ponderants are to one another as their diſtances from the center of the Scale.* 5,6. *The Moments of unequal Ponderants have their proportion to one another compounded of the proportions of their Waights and diſtances from the center of the Scale reciprocally taken.* 7 *If two Ponderants have their Moments and Diſtances from the Center of the Scale in reciprocal proportion, they are equally poiſed; and contrarily.* 8 *If the parts of any Ponderant preſs the Beam of the Scale every where equally, all the parts cut out off reckoned from the Center of the Scale, will have their Moments in the ſame proportion with that of the parts of a Triangle cut off from the Vertex by ſtraight Lines parallel to the baſe.* 9 *The Diameter of Equiponderation of Figures which are deficient according to commenſurable proportions of their altitudes and baſes, divides the Axis, ſo, that the part taken next the vertex is to the other part as the complete figure to the deficient figure.* 10 *The diameter of Equiponderation of the Complement of the half of any of the ſaid deficient figures, divides that line which is drawn through the vertex parallel to the baſe, ſo, that the part next the vertex is to the other part as the complete figure to the Complement.* 11 *The Center of Equiponderation of the half of any of the deficient figures in the firſt row of the Table of the 3d. Article of the 17th Chapter, may be found out by the numbers of the ſecond row.* 12 *The center of Equiponderation of the half of any of the figures in the ſecond row of the ſame Table, may be found out by the numbers of the fourth row.* 13 *The Center of Equiponderation of the half of any of the figures in the ſame Table, being known, the Center of the Exceſs of the ſame figure above a Triangle of the ſame altitude and baſe is alſo known.* 14 *The Center of Equiponderation of a ſolid Sector, is in*

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the Axis, ſo divided, that the part next the Vertex be to the whole Axis want half the Axis of the portion of the Sphere, as 3 to 4.

Definitions.

1 A *Scale*, is a ſtraight line, whoſe middle point is immoveable, all the reſt of its points being at liberty; and that part of the Scale which reaches from the center to either of the waights, is called the *Beam*.

2 *Equiponderation* is, when the endeavour of one Body which preſſes one of the Beams, reſiſts the endeavour of another Body preſſing the other Beam, ſo, that neither of them is moved; and the Bodies when neither of them is moved, are ſaid to be *Equally poiſed*.

3 *Waight*, is the aggregate of all the Endeavours, by which all the points of that Body which preſſes the Beam, tend downwards in lines parallel to one another; and the Body which preſſes, is called the *Ponderant*.

4 *Moment*, is the Power which the Ponderant has to move the Beam, by reaſon of a determined ſituation.

5 *The plain of Equiponderation*, is that, by which the Ponderant is ſo divided, that the Moments on both ſides remain equal.

6 *The Diameter of Equiponderation*, is the common Section of the two Plains of Equiponderation; and is in the ſtraight line by which the waight is hanged.

7 *The Center of Equiponderation*, is the common point of the two Diameters of Equiponderation.

Suppoſitions

1 When two Bodies are equally pois'd, if waight be added to one of them, and not to the other, their Equiponderation ceaſes.

2 When two Ponderants of equal magnitude, and of the ſame Species or matter, preſs the Beam on both ſides at equal diſtances from the center of the Scale, their Moments are equal. Alſo when two Bodies endeavour at equal diſtances from the center of the Scale, if they be of equal magnitude and of the ſame Species, their Moments are equal.

2 No two Plains of Equiponderation are parallel.

Let A B C D (in the first figure) be any Ponderant whatsoever; and in it let E F be a Plain of Equiponderation; parallel to which, let any other Plain be drawn, as G H. I say G H is not a Plain of Equiponderation. For seeing the parts A E F D and E B C F of the Ponderant A B C D, are equally pois'd; and the weight E G H F is added to the part A E F D, and nothing is added to the part E B C F, but the weight E G H F is taken from it; therefore (by the first Supposition) the parts A G H D and G B C H will not be equally pois'd; and consequently G H is not a Plain of Equiponderation. Wherefore, No two Plains of Equiponderation, &c. Which was to be proved.

3 The Center of Equiponderation is in every Plain of Equiponderation.

For if another Plain of Equiponderation be taken, it will not (by the last Article) be parallel to the former Plain; and therefore both those Plains will cut one another. Now that Section (by the 6th Definition) is the Diameter of Equiponderation. Again, if another Diameter of Equiponderation be taken, it will cut that former Diameter; and in that Section (by the 7th Definition) is the Center of Equiponderation. Wherefore the Center of Equiponderation is in that Diameter which lies in the said Plain of Equiponderation.

4 The Moment of any Ponderant applyed to one point of the Beam, to the Moment of the same, or an equal Ponderant applyed to any other point of the Beam, is as the distance of the former point from the Center of the Scale, to the distance of the later point from the same Center. Or thus, Those Moments are to one another, as the Arches of Circles which are made upon the Center of the Scale through those points, in the same time. Or lastly thus, They are, as the parallel bases of two Triangles, which have a common angle at the Center of the Scale.

Let A (in the 2d figure) be the Center of the Scale; and let the equal Poderants D and E press the Beam A B in the points B and C; also let the straight lines B D and C E be Diameters of Equiponderation; and the points D and E in the Ponderants D and E be their Centers of Equiponderation. Let A G F be drawn howsoever,

ſoever, cutting DB produced in F, and E C in G; and laſtly, upon the common Center A, let the two arches B H and C I be deſcribed, cutting A G F in H and I. I ſay the Moment of the Ponderant D to the Moment of the Ponderant E, is as A B to A C, or as B H to C I, or as B F to C G. For the effect of the Ponderant D in the point B, is circular motion in the arch B H; and the effect of the Ponderant E in the point C, circular motion in the arch C I; and by reaſon of the equality of the Ponderants D and E, theſe motions are to one another as the Quickneſſes or Velocities with which the points B and C deſcribe the arches B H and C I, that is, as the arches themſelves B H and C I, or as the ſtraight parallels B F and C G, or as the parts of the Beam A B and A C; for A B. A C:: B F. C G:: B H. C I. are proportionals; and therefore the effects, that is, (by the 4th Definition) the Moments of the equal Ponderants applyed to ſeveral points of the Beam, are to one another, as A B and A C; or as the diſtances of thoſe points from the center of the Scale; or as the parallel baſes of the Triangles which have a common angle at A; or as the concentrick arches B H and C I; which was to be demonſtrated.

5 Unequal Ponderants, when they are applyed to ſeveral points of the Beam, and hang at liberty (that is, ſo as the line by which they hang be the Diameter of Equiponderation, whatſoever be the figure of the Ponderant), have their Moments to one another in proportion compounded of the proportions of their diſtances from the center of the Scale, and of their Waights.

Let A (in the 3d figure) be the center of the Scale, and A B the Beam; to which let the two Ponderants C & D be applied at the points B and E. I ſay the proportion of the Moment of the Ponderant C, to the Moment of the Ponderant D, is compounded of the proportions of A B to A E and of the Waight C to the Waight D; or (if C and D be of the ſame ſpecies) of the magnitude C to the magnitude D.

Let either of them, as C, be ſuppoſed to be bigger then the other D. If therefore by the addition of F, F and D together be as one Body equal to C, the Moment of C to the Moment of F + D will be (by the laſt article) as B G is to E H. Now as F + D is to D, ſo let E H be to another E I; and the moment of F + D, that is

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of C, to the moment of D, will be as B G to E I. But the proportion of B G to E I is compounded of the proportions (of B G to E H that is) of A B to A E, and (of E H to E I, that is) of the waight C to the waight D. Wherefore unequal Ponderants, when they are applied, &c. which was to be proved.

6 The same figure remaining, if I K be drawn parallel to the Beame A B, and cutting A G in K; and K L be drawn parallel to to B G, cutting A B in L, the distances A B and A L from the center, will be proportional to the moments of C and D. For the moment of C is B G, and the moment of D is E I, to which K L is equal. But as the distance A B from the center, is to the distance A L from the center, so is B G the moment of the Ponderant C, to L K, or E I the moment of the Ponderant D.

7 If two Ponderants have their waights and distances from the center in reciprocal proportion, and the center of the Scale be between the points to which the Ponderants are applied, they will be equally poised. And contrarily, if they be equally poised, their waights and distances from the center of the Scale will be in reciprocall proportion.

Let the center of the Scale (in the same 3d figure) be A, the Beam A B; and let any Ponderant C, having B G for its moment, be applied to the point B; also let any other Ponderant D, whose moment is E I, be applied to the point E. Through the point I, let I K be drawn parallel to the Beam A B, cutting A G in K; also let K L be drawn parallel to B G. K L will then be the Moment of the Ponderant D; and (by the last Article) it will be as B G the Moment of the Ponderant C in the point B, to L K the Moment of the Ponderant D in the point E, so A B to A L. On the other side of the center of the Scale, let A N be taken equal to A L; and to the point N let there be applyed the Ponderant O, having to the Ponderant C the proportion of A B to A N. I say the Ponderants in B and N will be equally poised. For the proportion of the Moment of the Ponderant O in the point N, to the Moment of the Ponderant C in the point B, is (by the 5th Article) cõpounded of the proportions of the waight O to the waight B, & of the distance (from the center of the Scale) A N or A L to the distãce (frõ the center of the Scale) A B. But seeing we have supposed, that the distance A B to the distance A N, is in reciprocal propor-

tion of the Waight O to the waight C, the proportion of the Moment of the Ponderant O in the point N, to the Moment of the Ponderant C in the point B, will be compounded of the proportions of A B to A N, and of A N to A B. Wherefore, ſetting in order A B, A N, A B, the Moment of O to the Moment of C will be as the firſt to the laſt, that is, as A B to A B. Their Moments therefore are equal; and conſequently the Plain which paſſes through A, will (by the fifth Definition) be a Plain of Equiponderation. Wherefore they will be equally poiſed; as was to be proved.

Now the converſe of this is manifeſt. For if there be Equiponderation, and the proportion of the Waights and Diſtances be not reciprocal, then both the Waights will alwayes have the ſame Moments, although one of them have more waight added to it, or its diſtance changed.

Corollary. When Ponderants are of the ſame Species, and their Moments be equal; their Magnitudes and Diſtances from the center of the Scale will be reciprocally proportional. For in Homogeneous Bodies, it is as Waight to Waight, ſo Magnitude to Magnitude.

8 If to the whole length of the Beam there be applyed a Parallelogram, or a *Parallelopipedum*, or a *Priſma*, or a Cylinder, or the Superficies of a Cylinder, or of a Sphere, or of any portion of a Sphere or *Priſma*; the parts of any of them cut off with plains parallel to the baſe, will have their Moments in the ſame proportion with the parts of a Triangle which has its Vertex in the center of the Scale, and for one of its ſides the Beam it ſelf, which parts are cut off by Plains parallel to the baſe.

Firſt, let the rectangled Parallelogram A B C D (in the 4th figure) be applyed to the whole length of the Beam A B; and producing C B howſoever to E, let the Triangle A B E be deſcribed. Let now any part of the Parallelogram, as A F, be cut off by the plain F G, parallel to the baſe C B; and let F G be produced to A E in the point H. I ſay the Moment of the whole A B C D to the Moment of its part A F, is as the Triangle A B E to the Triangle A G H, that is, in proportion duplicate to that of the diſtances from the center of the Scale.

For, the Parallelogram A B C D being divided into equal parts in-

infinite in number, by ſtraight lines drawn parallel to the baſe; and ſuppoſing the Moment of the ſtraight line C B to be B E; the Moment of the ſtraight line F G, will (by the 7th Article) be G H; and the Moments of all the ſtraight lines of that Parallelogram, will be ſo many ſtraight lines in the Triangle ABE drawn parallel to the baſe B E; all which parallels together taken are the Moment of the whole Parallelogram A B C D; and the ſame parallels do alſo conſtitute the ſuperficies of the Triangle A B E. Wherefore the Moment of the Parallelogram A B C D, is the Triangle A B E; and for the ſame reaſon, the Moment of the Parallelogram A F, is the Triangle A G H; and therefore the Moment of the whole Parallelogram, to the Moment of a Parallelogram which is part of the ſame, is as the Triangle A B E, to the Triangle A G H, or in proportion duplicate to that of the Beams to which they are applyed. And what is here demonſtrated in the caſe of a Parallelogram, may be underſtood to ſerve for that of a Cylinder, and of a *Priſma*, and their Superficies; as alſo for the Superficies of a Sphere, of an Hemiſphere, or any portion of a Sphere, (for the parts of the Superficies of a Sphere, have the ſame proportion with that of the parts of the Axis cut off by the ſame parallels by which the parts of the Superficies are cut off, as *Archimedes* has demonſtrated); and therefore when the parts of any of theſe figures are equal and at equal diſtances from the Center of the Scale, their Moments alſo are equal, in the ſame manner as they are in Parallelograms.

Secondly, let the Parallelogram A K I B not be rectangled; the ſtraight line I B wil nevertheleſs preſs the point B perpendicularly in the ſtraight line B E; & the ſtraight line L G wil preſs the point G perpendicularly in the ſtraight line G H; and all the reſt of the ſtraight lines which are parallel to I B will do the like. Whatſoever therefore the Moment be which is aſſigned to the ſtraight line I B, as here (for example) it is ſuppoſed to be B E, if A E be drawn, the Moment of the whole Parallelogram A I will be the Triangle A B E; and the Moment of the part A L will be the Triangle A G H. Wherefore the Moment of any Ponderant, which has its ſides equally applyed to the Beam, (whether they be applyed perpendicularly or obliquely) will be always to the Moment of a part of the ſame, in ſuch proportion, as the whole Triangle has to a part

of the ſame cut off by a plain which is parallel to the baſe.

9 The Center of Equiponderation of any figure which is deficient according to commenſurable proportions of the altitude and baſe diminiſhed, and whoſe complete figure is either a Parallelogram, or a Cylinder, or a *Parallelopipedum*, divides the Axis, ſo, that the part next the Vertex, to the other part, is as the complete figure to the deficient figure.

For let C I A P E (in the 5th figure) be a deficient figure, whoſe Axis is A B, and whoſe complete figure is C D F E; and let the Axis A B be ſo divided in Z, that A Z be to Z B as C D F E is to C I A P E. I ſay the center of Equiponderation of the figure C I A P E will be in the point Z.

Firſt, that the Center of Equiponderation of the figure C I A P E is ſomewhere in the Axis A B, is manifeſt of it ſelf; and therefore A B is a Diameter of Equiponderation. Let A E be drawn, and let B E be put for the Moment of the ſtraight line C E; the Triangle A B E will therefore (by the 3d Article) be the Moment of the complete figure C D F E. Let the Axis A B be equally divided in L, and let G L H be drawn parallel and equal to the ſtraight line C E, cutting the crooked line C I A P E in I and P, and the ſtraight lines A C and A E in K and M. Moreover, let Z O be drawn parallel to the ſame C E; and let it be, as L G to L I, ſo L M to another L N; and let the ſame be done in all the reſt of the ſtraight lines poſſible, parallel to the baſe; and through all the points N, let the line A N E be drawn; the three-ſided figure A N E B will therefore be the Moment of the figure C I A P E. Now the Triangle A B E is (by the 9th Article of the 17th Chapter) to the three-ſided figure A N E B, as A B C D + A I C B is to A I C B twice taken, that is, as C D F E + C I A P E is to C I A P E twice taken. But as C I A P E is to C D F E, that is, as the waight of the deficient figure, is to the waight of the complete figure, ſo is C I A P E twice taken, to C D F E twice taken. Wherefore, ſetting in order C D F E + C I A P E. 2 C I A P E. 2 C D F E; the proportion of C D F E + C I A P E to C D F E twice taken, will be compounded of the proportion of C D F E + C I A P E to C I A P E twice taken, that is, of the proportion of the Triangle A B E to the threeſided figure A N E B, that is, of the Moment of the complete figure to the Moment of

the

the deficient figure, and of the proportion of C I A P E twice taken, to C D F E twice taken, that is, to the proportion reciprocally taken of the waight of the deficient figure to the waight of the complete figure.

Again, seeing by supposition A Z. Z B :: C D F E. C I A P E are proportionals; A B. A Z :: C D F E + C I A P E. C D F E will also (by cõpounding) be proportionals. And seeing A L is the half of A B, A L. A Z :: C D F E + C I A P E. 2 C D F E will also be proportionals. But the proportion of C D F E + C I A P E to 2 C D F E is compounded (as was but now shewn) of the proportions of Moment to Moment &c. and therefore the proportion of A L to A Z is compounded of the proportion of the Momen t of the complete figure C D F E to the Moment of the deficient figure C I A P E, and of the proportion of the waight of the deficient figure C I A P E, to the waight of the complete figure C D F E; But the proportion of A L to A Z is compounded of the proportions of A L to B Z and of B Z to A Z. Now the proportion of B Z to A Z is the proportion of the Waights reciprocally taken, that is to say, of the waight C I A P F to the waight C D F E. Therefore the remayning proportion of A L to B Z, that is, of L B to B Z is the proportion of the Moment of the waight C D F E to the Moment of the waight C I A P E. But the proportion of A L to B Z is compounded of the proportions of A L to A Z and of A Z to Z B; of which proportions that of A Z to Z B is the proportion of the waight C D F E to the waight C I A P E. Wherefore (by the 5th Article of this Chapter) the remayning proportion of A L to A Z is the proportion of the distances of the points Z and L from the center of the Scale, which is A. And therefore (by the 6th Article) the waight C I A P E shall hang from O in the straight line O Z. So that O Z is one Diameter of Equiponderation of the waight C I A P E. But the straight line A B is the other Diameter of Equiponderation of the same waight C I A P E. Wherefore (by the 7th Definition) the point Z is the center of the same Equiponderation; which point (by construction) divides the axis so, that the part A Z which is the part next the vertex, is to the other part Z B, as the complete figure C D F E is to the deficient figure C I A P E; which is that which was to be demonstrated.

Corol-

Corollary. The Center of Equiponderation of any of thoſe plain three-ſided figures, which are compared with their complete figures in the Table of the third Article of the 17th Chapter, is to be found in the ſame Table, by taking the Denominator of the fraction for the part of the axis cut off next the vertex, and the Numerator for the other part next the baſe. For example, if it be required to find the Center of Equiponderation of the ſecond three-ſided figure of foure Meanes, there is in the concourſe of the ſecond columne with the row of three-ſided figures of four Meanes this fraction $\frac{5}{7}$, which ſignifies that that figure is to its parallelogrā or compleat figure as $\frac{5}{7}$ to Unity, that is, as $\frac{5}{7}$ to $\frac{7}{7}$, or as 5 to 7; and therefore the Center of Equiponderation of that figure, divides the axis, ſo, that the part next the vertex is to the other part as 7 to 5.

2 *Corallary.* The Center of Equiponderation of any of the Solids of thoſe figures which are contained in the Table of the 8th Article of the ſame 17th Chapter, is exhibited in the ſame Table. For example, if the Center of Equiponderation of a Cone be ſought for; the Cone will be found to be $\frac{1}{3}$ of its Cylinder; and therefore the Center of its Equiponderation will ſo divide the axis, that the part next the vertex, to the other part, will be as 3 to 1. Alſo the Solid of a three-ſided figure of one Meane, that is, a parabolical Solid, ſeeing it is $\frac{2}{4}$, that is $\frac{1}{2}$ of its Cylinder, will have its Center of Equiponderation in that point, which divides the axis, ſo, that the part towards the vertex be double to the part towards the baſe.

10 The Diameter of Equiponderation of the Complement of the half of any of thoſe figutes which are contained in the Table of the 3d article of the 17th Chapter, divides that line which is drawne through the Vertex parallel and equall to the baſe, ſo, that the part next the Vertex, will be to the other part, as the Complete figure to the Complement.

For let A I C B (in the ſame 5 fig.) be the halfe of a Parabola, or of any other of thoſe three-ſided figures which are in the Table of the 3d article of the 17th Chap. whoſe Axis is A B, and baſe B C; having A D drawn from the Vertex, equall and parallel to the baſe B C; and whoſe complete figure is the parallelogramme A B C D. Let I Q be drawne, at any diſtance from the ſide C D,

but

but parallel to it; and let A D be the altitude of the Complement A I C D, and Q I a line ordinately applyed in it. Wherefore the altitude A L in the deficient figure A I C B, is equal to Q I the line ordinately applyed in its Complement; and contrarily, L I the line ordinately applyed in the figure A I C B, is equall to the altitude A Q in its Complement; and so in all the rest of the ordinate lines and altitudes, the mutation is such, that that line which is ordinately applyed in the figure, is the altitude of its Complement. And therefore the proportion of the altitudes decreasing, to that of the ordinate lines decreasing, being multiplicate according to any number in the deficient figure, is submultiplicate according to the same number in its Complement. For example, if A I C B be a Parabola, seeing the proportion of A B to A L is duplicate to that of B C to L I, the proportion of A D to A Q in the Complement A I C D (which is the same with that of B C to L I) will be subduplicate to that of C D to Q I (which is the same with that of A B to A L); and consequently, in a Parabola, the Complement will be to the Parallelogramme as 1 to 3; in a three-sided figure of two Meanes, as 1 to 4; in a three-sided figure of three Meanes, as 1 to 5, &c. But all the ordinate lines together in A I C D are its moment; and all the ordinate lines in A I C B are its moment. Wherefore the moments of the Complements of the halves of Deficient figures in the Table of the 3d article of the 17th Chap. being compared, are as the Deficient figures themselves; and therefore the Diameter of Equiponderation will divide the straight line A D in such proportion, that the part next the Vertex be to the other part, as the complete figure A B C D is to the Complement A I C D.

Coroll. The diameter of Equiponderation of these halves, may be found by the Table of the 3d article of the 17th Chapter in this manner. Let there be propounded any deficient figure, namely the second three-sided figure of two Meanes. This figure is to the complete figure as $\frac{3}{5}$ to 1, that is as 3 to 5. Wherefore the Complement to the same complete figure is as 2 to 5; and therefore the diameter of Equiponderation of this Complement will cut the straight line drawne from the Vertex parallel to the base, so, that the part next the Vertex will be to the other part as 5 to 2.

And

And in like manner, any other of the ſaid three-ſided figures being propounded, if the numerator of its fraction (found out in the Table) be taken from the denominator, the ſtraight line drawn from the Vertex is to be divided, ſo, that the part next the Vertex be to the other part, as the denominator is to the remainder which that ſubſtraction leaves.

11 The center of Equiponderation of the halfe of any of thoſe crooked-lined figures which are in the firſt row of the Table of the 3d article of the 17th chapter, is in that ſtraight line, which being parallel to the Axis, divides the baſe according to the numbers of the fraction next below it in the ſecond row, ſo, that the Numerator be anſwerable to that part which is towards the Axis.

For example, let the firſt figure of three Means be taken, whoſe half is A B C D (in the 6th figure), and let the rectangle A B E D be completed. The Complement therefore will be B C D E. And ſeeing A B E D is to the figure A B C D (by the Table) as 5 to 4, the ſame A B E D will be to the Complement B C D E as 5 to 1. Wherefore if F G be drawn parallel to the baſe D A, cutting the axis, ſo, that A G be to G B as 4 to 5, the center of Equiponderation of the figure A B C D, will (by the precedent article) be ſomewhere in the ſame F G. Again, ſeeing (by the ſame article) the complete figure A B E D, is to the Complement B C D E as 5 to 1, therefore if B E and A D be divided in H and I as 5 to 1, the center of Equiponderation of the Complement B C D E will be ſomewhere in the ſtraight line which connects H and I. Let now the ſtraight line L K be drawn through M the center of the complete figure, parallel to the baſe; and the ſtraight line N O, through the ſame center M, perpendicular to it; and let the ſtraight lines L K and F G cut the ſtraight line H I in P and Q. Let P R be taken quadruple to P Q; and let R M be drawn and produced to F G in S. R M therefore will be to M S as 4 to 1, that is, as the figure A B C D to its Complement B C D E. Wherefore ſeeing M is the center of the Complete figure A B E D; and the diſtances of R and S from the center M be in proportion reciprocall to that of the waight of the Complement B C D E to the waight of the figure A B C D, R and S will either be the centers of Equiponderation of their own

figures, or thoſe centers will be in ſome other points of the diameters of Equiponderation H I and F G. But this laſt is impoſſible. For no other ſtraight line can be drawn through the point M terminating in the ſtraight lines H I and F G, and retaining the proportion of M R to M S, that is, of the figure A B C D to its complement B C D E. The center therefore of Equiponderation of the figure A B C D is in the point S. Now ſeeing P M hath the ſame proportion to Q S which R P hath to R Q, Q S will be 5 of thoſe parts of which P M is 4, that is, of which I N is 4. But I N or P M is 2 of thoſe parts of which E B or F G is 6; and therefore if it be, as 4 to 5, ſo 2 to a fourth, that fourth will be $2\frac{1}{2}$. Wherefore Q S is $2\frac{1}{2}$ of thoſe parts of which F G is 6. But F Q is 1; and therefore F S is $3\frac{1}{2}$. Wherefore the remayning part G S is $2\frac{1}{2}$. So that F G is ſo divided in S, that the part towards the Axis, is in proportion to the other part as $2\frac{1}{2}$ to $3\frac{1}{2}$, that is, as 5 to 7; which anſwereth to the fraction $\frac{5}{7}$ in the ſecond row, next under the fraction $\frac{4}{5}$ in the firſt row. Wherefore drawing S T parallel to the Axis, the baſe wil be divided in like manner.

By this Method it is manifeſt, that the baſe of a Semiparabola will be divided into 3 and 5; and the baſe of the firſt three-ſided figure of two Means, into 4 and 6; and of the firſt three-ſided figure of four Means, into 6 and 8. The fractions therefore of the ſecond row denote the proportions into which the baſes of the figures of the firſt row are divided by the diameters of Equiponderation. But the firſt row begins one place higher then the ſecond row.

12 The center of Equiponderation of the half of any of the figures in the ſecond row of the ſame Table of the 3d article of the 17th Chapter, is in a ſtraight line parallel to the Axis, and dividing the baſe according to the nũbers of the fraction in the fourth row, two places lower, ſo, as that the Numerator be anſwerable to that part which is next the Axis.

Let the half of the ſecond three-ſided figure of two Means be taken; and let it be A B C D (in the 7th Figure); whoſe complement is B C D E, and the rectangle completed A B E D. Let this rectangle be divided by the two ſtraight lines L K & N O, cutting one another in the center M at right angles; and becauſe A B E D

is to A B C D as 5 to 3, let A B be divided in G, ſo, that A G to B G be as 3 to 2; and let F G be drawn parallel to the baſe. Alſo becauſe A B E D is (by the 9th article) to B C D E as 5 to 2, let B E be divided in the point I, ſo, that B I be to I E as 5 to 2; and let I H be drawn parallel to the Axis, cutting L K and F G in P and Q. Let now P R be ſo taken, that it be to P Q as 3 to 2, and let R M be drawn and produced to F G in S. Seeing therefore R P is to R Q, that is, R M to M S, as A B C D is to its complement B C D E, and the centers of Equiponderation of A B C D and B C D E are in the ſtraight lines F G and H I, and the center of Equiponderation of them both together in the point M; R will be the center of the Complement B C D E, and S the center of the Figure A B C D. And ſeeing P M, that is I N, is to Q S, as R P is to R Q; and I N, or P M is 3 of thoſe parts, of which B E, that is, F G is 14; therefore Q S is 5 of the ſame parts; and E I, that is F Q, 4; and F S, 9; and G S, 5. Wherefore the ſtraight line S T being drawn parallel to the Axis, will divide the baſe A D into 5 and 9. But the fraction $\frac{5}{9}$ is found in the fourth row of the Table, two places below the fracti on $\frac{3}{5}$ in the ſecond row.

By the ſame method, if in the ſame ſecond row, there be taken the ſecond three-ſided Figure of three Meanes, the center of Equiponderation of the half of it, will be found to be in a ſtraight line parallel to the Axis, dividing the baſe according to the numbers of the fraction $\frac{5}{10}$, two places below in the fourth row. And the ſame way ſerves for all the reſt of the Figures in the ſecond row. In like manner, the center of Equiponderation of the third three-ſided Figure of three Means, will be found to be in a ſtraight line parallel to the Axis, dividing the baſe, ſo, that the part next the Axis, be to the other part, as 7 to 13, &c.

Coroll. The Centers of Equiponderation of the halves of the ſaid Figures are known, ſeeing they are in the interſection of the ſtraight lines S T and F G, which are both known.

13 The center of Equiponderation of the half of any of the Figures, which (in the Table of the 3d Article of the 17th Chap.) are compared with their Parallelograms, being known; the center of Equiponderation of the exceſs of the ſame Figure above its triangle, is alſo known.

For

For example, let the Semiparabola A B C D (in the 8th Fig.) be taken; whoſe Axis is A B; whoſe complete Figure is A B E D; and whoſe exceſs above its triangle is B C D B. Its center of Equiponderation may be fonnd out in this manner. Let FG be drawn parallel to the baſe, ſo, that A F be a third part of the Axis; and let HI be drawn parallel to the Axis, ſo, that A H be a third part of the baſe. This being done, the center of Equiponderation of the triangle A B D, will be I. Again, let K L be drawn parallel to the baſe, ſo, that A K be to A B as 2 to 5; and M N parallel to the Axis, ſo, that A M be to A D as 3 to 8; and let M N terminate in the ſtraight line K L. The center therefore of Equiponderation of the Parabola A B C D is N; and therefore we have the centers of Equiponderation of the Semiparabola A B C D, and of its part the triangle A B D. That we may now finde the Center of Equiponderation of the remayning part B C D B, let I N be drawn and produced to O, ſo, that N O be triple to I N; and O will be the center ſought for. For ſeeing the waight of A B D, to the waight of B C D B is in proportion reciprocall to that of the ſtraight line N O to the ſtraight line I N; and N is the center of the whole, and I the center of the triangle A B D; O will be the center of the remaining part, namely, of the figure B D C B; which was to be found.

Coroll. The Center of Equiponderation of the figure B D C B, is in the concourſe of two ſtraight lines, whereof one is parallel to the baſe, and divides the Axis, ſo, that the part next the baſe be $\frac{2}{5}$ or $\frac{6}{15}$ of the whole Axis; the other is parallel to the Axis, and ſo divides the baſe, that the part towards the Axis be $\frac{1}{2}$ or $\frac{12}{24}$ of the whole baſe. For drawing O P parallel to the baſe, it will be as I N to N O, ſo F K to K P, that is, ſo 1 to 3, or 5 to 15. But A F is $\frac{5}{15}$ or $\frac{1}{3}$ of the whole A B; and A K is $\frac{6}{15}$ or $\frac{2}{5}$; and F K $\frac{1}{15}$; and K P $\frac{3}{15}$; and therefore A P is $\frac{9}{15}$ of the Axis A B. Alſo A H is $\frac{1}{3}$ or $\frac{8}{24}$; and A M $\frac{3}{8}$ or $\frac{9}{24}$ of the whole baſe; and therefore O Q being drawn parallel to the Axis, M Q (which is triple to H M) will be $\frac{3}{24}$. Wherefore A Q is $\frac{12}{24}$ or $\frac{1}{2}$ of the baſe A D.

The exceſſes of the reſt of the three-ſided figures in the firſt row of the Table of the 3d article of the 17th Chapter, have their centers of Equiponderation in two ſtraight lines which divide the

Axis and baſe according to thoſe fractions, which adde 4 to the numerators of the fractions of a Parabola $\frac{2}{15}$ and $\frac{13}{24}$; and 6 to the denominators, in this mannner,

In a Parabola, The Axis $\frac{2}{15}$, The Baſe $\frac{13}{24}$
In the firſt three-ſided figure, The Axis $\frac{13}{21}$, The Baſe $\frac{16}{30}$
In the ſecond three-ſided figure, The Axis $\frac{17}{27}$, The Baſe $\frac{20}{36}$ &c.

And by the ſame method, any man (if it be worth the paines) may find out the centers of Equiponderation of the exceſſes above their triangles of the reſt of the figures in the ſecond & third row, &c.

14 The center of Equiponderation of the Sector of a Sphere (that is, of a figure compounded of a right Cone whoſe Vertex is the center of the Sphere, and the portion of the Sphere whoſe baſe is the ſame with that of the Cone), divides the ſtraight line which is made of the Axis of the Cone and halfe the Axis of the portion together taken, ſo, that the part next the Vertex be triple to the other part, or to the whole ſtraight line, as 3 to 4.

For let A B C (in the 9th fig.) be the Sector of a Sphere, whoſe Vertex is the center of the Sphere A; whoſe Axis is A D; and the circle upon B C is the common baſe of the portion of the Sphere and of the Cone whoſe Vertex is A; the Axis of which portion is E D, and the halfe thereof F D; and the Axis of the Cone, A E. Laſtly let A G be $\frac{3}{4}$ of the ſtraight line A F. I ſay G is the center of Equiponderation of the Sector A B C.

Let the ſtraight line F H be drawne of any length, making right angles with A F at F; and drawing the ſtraight line A H, let the triangle A F H be made. Then upon the ſame center A let any arch I K be drawne, cutting A D in L; and its chord, cutting A D in M; and dividing M L equally in N, let N O be drawne parallel to the ſtraight line F H, and meeting with the ſtraight line A H in O.

Seeing now B D C is the Spherical Superficies of the portion cut off with a plain paſſing through B C, and cutting the Axis at right angles; and ſeeing F H divides E D the Axis of the portion into two equal parts in F; the center of Equiponderation of the Superficies

ficies B D C will be in F (by the 8th article); and for the same reason the center of Equiponderation of the Superficies I L K (K being in the straight line A C) will be in N. And in like manner, if there were drawne between the center of the Sphere A and the outermost Spherical Superficies of the Sector, arches infinite in number, the centers of Equiponderation of the Sphericall Superficies in which those arches are,, would be found to be in that part of the Axis, which is intercepted between the Superficies it selfe and a plaine passing along by the chord of the arch, and cutting the Axis in the middle at right angles.

Let it now be supposed that the moment of the outermost sphericall Superficies B D C is F H. Seeing therefore the Superficies B D C is to the Superficies I L K in proportion duplicate to that of the arch B D C to the arch I L K, that is, of B E to I M, that is, of F H to N O; let it be as F H to N O, so N O to another N P; and again, as N O to N P, so N P to another N Q; and let this be done in all the straight lines parallel to the base F H that can possibly be drawn between the base and the vertex of the triangle A F H. If then through all the points Q there be drawn the crooked line A Q H, the figure A F H Q A will be the complement of the first three-sided figure of two Meanes; and the same will also be the moment of all the Sphericall Superficies of which the Solid Sector A B C D is compounded; and by consequent, the moment of the Sector it selfe. Let now F H be understood to be the semidiameter of the base of a right Cone, whose side is A H, and Axis A F. Wherfore seeing the bases of the Cones which passe through F and N and the rest of the points of the Axis, are in proportion duplicate to that of the straight lines F H and N O, &c. the moment of all the bases together, that is, of the whole Cone, will be the figure it self A F H Q A; and therefore the center of Equiponderation of the Cone AFH is the same with that of the solid Sector. Wherefore seeing A G is $\frac{3}{4}$ of the Axis A F, the center of Equiponderation of the Cone A F H is in G; and therefore the center of the solid Sector is in G also, and divides the part A F of the Axis, so, that A G is triple to G F; that is, A G is to A F as 3 to 4; which was to be demonstrated.

Note, that when the Sector is a Hemisphere, the Axis of the Cone

Cone vaniſheth into that point which is the center of the Sphere; and therefore it addeth nothing to half the Axis of the portion. Wherefore, if in the Axis of the Hemiſphere, there be taken from the center, $\frac{3}{4}$ of halfe the Axis, that is, $\frac{3}{8}$ of the Semidiameter of the Sphere, there will be the center of Equiponderation of the Hemiſphere.

AD CAP. XXIII

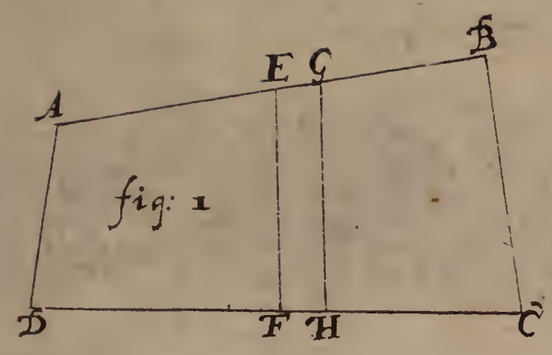


fig: 1

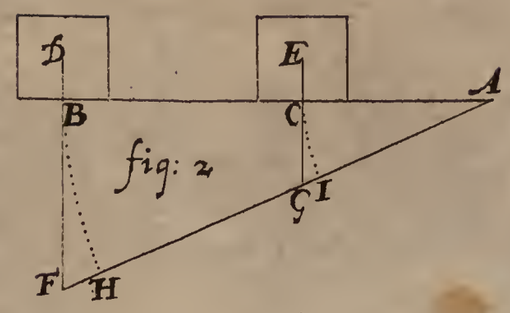


fig: 2

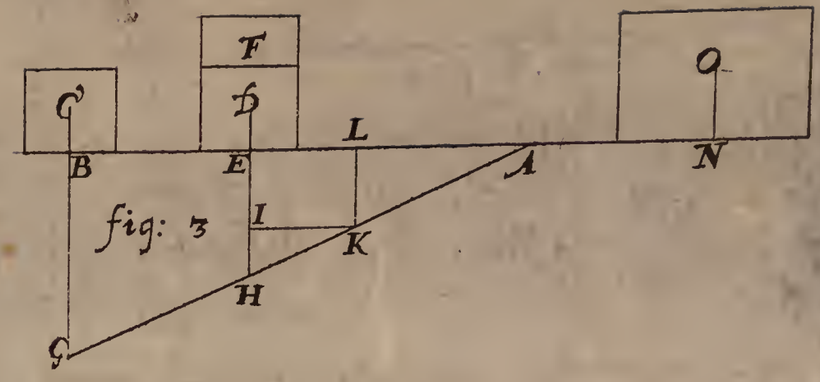


fig: 3

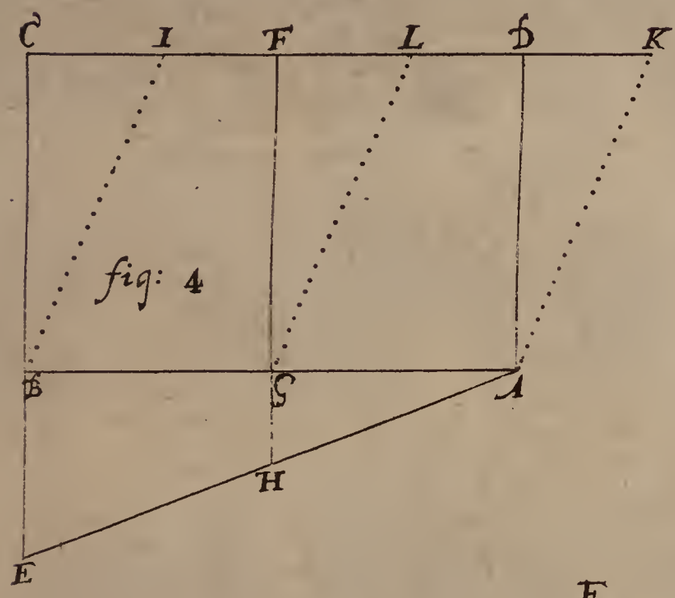


fig: 4

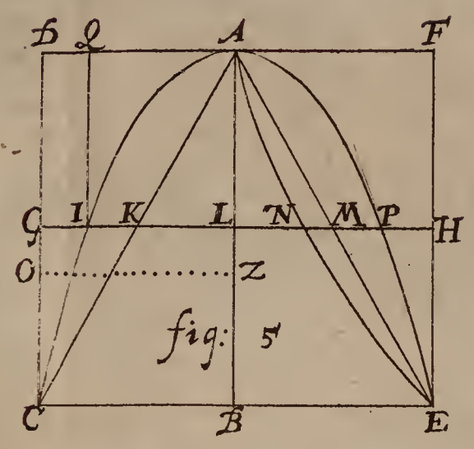


fig: 5

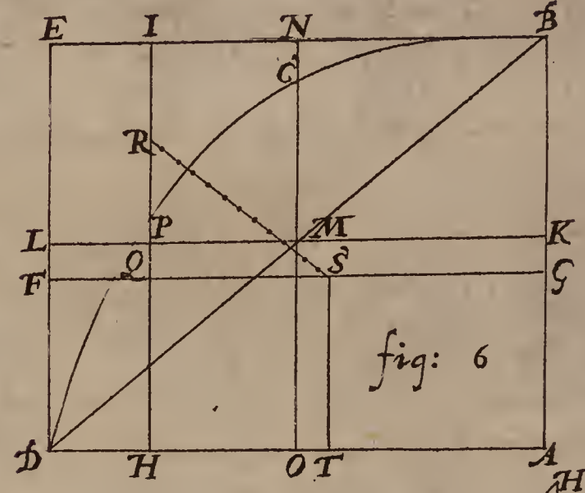


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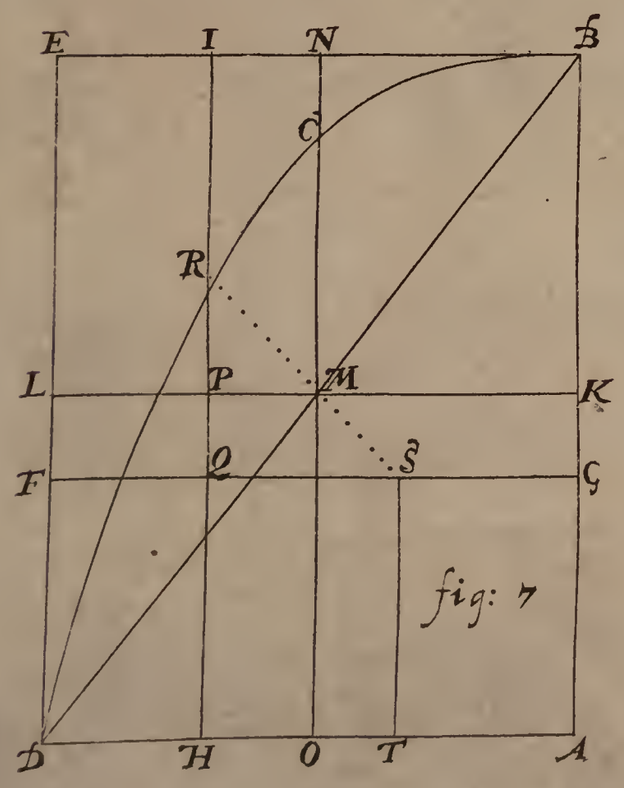


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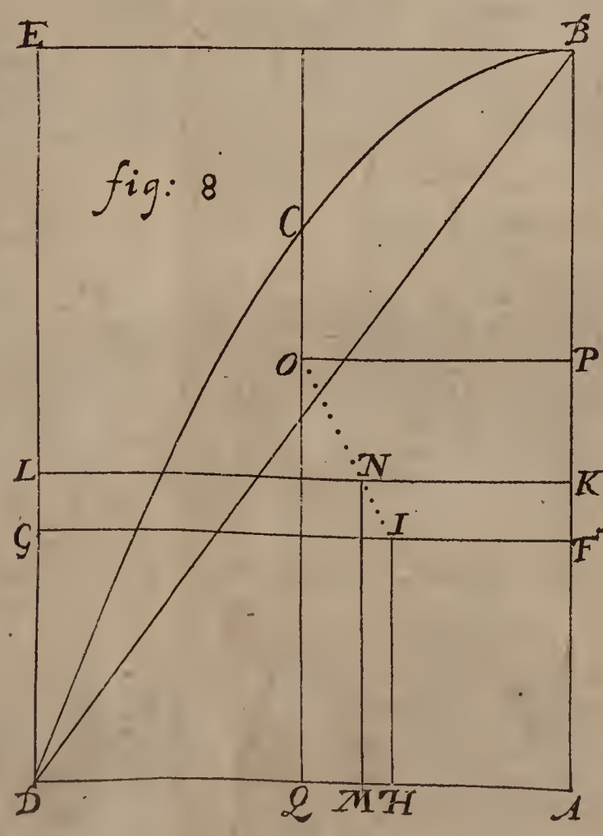


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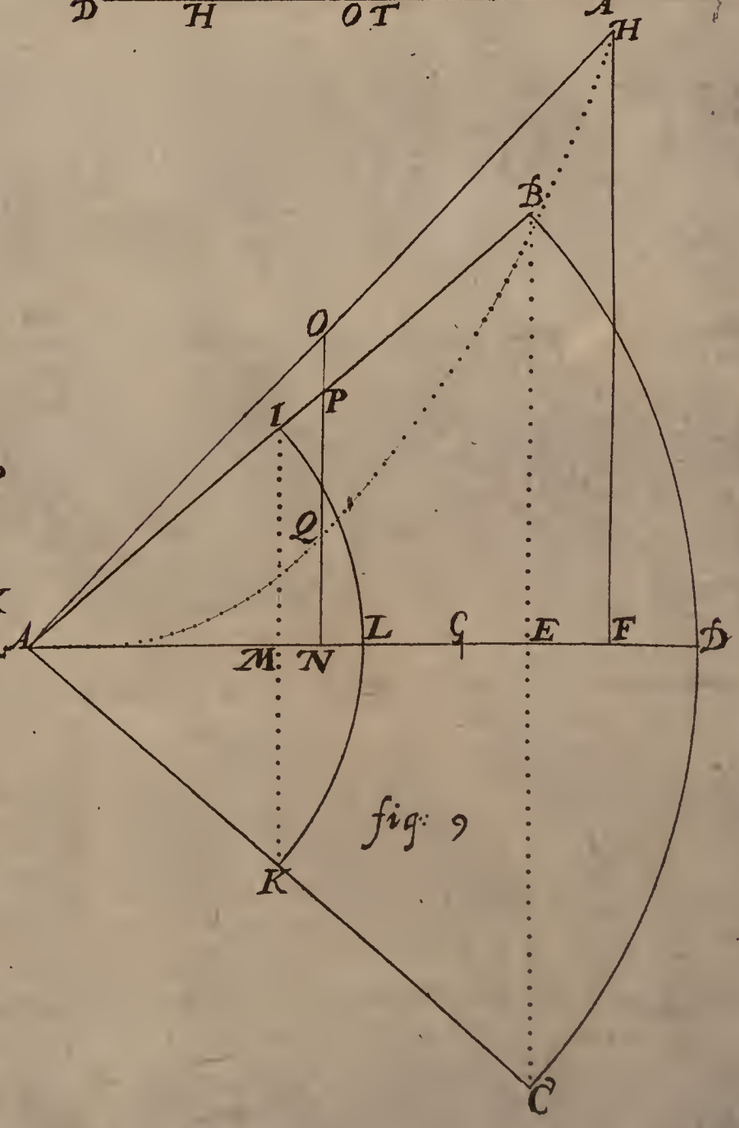


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CHAP. XXIV.

Of Refraction *and* Reflection.

1 *Definitions.* 2 *In perpendicular Motion there is no Refraction.* 3 *Things thrown out of a thinner into a thicker Medium, are ſo refracted, that the Angle Refracted is greater then the Angle of Inclination.* 4 *Endeavour which from one point tendeth every way, will be ſo Refracted, as that the ſine of the Angle Refracted, will be to the ſine of the Angle of Inclination, as the Denſity of the firſt Medium is to the Denſity of the ſecond Medium, reciprocally taken.* 5 *The ſine of the Refracted Angle in one Inclination is to the ſine of the Refracted Angle in another Inclination, as the ſine of the Angle of that Inclination is to the ſine of the Angle of this Inclination.* 6 *If two lines of Incidence, having equal Inclination, be the one in a thinner, the other in a thicker Medium, the ſine of the angle of Inclination will be a Mean proportional between the two ſines of the Refracted angles.* 7 *If the angle of Inclination be ſemirect, and the line of Inclination be in the thicker Medium, and the proportion of their Denſities be the ſame with that of the Diagonal to the ſide of a Square, and the ſeparating Superficies be plain, the Refracted line will be in the ſeparating Superficies.* 8 *If a Body be carried in a ſtraight line upon another Body, and do not penetrate the ſame, but be reflected from it, the angle of Reflexion will be equal to the Angle of Incidence.* 9 *The ſame happens in the generation of Motion in the line of Incidence.*

1 *Definitions.*

1 REFRACTION, is the *breaking* of that ſtraight Line, in which a Body is moved, or its Action would proceed in one and the ſame Medium, into two ſtraight lines, by reaſon of the different natures of the two Mediums.

2 The former of theſe is called the *Line of Incidence*; the later the *Refracted Line.*

3 The *Point of Refraction*, is the common point of the Line of Incidence and of the Refracted Line.

4 The

4 The *Refracting Superficies*, which also is the *Separating Superficies* of the two Mediums, is that in which is the point of Refraction.

5 The *Angle Refracted*, is that which the Refracted Line makes in the point of Refraction, with that Line which from the same point is drawn perpendicular to the separating Superficies in a different Medium.

6 The *Angle of Refraction*, is that which the Refracted line makes with the Line of Incidence produced.

7 The *Angle of Inclination*, is that which the Line of Incidence makes with that Line which from the point of Refraction is drawn perpendicular to the separating Superficies.

8 The *Angle of Incidence*, is the Complement to a right Angle of the Angle of Inclination.

And so, (in the first Figure) the *Refraction* is made in A B F. The *Refracted Line* is B F. The *Line of Incidence* is A B. The *Point of Incidence, and of Refraction* is B. The *Refracting* or *Separating Superficies* is D B E. The *Line of Incidence produced directly* is A B C The *Perpendicular* to the separating Superficies is B H. The *Angle of Refraction* is C B F. The *Angle Refracted* is H B F. The *Angle of Inclination* is A B G or H B C. The *Angle of Incidence* is A B D.

9 Moreover the *Thinner Medium*, is understood to be that in which there is less resistance to Motion or to the generation of Motion; & the *Thicker*, that wherin there is greater resistance.

10 And that Medium in which there is equal resistance every where, is a *Homogeneous Medium*. All other Mediums are *Heterogeneous*.

2 If a Body pass, or there be generation of Motion, from one Medium to another of different Density, in a line perpendicular to the Separating Superficies; there will be no Refraction.

For seeing on every side of the perdendicular all things in the Mediums are supposed to be like and equal; if the Motion it self be supposed to be perpendicular, the Inclinations also will be equal, or rather none at all; and therefore there can be no cause, from which Refraction may be inferred to be on one side of the per-

perpendicular, which wil not cõclude the ſame Refraction to be on the other ſide. Which being ſo, Refraction on one ſide will deſtroy Refraction on the other ſide; and conſequently, either the Refracted line will be every where, (which is abſurd), or there will be no Refracted line at all; which was to be demonſtrated.

Corol. It is manifeſt from hence, that the cauſe of Refraction conſiſteth onely in the obliquity of the line of Incidence, whether the Incident Body penetrate both the Mediums, or without penetrating, propagate motion by Preſſure onely.

3 If a Body, without any change of ſituation of its internal parts, as a ſtone, be moved obliquely out of the thinner Medium, and proceed penetrating the thicker Medium; and the thicker Medium be ſuch, as that its internal parts being moved, reſtore themſelves to their former ſituation; the angle Refracted will be greater then the angle of Inclination.

For let D B E (in the ſame firſt figure) be the ſeparating Superficies of two Mediums; and let a Body, as a ſtone thrown, be underſtood to be moved as is ſuppoſed in the ſtraight line A B C; and let A B be in the thinner Medium, as in the Aire; and B C in the thicker, as in the Water. I ſay the ſtone, wch being thrown, is moved in the line A B, will not proceed in the line B C, but in ſome other line, namely that, with which the perpendicular B H makes the Refracted angle H B F greater then the angle of Inclination H B C.

For ſeeing the ſtone coming from A, and falling upon B, makes that which is at B proceed towards H, and that the like is done in all the ſtraight lines which are parallel to B H; and ſeeing the parts moved reſtore themſelves by contrary motion in the ſame line; there will be contrary motion generated in H B, and in all the ſtraight lines which are parallel to it. Wherefore the motion of the ſtone will be made by the concourſe of the motions in A G, that is, in D B, and in G B, that is, in B H, and laſtly, in H B, that is, by the concourſe of three motions. But by the concourſe of the motions in A G and B H, the ſtone will be carried to C; and therefore by adding the motion in H B, it will be carried higher in ſome other line, as in B F, and make the angle H B F greater then the angle H B C.

And from hence may be derived the cauſe, why Bodies which

are thrown in a very oblique line, if either they be any thing flat, or be thrown with great force, will when they fall upon the water, be caſt up again from the water into the aire.

For let A B (in the 2d figure) be the ſuperficies of the water; into which from the point C, let a ſtone be thrown in the ſtraight line C A, making with the line B A produced a very little angle C A D; and producing B A indefinitely to D, let C D be drawn perpendicular to it, and A E parallel to C D. The ſtone therefore will be moved in C A by the concourſe of two motions in C D and D A, whoſe velocities are as the lines themſelves C D and D A. And from the motion in C D and all its parallels downwards, as ſoon as the ſtone falls upon A, there will be Reaction upwards, becauſe the water reſtores it ſelf to its former ſituation. If now the ſtone be thrown with ſufficient obliquity; that is, if the ſtraight line C D be ſhort enough, that is, if the endeavour of the ſtone downwards be leſs then the Reaction of the water upwards, that is, leſs then the endeavour it hath from its own gravity, (for that may be), the ſtone will (by reaſon of the exceſs of the endeavour which the water hath to reſtore it ſelf, above that which the ſtone hath downwards) be raiſed again above the Superficies A B, and be carried higher, being reflected in a line which goes higher, as the line A G.

4 If from a point, whatſoever the Medium be, Endeavour be propagated every way into all the parts of that Medium; and to the ſame Endeavour there be obliquely oppoſed another Medium of a different nature, that is, either thinner or thicker; that Endeavour will be ſo refracted, that the ſine of the angle Refracted, to the ſine of the angle of Inclination, will be as the denſity of the firſt Medium to the denſity of the ſecond Medium, reciprocally taken.

Firſt, let a Body be in the thinner Medium in A (Figure 3d.); and let it be underſtood to have endeavour every way, and conſequently that its endeavour proceed in the lines A B and A *b*; to which let B *b* the ſuperficies of the thicker Medium be obliquely oppoſed in B and *b*, ſo that A B and A *b* be equal; and let the ſtraight line B *b* be produced both wayes. From the points B and *b* let the perpendiculars B C and *b c* be drawn; and upon the centers

B and *b*, and at the equal distances B A and *b* A, let the Circles A C and A *c* be described, cutting B C and *b c* in C and *c*, and the same C B and *c b* produced in D and *d*, as also A B and A *b* produced in E and *e*. Then from the point A to the straight lines B C and *b c* let the perpendiculars A F and A *f* be drawn. A F therefore will be the sine of the angle of Inclination of the straight line A B, and A *f* the sine of the angle of Inclination of the straight line A *b*, which two Inclinations are by construction made equal. I say, as the density of the Medium in which are B C and *b c*, is to the density of the Medium in which are B D and *b d*, so is the sine of the angle Refracted, to the sine of the angle of Inclination.

Let the straight line F G be drawn parallel to the straight line A B, meeting with the straight line *b* B produced in G.

Seeing therefore A F and B G are also parallels, they will be equal; and consequently, the endeavour in A F is propagated in the same time, in which the endeavour in B G would be propagated if the Medium were of the same density. But because B G is in a thicker Medium, that is, in a Medium which resists the endeavour more then the Medium in which A F is, the endeavour will be propagated less in B G then in A F, according to the proportion which the density of the Medium in which A F is, hath to the density of the Medium in which B G is. Let therefore the density of the Medium in which B G is, be to the density of the Medium in which A F is, as B G is to B H; and let the measure of the time be the Radius of the Circle. Let H I be drawn parallel to B D, meeting with the circumference in I; and from the point I let I K be drawn perpendicular to B D; which being done, B H and I K will be equal; and I K will be to A F, as the density of the Medium in which is A F, is to the density of the Medium in which is I K. Seeing therefore in the time A B (which is the Radius of the Circle) the endeavour is propagated in A F in the thinner Medium, it will be propagated in the same time, that is, in the time B I in the thicker Medium from K to I. Therefore B I is the Refracted line of the line of Incidence A B; and I K is the sine of the angle Refracted; and A F, the sine of the angle of Inclination. Wherefore seeing I K is to A F, as the density of the Medium in which is A F to the density of the Medium in which is I K; it will be as the density of the Medium in which is A F, (or

B C) to the denſity of the Medium in which is I K (or B D), ſo the ſine of the angle Refracted to the ſine of the angle of Inclination. And by the ſame reaſon it may be ſhewn, that as the denſity of the thinner Medium is to the denſity of the thicker Medium, ſo will K I the ſine of the angle Refracted be to A F the ſine of the Angle of Inclination.

Secondly, let the Body which endeavoureth every way, be in the thicker Medium at I. If therefore both the Mediums were of the ſame denſity, the endeavour of the Body in I B would tend directly to L; and the ſine of the angle of Inclination L M would be equal to I K or B H. But becauſe the denſity of the Medium in which is I K, to the denſity of the Medium in which is L M, is as B H to B G, that is, to A F, the endeavour will be propagated further in the Medium in which L M is, then in the Medium in which I K is, in the proportion of denſity to denſity, that is, of M L to A F. Wherefore B A being drawn, the angle Refracted will be C B A, and its ſine A F. But L M is the ſine of the angle of Inclination; and therefore again, as the denſity of one Medium is to the denſity of the different Medium, ſo reciprocally is the ſine of the angle Refracted to the ſine of the angle of Inclination, which was to be demonſtrated.

In this Demonſtration, I have made the ſeparating Superficies B *b* plain by conſtruction. But though it were concave or convex, the Theoreme would nevertheleſs be true. For the Refraction being made in the point B of the plain ſeparating Superficies, if a crooked line, as P Q be drawn, touching the ſeparating line in the point B; neither the Refracted line B I, nor the perpendicular B D will be altered; and the Refracted angle K B I, as alſo its ſine K I will be ſtill the ſame they were.

5 The ſine of the angle Refracted in one Inclination, is to the ſine of the angle Refracted in another Inclination, as the ſine of the angle of that Inclination to the ſine of the angle of this Inclination.

For ſeeing the ſine of the Refracted angle is to the ſine of the angle of Inclination, (whatſoever that Inclination be) as the denſity of one Medium, to the denſity of the other Medium; the proportion of the ſine of the Refracted angle, to the ſine of the angle of Inclination, will be compounded of the proportions of denſity to denſity,

ſity, and of the ſine of the angle of one Inclination to the ſine of the angle of the other Inclination. But the proportions of the denſities in the ſame Homogeneous Body, are ſuppoſed to be the ſame. Wherefore Refracted angles in different Inclinations, are as the ſines of the angles of thoſe Inclinations; which was to be demonſtrated.

6 If two lines of Incidence having equal inclination, be the one in a thinner, the other in a thicker Medium; the ſine of the angle of their Inclination, will be a mean proportional between the two ſines of their angles Refracted.

For let the ſtraight line AB (in the ſame 3d figure) have its Inclination in the thinner Medium, and be refracted in the thicker Medium in B I; and let E B have as much Inclination in the thicker Medium, and be refracted in the thinner Medium in B S; and let R S the ſine of the angle Refracted be drawn. I ſay the ſtraight lines R S, A F and I K are in continual proportion. For it is, as the denſity of the thicker Medium to the denſity of the thinner Medium, ſo R S to A F. But it is alſo, as the denſity of the ſame thicker Medium, to that of the ſame thinner Medium, ſo AF to IK. Wherefore R S. A F :: A F. I K are propoortionals; that is, R S, A F and I K are in continual proportion, and A F is the Mean proportional; which was to be proved.

7 If the angle of Inclination be ſemirect, and the line of Inclination be in the thicker Medium, and the proportion of the Denſities be as that of a Diagonal to the ſide of its Square, and the ſeparating Superficies be plain, the Refracted line will be in that ſeparating Superficies.

For in the Circle A C (in the 4th figure) let the angle of Inclination A B C be an angle of 45 degrees. Let C B be produced to the Circumference in D; & let C E (the ſine of the angle EBC) be drawn; to which, let B F be taken equal in the ſeparating line B G. B C E F will therefore be a Parallelogram, & F E & B C, that is, F E and B G equal. Let AG be drawn, namely, the Diagonal of the Square whoſe ſide is B G; and it will be, as A G to E F, ſo B G to B F; & ſo (by ſuppoſition) the denſity of the Medium in which C is, to the denſity of the Medium in which D is; and ſo alſo the ſine of the angle Refracted to the ſine of the angle of Inclination.

Draw-

Drawing therefore F D, & from D the line D H perpendicular to A B produced, DH will be the ſine of the angle of Inclination. And ſeeing the ſine of the angle Refracted is to the ſine of the angle of Inclination, as the denſity of the Medium in which is C; is to the denſity of the Medium in which is D; that is, (by ſuppoſition) as A G is to F E, that is, as D H is to B G; and ſeeing D H is the ſine of the angle of Inclination, B G will therefore be the ſine of the angle Refracted. Wherefore B G will be the Refracted line, and lye in the plain ſeparating Superficies; which was to be demonſtrated.

Coroll. It is therefore manifeſt, that when the Inclination is greater then 45 degrees, as alſo when it is leſs, provided the denſity be greater, it may happen that the Refraction will not enter the thinner Medium at all.

8 If a Body fall in a ſtraight line upon another Body, and do not penetrate it, but be reflected from it, the angle of Reflexion will be equal to the angle of Incidence.

Let there be a Body at A (in the 5th figure), which falling with ſtraight motion in the line A C upon another Body at C, paſſeth no further, but is reflected; and let the angle of Incidence be any angle, as A C D. Let the ſtraight line C E be drawn, making with D C produced the angle E C F equall to the angle A C D; and let A D be drawn perpendicular to the ſtraight line D F. Alſo in the ſame ſtraight line D F let C G be taken equall to C D; and let the perpendicular G E be raiſed, cutting C E in E. This being done, the triangles A C D and E C G will be equall and like. Let C H be drawn equal and parallel to the ſtraight line A D; and let H C be produced indefinitely to I. Laſtly let E A be drawn, which will paſſe through H, and be parallel and equall to G D. I ſay the motion from A to C in the ſtraight line of Incidence AC, will be reflected in the ſtraight line C E.

For the motion from A to C is made by two coefficient or concurrent motions, the one in A H parallel to D G, the other in A D perpendicular to the ſame D G; of which two motions, that in A H workes nothing upon the Body A after it has been moved as farre as C, becauſe (by ſuppoſition) it doth not paſſe the ſtraight line D G; whereas the endeavour in A D, that is in H C, worketh further towards I. But ſeeing it doth onely preſſe and not penetrate

netrate, there will be reaction in H, which cauſeth motion from C towards H; and in the mean time the motion in H E remaines the ſame it was in A H; and therefore the Body will now be moved by the concourſe of two motions in C H and H E, which are equall to the two motions it had formerly in A H and H C. Wherefore it will be carried on in C E. The angle therefore of Reflection will be E C G, equall (by conſtruction) to the angle A C D; which was to be demonſtrated.

Now when the Body is conſidered but as a point, it is all one, whether the Superficies or line in which the Reflection is made, be ſtraight or crooked; for the point of Incidence and Reflexion C, is as well in the crooked line which toucheth D G in C, as in D G it ſelfe.

9 But if we ſuppoſe that not a Body be moved, but ſome Endeavour onely be propagated from A to C, the Demonſtratio n will nevertheleſſe be the ſame. For all Endeavour is motion; and when it hath reached the Solid Body in C, it preſſeth it, and endeavoureth further in C I. Wherefore the reaction will proceed in C H; and the endeavour in C H concurring with the endeavour in H E, will generate the endeavour in C E, in the ſame manner as in the repercuſſion of Bodies moved.

If therefore Endeavour be propagated from any point to the concave Superficies of a Spherical Body, the Reflected line with the circumference of a great circle in the ſame Sphere, will make an angle equall to the angle of Incidence.

For if Endeavour be propagated from A (in the 6 fig.) to the circumference in B, and the center of the Sphere be C, and the line C B be drawne, as alſo the Tangent D B E; and laſtly if the angle F B D be made equall to the angle A B E, the Reflexion will be made in the line B F, as hath been newly ſhewn. Wherefore the angles which the ſtraight lines A B and F B make with the circumference, will alſo be equall. But it is here to be noted that if C B be produced howſoever to G, the endeavour in the line G B C will proceed onely from the perpendicular reaction in G B; and that therefore there will be no other endeavour in the point B towards the parts which are within the Sphere, beſides that which tends towards the center.

And

And here I put an end to the third part of this Diſcourſe; in which I have conſidered Motion and Magnitude by themſelves in the abſtract. The fourth and laſt part, concerning the *Phænomena of Nature*, that is to ſay, concerning the Motions and Magnitudes of the Bodies which are parts of the World, reall and exiſtent, is that which followes.

AD CAP. XXIIII

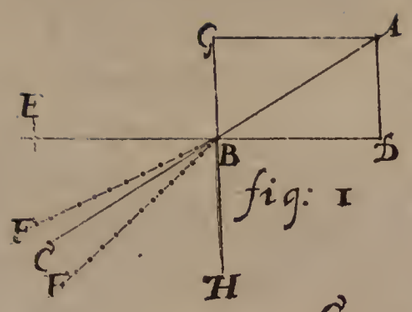


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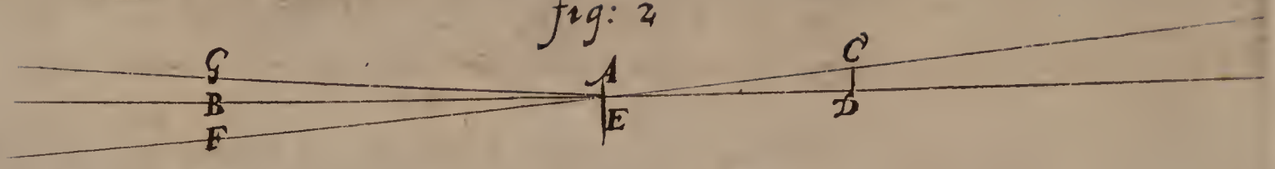


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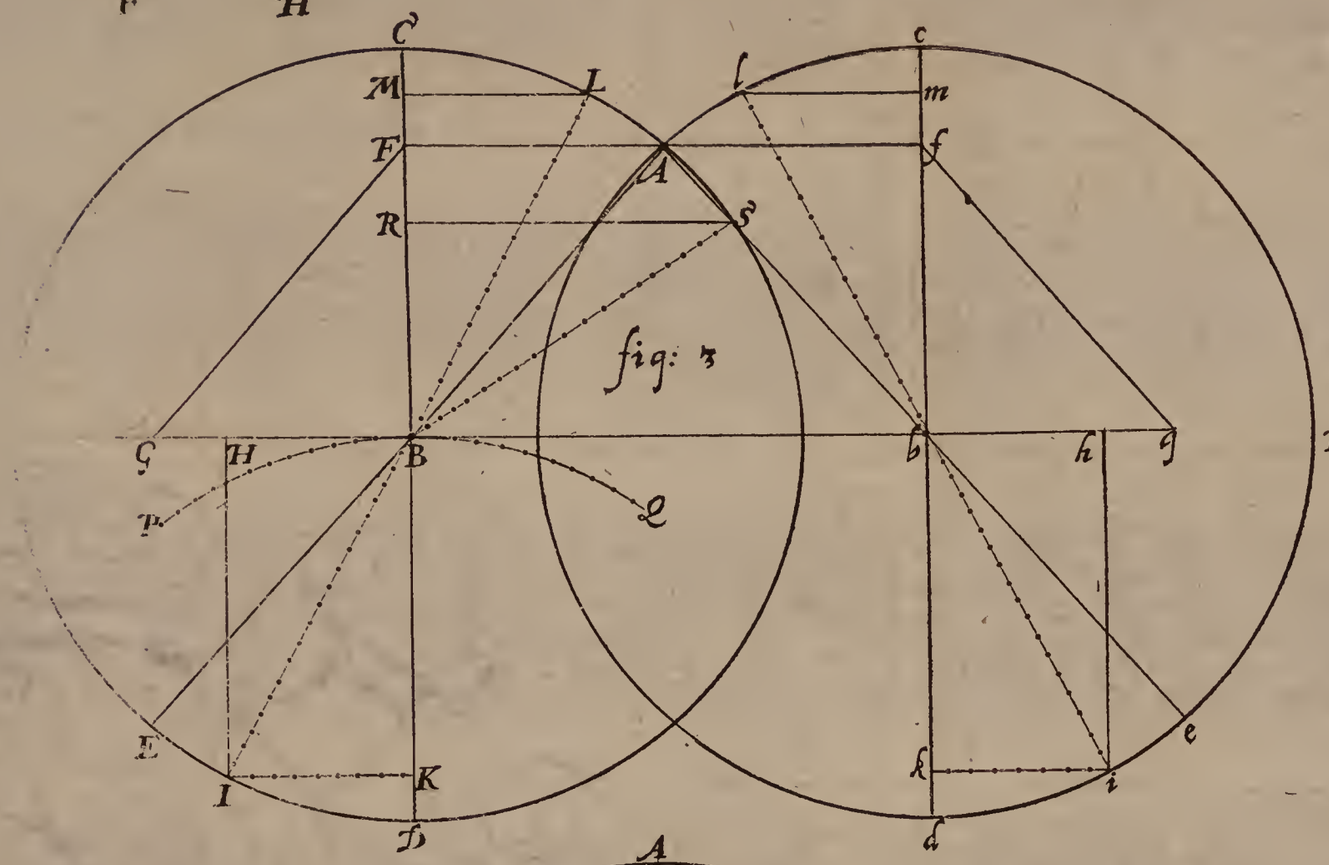


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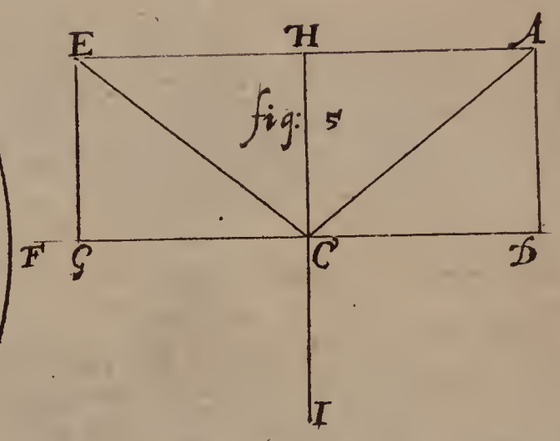


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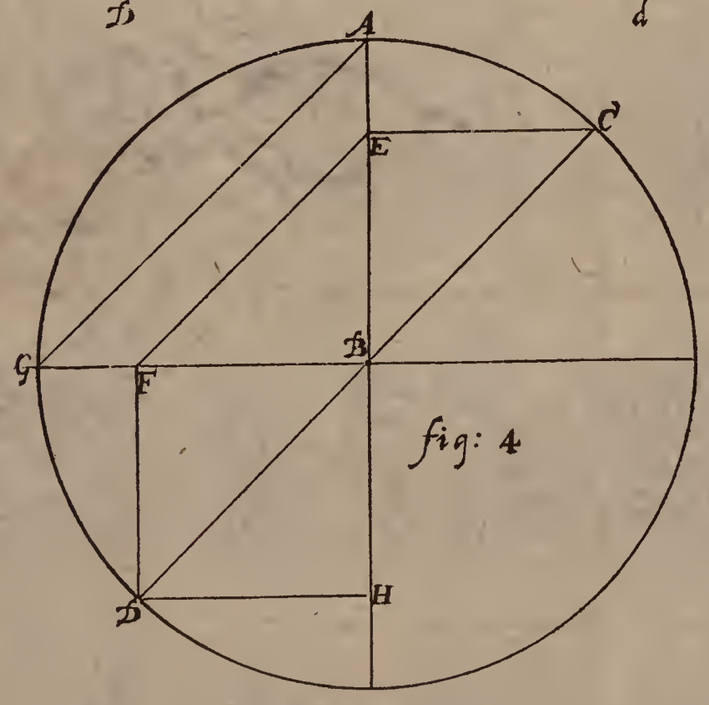


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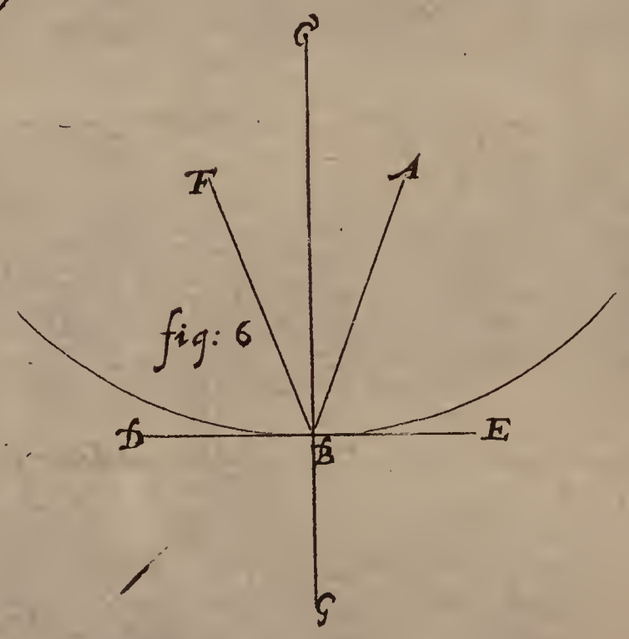


fig: 6

PHYSIQVES, or the PHÆNOMENA of NATVRE.

CHAP. XXV.

Of Senſe *and* Animal Motion.

1 *The connexion of what hath been ſaid with that which followeth.* 2 *The inveſtigation of the nature of Senſe, and the Definition of Senſe.* 3 *The Subject and Object of Senſe.* 4 *The Organ of Senſe.* 5 *All Bodies are not indued with Senſe.* 6 *But one Phantaſme at one and the ſame time.* 7 *Imagination, the Remayns of paſt Senſe, (which alſo is Memory). Of Sleep.* 8 *How Phantaſmes ſucceed one another.* 9 *Dreames, whence they proceed.* 10 *Of the Senſes, their kindes, their Organs, and Phantaſmes proper and common.* 11 *The Magnitude of Images, how and by what it is determined.* 12 *Pleaſure, Pain, Appetite, and Averſion, what they are.* 13 *Deliberation and Will, what.*

1 I Have (in the firſt Chapter) defined *Philoſophy* to be *Knowledge of Effects acquired by true Ratiocination, from knowledge firſt had of their Cauſes and Generation; and of ſuch Cauſes or Generations as may be, from former knowledge of their Effects or Appearances.* There are therefore two Methods of Philoſophy, One from the Generation of things to their poſſible Effects, and the other from their Effects or Appearances to ſome poſſible Generation of the ſame. In the former of theſe, the Truth of the firſt Principles of our ratiocination (namely Definitions) is made and conſtituted by our ſelves, whileſt we conſent and agree about the Appellations of things. And this part I have finiſhed in the foregoing Chapters; in which (if I am not deceived) I have affirmed nothing (ſaving

the Definitions themſelves) which hath not good coherence with the Definitions I have given; that is to ſay, which is not ſufficiently demonſtrated to all thoſe that agree with me in the uſe of Words and Appellations; for whoſe ſake onely I have written the ſame. I now enter upon the other part; which is the finding out by the Appearances or Effects of Nature which we know by Senſe, ſome wayes and means by which they may be (I do not ſay, they are) generated. The Principles therefore, upon which the following diſcourſe depends, are not ſuch as we our ſelves make and pronounce in general terms, as Definitions; but ſuch, as being placed in the things themſelves by the Authour of Nature, are by us obſerved in them; and we make uſe of them in ſingle and particular, not univerſal propoſitions. Nor do they impoſe upon us any neceſſity of conſtituting Theoremes; their uſe being onely (though not without ſuch general Propoſitions as have been already demonſtrated) to ſhew us the poſſibility of ſome production or generation. Seeing therefore the Science which is here taught, hath its Principles in the Appearances of Nature, and endeth in the attayning of ſome knowledge of Natural cauſes, I have given to this Part, the title of PHYSIQUES, or the PHÆNOMENA of NATURE. Now ſuch things as *appear*, or are *ſhewn* to us by Nature, we call *Phænomena* or Appearances.

Of all the *Phænomena*, or *Appearances* which are neer us, the moſt admirable is *Apparition* it ſelf, τὸ φαίνεσθαι; namely, that ſome Natural Bodies have in themſelves the patterns almoſt of all things, and others of none at all. So that if the *Appearances* be the Principles by which we know all other things, we muſt needs acknowledge *Senſe* to be the Principle by which we know thoſe Principles; & that all the knowledge we have is derived from it. And as for the cauſes of *Senſe*, we cannot begin our ſearch of them from any other *Phænomenon* then that of *Senſe* it ſelf. But you will ſay, by what Senſe ſhall we take notice of Senſe? I anſwer, by Senſe it ſelf, namely, by the Memory which for ſome time remains in us of things ſenſible, though they themſelves paſs away. For he that perceiues that he hath perceived, *remembers*.

In the firſt place therefore the cauſes of our Perception, that is, the cauſes of thoſe Ideas and Phantaſmes which are perpetually gene-

generated within us whileſt we make uſe of our Senſes, are to be enquired into; and in what manner their generation proceeds. To help which Inquiſition, we may obſerve firſt of all, that our Phantaſmes or Ideas are not alwayes the ſame; but that new ones appear to us, and old ones vaniſh, according as we apply our Organs of Senſe, now to one Object, now to another. Wherefore they are generated, and periſh. And from hence it is manifeſt, that they are ſome change or mutation in the Sentient.

2 Now that all Mutation or Alteration is Motion, or Endeavour (and Endeavour alſo is Motion) in the internal parts of the thing that is altered, hath been proved (in the 9th Article of the 8th Chapter) from this, that whileſt even the leaſt parts of any Body remain in the ſame ſituation in reſpect of one another, it cannot be ſaid that any alteration (unleſs perhaps that the whole Body together hath been moved) hath hapned to it; but that it both appeareth and is the ſame it appeared & was before. Senſe therefore in the Sentient, can be nothing elſe but motion in ſome of the internal parts of the Sentient; and the parts ſo moved, are parts of the Organs of Senſe. For the parts of our Body by which we perceive any thing, are thoſe we commonly call the *Organs of Senſe*. And ſo we find what is the Subject of our Senſe, namely, that in which are the Phantaſmes; and partly alſo we have diſcovered the nature of Senſe, namely, that it is ſome internal Motion in the Sentient.

I have ſhewn beſides (in the 8th Chap. at the 7th Article) that no Motion is generated but by a Body contiguous and Moved. From whence it is manifeſt, that the immediate cauſe of Senſe or Perception conſiſts in this, that the firſt Organ of Senſe is touched and preſſed. For when the uttermoſt part of the Organ is preſſed, it no ſooner yeilds, but the part next within it, is preſſed alſo; and in this manner, the preſſure or Motion is propagated through all the parts of the Organ to the innermoſt. And thus alſo the preſſure of the uttermoſt part proceeds from the preſſure of ſome more remote Body, and ſo continually, till we come to that from which, as from its fountain we derive the Phantaſme or Idea that is made in us by our Senſe. And this, whatſoever it be, is that we commonly call the *Object*. Senſe therefore is ſome internal Motion

in the Sentient, generated by ſome internal Motion of the parts of the Object, and propagated through all the *Media* to the innermoſt part of the Organ. By which words I have almoſt defined what Senſe is.

Moreover, I have ſhewn (in the 2d Article of the 15 Chapter) that all Reſiſtance is Endeavour oppoſite to another Endeavour, that is to ſay, Reaction. Seeing therefore there is in the whole Organ by reaſon of its own internal natural Motion; ſome Reſiſtance or Reaction againſt the Motion which is propagated from the Object to the innermoſt part of the Organ, there is alſo in the ſame Organ an Endeavour oppoſite to the Endeavour which proceeds from the Object; ſo that when that Endeavour inwards is the laſt action in the act of Senſe, then from the Reaction, how little ſoever the duration of it be, a Phantaſme or Idea hath its being; which by reaſon the Endeavour is now outwards, doth alwayes appear as ſomething ſituate without the Organ. So that now I ſhall give you the whole Definition of Senſe, as it is drawn from the explication of the cauſes thereof, and the order of its generation, thus; SENSE *is a Phantaſme, made by the Reaction and endeavour outwards in the Organ of Senſe, cauſed by an Endeavour inwards from the Object, remayning for ſome time more or leſs.*

3 The *Subject* of Senſe, is the *Sentient* it ſelf, namely, ſome living Creature; and we ſpeak more correctly, when we ſay a *Living Creature* ſeeth, then when we ſay the *Eye* ſeeth. The *Object*, is the thing *Perceived*; and it is more accurately ſaid, that we ſee the Sun, then that we ſee the Light. For Light & Colour & Heat & Sound, and other qualities which are commonly called *Senſible*, are not *Objects*, but Phantaſms in the Sentients. For a Phantaſm is the act of Senſe, and differs no otherwiſe from Senſe then *fieri* (that is, Being a doing) differs from *Factum eſſe*, (that is, Being done;) which difference, in things that are done in an Inſtant, is none at all; and a Phantaſme is made in an Inſtant. For in all Motion which proceeds by perpetual propagation, the firſt part being moved moves the ſecond, the ſecond the third, and ſo on to the laſt, and that to any diſtance how great ſoever. And in what point of time the firſt or formoſt part proceeded to the place of the ſecond, which is thruſt on; in the ſame point of time the laſt ſave

one

one proceeded into the place of the laſt yeilding part; which by reaction, in the ſame inſtant, if the reaction be ſtrong enough, makes a Phantaſme; and a Phantaſme being made, Perception is made together with it,

4 The *Organs* of Senſe, which are in the Sentient, are ſuch parts thereof, that if they be hurt, the very generation of Phantaſmes is thereby deſtroyed, though all the reſt of the parts remain intire. Now theſe parts in the moſt of Living Creatures are found to be certain Spirits and Membranes, which proceeding from the *Pia Mater*, involve the Brain and all the Nerves; alſo the Brain it ſelf, and the Arteries which are in the Brain; and ſuch other parts, as being ſtirred, the Hart alſo, which is the fountain of all Senſe is ſtirred together with them. For whenſoever the action of the Object reacheth the Body of the Sentient, that action is by ſome Nerve propagated to the Brain; and if the Nerve leading thither be ſo hurt or obſtructed, that the Motion can be propagated no further, no Senſe follows. Alſo if the motion be intercepted between the Brain and the Heart by the defect of the Organ by which the action is propagated, there will be no perception of the Object.

5 But though all Senſe, as I have ſaid, be made by Reaction, nevertheleſs it is not neceſſary that every thing that Reacteth ſhould have Senſe. I know there have been Philoſophers, & thoſe learned men, who have maintained that all Bodies are endued with Senſe. Nor do I ſee how they can be refuted, if the nature of Senſe be placed in Reaction onely. And, though by the Reaction of Bodies inanimate a Phantaſme might be made, it would nevertheleſs ceaſe, as ſoon as ever the Object were removed. For unleſs thoſe Bodies had Organs, (as living Creatures have) fit for the retaining of ſuch Motion as is made in them, their Senſe would be ſuch, as that they ſhould never remēber the ſame. And therefore this hath nothing to do with that Senſe which is the ſubject of my diſcourſe. For by Senſe we commonly underſtand the judgement we make of Objects by their Phantaſmes; namely, by comparing and diſtinguiſhing thoſe Phantaſmes; which we could never do, if that motion in the Organ, by which the Phantaſme is made, did not remain there for ſome time, and make the ſame Phantaſme re-

turn

turn. Wherefore Senſe, as I here underſtand it, and which is commonly ſo called, hath neceſſarily ſome memory adhering to it; by which former and later Phantaſmes may be compared together, and diſtinguiſhed from one another.

Senſe therefore properly ſo called, muſt neceſſarily have in it a perpetual variety of Phantaſmes, that they may be diſcerned one from another. For if we ſhould ſuppoſe a man to be made with cleer Eyes, and all the reſt of his Organs of Sight well diſpoſed, but endued with no other Senſe; and that he ſhould look onely upon one thing, which is alwayes of the ſame colour and figure without the leaſt appearance of variety, he would ſeem to me, whatſoever others may ſay, to ſee, no more then I ſeem to my ſelf to feel the Bones of my own Limbs by my Organs of Feeling; and yet thoſe Bones are alwayes, and on all ſides touched by a moſt ſenſible Membrane. I might perhaps ſay he were aſtoniſhed, and looked upon it; but I ſhould not ſay he ſaw it; it being almoſt all one for a man to be alwayes ſenſible of one and the ſame thing, and not to be ſenſible at all of any thing.

6 And yet ſuch is the nature of Senſe, that it does not permit a man to diſcern many things at once. For ſeeing the nature of Senſe conſiſts in Motion; as long as the Organs are employed about one Object, they cannot be ſo Moved by another at the ſame time, as to make by both their Motions one ſincere Phantaſme of each of them at once. And therefore two ſeveral Phantaſmes will not be made by two Objects working together, but onely one Phantaſme compounded from the action of both.

Beſides, as when we divide a Body, we divide its place; and when we reckon many Bodies, we muſt neceſſarily reckon as many places; and contrarily, as I have ſhewn in the firſt Article of the 7th Chapter; ſo what number ſoever we ſay there be of Times, we muſt underſtand the ſame number of Motions alſo; and as oft as we count many Motions, ſo oft we reckon many times. For though the object we looke upon be of diverſe colours, yet with thoſe diverſe colours it is but one varied Object, and not variety of Objects.

Moreover, whileſt thoſe Organs which are common to all the Senſes (ſuch as are thoſe parts of every Organ which proceed (in

Men)

Men) from the root of the Nerves, to the Hart) are vehemently ſtirred by a ſtrong action from ſome one Object, they are (by reaſon of the contumacy which the motion they have already gives them againſt the reception of all other motion) made the leſſe fit to receive any other impreſſion from whatſoever other Objects, to what ſenſe ſoever thoſe Objects belong. And hence it is, that an earneſt ſtudying of one Object, takes away the Senſe of all other Objects for the preſent. For *Study* is nothing elſe but a poſſeſſion of the mind, that is to ſay, a vehement motion made by ſome one Object in the Organs of Senſe, which are ſtupid to all other motions as long as this laſteth; according to what was ſaid by *Terence*, *Populus ſtudio ſtupidus in funambulo animum occuparat.* For what is *Stupor* but that which the Greekes call ἀναισθησία, that is, a ceſſation from the Senſe of other things? Wherefore at one and the ſame time, we cannot by Senſe perceive more then one ſingle Object; as in reading, we ſee the letters ſucceſſively one by one, and not all together, though the whole Page be preſented to our eye; and though every ſeverall letter be diſtinctly written there, yet when we looke upon the whole page at once, we read nothing.

From hence it is manifeſt, that every endeavour of the Organ outwards, is not to be called Senſe, but that onely which at ſeverall times is by Vehemence made ſtronger and more prædominant than the reſt; which deprives us of the Senſe of other Phantaſmes, no otherwiſe then the Sun deprives the reſt of the ſtarres of light, not by hindering their action, but by obſcuring and hiding them with his exceſſe of brightneſſe.

7. But the motion of the Organ, by which a Phantaſme is made, is not commonly called Senſe, except the Object be preſent. And the Phantaſme remaining after the Object is removed or paſt by, is called *Fancy*, and in latine *Imaginatio*; which word (becauſe all Phantaſmes are not Images) doth not fully anſwer the ſignification of the word *Fancy* in its generall acceptation. Nevertheleſſe I may uſe it ſafely enough, by underſtanding it for the Greek φαντασία.

IMAGINATION therefore is nothing elſe but *Senſe decaying*, or *weakned*, by the abſence of the Object. But what may be the cauſe of this decay or weakning? Is the Motion the weaker becauſe the Ob-

Object is taken away? If it were, then Phantasmes would alwayes and necessarily be less cleare in the Imagination, then they are in Sense; which is not true. For in Dreams (which are the Imaginations of those that sleep) they are no less clear then in Sense it self. But the reason why in men Waking the Phantasms of things past are more obscure then those of things present, is this, that their Organs being at the same time moved by other present Objects, those Phantasmes are the lesse prædominant. Whereas in Sleep, the passages being shut up, externall action doth not at all disturbe or hinder internall motion.

If this be true, the next thing to be considered, will be, whether any cause may be found out, from the supposition whereof it will follow, that the passage is shut up from the externall Objects of Sense to the internall Organ. I suppose therefore, that by the continuall action of Objects, (to which a Reaction of the Organ, and more esqecially of the Spirits, is necessarily consequent) the Organ is wearied, that is, its parts are no longer moved by the Spirits without some pain; and consequently the Nerves being abandoned and grown slack, they retire to their fountain which is the cavity either of the Brain or of the Heart; by which means the action which proceeded by the Nerves is necessarily intercepted. For Action, upon a Patient that retires from it, makes but little Impression at the first; and at last, when the Nerves are by little and little slackned, none at all. And therefore there is no more Reaction, that is, no more Sense, till the Organ being refreshed by Rest, and by a supply of new Spirits recovering strength and motion, the Sentient awaketh. And thus it seems to be alwayes, unless some other præternatural cause intervene; as Heat in the internal parts from lassitude, or from some disease stirring the Spirits and other parts of the Organ in some extraordinary manner.

8 Now it is not without cause, nor so casual a thing as many perhaps think it, that Phantasmes in this their great variety, proceed from one another; and that the same Phantasmes sometimes bring into the mind other Phantasmes like themselves, and at other times extreamly unlike. For in the motion of any continued Body, one part followes another by cohæsion; and therefore, whilst we turne our Eies and other Organs successively to many

Objects

Objects, the motion which was made by every one of them remayning, the Phantaſmes are renewed as often as any of thoſe motions comes to be prædominant above the reſt; and they become prædominant in the ſame order, in which at any time formerly they were generated by Senſe. So that when by length of time very many Phantaſmes have been generated within us by Senſe, then allmoſt any thought may ariſe from any other thought; in ſo much that it may ſeeme to be a thing indifferent and caſuall, which thought ſhall follow which. But for the moſt part this is not ſo uncertain a thing to waking as to ſleeping men. For the thought or Phantaſme of the deſired End, brings in all the Phantaſmes that are meanes conducing to that end, and that in order backewards from the laſt to the firſt, and againe forwards from the beginning to the End. But this ſuppoſes both Appetite, and Judgement to diſcerne what meanes conduce to the end; which is gotten by Experience; and Experience is ſtore of Phantaſmes, ariſing from the ſenſe of very many things. For φανταζεσθαι and *Meminiſſe*, *Fancy* and *Memory* differ onely in this, that Memory ſuppoſeth the time paſt, which Fancy doth not. In Memory, the Phantaſmes we conſider are as if they were worne out with time; but in our Fancy we conſider them as they are; which diſtinction is not of the things themſelves, but of the conſiderations of the Sentient. For there is in Memory, ſomething like that which happens in looking upon things at a great diſtance; in which, as the ſmall parts of the Object are not diſcerned by reaſon of their remoteneſſe; ſo in Memory, many accidents and places and parts of things, which were formerly perceived by Senſe, are by length of time decayed and loſt.

The perpetuall ariſing of Phantaſmes, both in Senſe and Imagination, is that which we commonly call *Diſcourſe of the Mind*, and is common to men with other living Creatures. For he thta thinketh, compareth the Phantaſmes that paſſe, that is, taketh notice of their likeneſſe or unlikeneſſe to one another. And as he that obſerves readily the likeneſſes of things of different natures, or that are very remote from one another, is ſaid to have a good Fancy; ſo he is ſaid to have a good Judgement, that finds out the unlikeneſſes or differences of things that are like one another.

Now this obſervation of differences, is not perception made by a common Organ of Senſe, diſtinct from Senſe or Perception properly ſo called; but is Memory of the differences of particular Phantaſmes remaining for ſome time; as the diſtinction between Hot and Lucid, is nothing elſe but the Memory both of a Heating, and of an Enlightning Object.

9 The Phantaſmes of men that ſleep, are DREAMS. Concerning which we are taught by experience theſe five things. Firſt, that for the moſt part there is neither order nor coherence in them. Secondly, that we dream of nothing but what is compounded and made up of the Phantaſmes of Senſe paſt. Thirdly, that ſomtimes they proceed (as in thoſe that are drowſy) from the interruption of their Phantaſmes by little and little broken and altered through ſleepineſs; and ſometimes alſo they begin in the midſt of ſleep. Fourthly, that they are clearer then the Imaginations of waking men, except ſuch as are made by Senſe it ſelf, to which they are equal in clearneſs. Fifthly, that when we dream, we admire neither the places nor the looks of the things that appear to us. Now from what hath been ſaid, it is not hard to ſhew what may be the cauſes of theſe *Phænomena*. For as for the firſt, ſeeing all Order and Coherence proceeds from frequent looking back to the End, that is, from Conſultation; it muſt needs be, that ſeeing in ſleep we loſe all thought of the End, our Phantaſmes ſucceed one another, not in that order which tends to any End, but as it happeth, and in ſuch manner, as Objects preſent themſelves to our Eyes when we look indifferently upon all things before us, and ſee them, not becauſe we would ſee them, but becauſe we do not ſhut our Eyes; for then they appear to us without any order at all. The ſecond proceeds from this, that in the ſilence of Senſe, there is no new motion from the Objects, and therefore no new Phantaſme, unleſs we call that new, which is compounded of old ones, as a *Chimæra*, a golden Mountain, and the like. As for the third, why a Dream is ſometimes as it were the continuation of Senſe, made up of broken Phantaſmes, as in men diſtempered with ſickneſs, the reaſon is manifeſtly this, that in ſome of the Organs Senſe remains, and in others it faileth. But how ſome Phantaſmes may be revived, when all the exteriour Organs are benummed with ſleep, is not ſo eaſily ſhewn. Nevertheleſs, that which

hath

hath already been ſaid, contains the reaſon of this alſo. For whatſoever ſtrikes the *Pia Mater*, reviveth ſome of thoſe Phantaſmes that are ſtill in motion in the Brain; and when any internal motion of the Heart reacheth that Membrane, then the prædominant motion in the Brain makes the Phantaſme. Now the Motions of the Heart are Appetites and Averſions, of which I ſhall preſently ſpeak further. And as Appetites and Averſions are generated by Phantaſmes, ſo reciprocally Phantaſmes are generated by Appetites and Averſions. For example, Heat in the Heart proceeds from Anger and Fighting; and again from Heat in the Heart, (whatſoever be the cauſe of it) is generated Anger, and the Image of an Enemy in Sleep. And as Love and Beauty ſtirre up heat in certain Organs; ſo Heat in the ſame Organs, from whatſoever it proceeds, often cauſeth Deſire, and the Image of an unreſiſting Beauty. Laſtly, Cold doth in the ſame manner generate Feare in thoſe that ſleep, and cauſeth them to dream of Ghoſts, and to have Phantaſmes of horrour and danger; as Fear alſo cauſeth Cold in thoſe that wake; ſo reciprocal are the motions of the Heart and Brain. The fourth, namely, that the things we ſeem to ſee and feel in ſleep, are as clear as in ſenſe it ſelf, proceeds from two cauſes; one, that having then no ſenſe of things without us, that internal motion which makes the Phantaſme, in the abſence of all other impreſſions, is prædominant; and the other, that the parts of our Phantaſms which are decayed and worn out by time, are made up with other fictitious parts. To conclude, when we dream, we do not wonder at ſtrange places, and the appearances of things unknown to us, becauſe Admiration requires that the things appearing be new and unuſual, which can happen to none but thoſe that remember former appearances; whereas in ſleep, all things appear as preſent.

But it is here to be obſerved, that certain Dreams, eſpecially ſuch as ſome men have when they are between ſleeping and waking, and ſuch as happen to thoſe that have no knowledge of the nature of Dreams, and are with all ſuperſtitious, were not heretofore, nor are now accounted Dreams. For the Apparitions men thought they ſaw, and the Voices they thought they heard in ſleep, were not believed to be Phantaſmes, but things ſubſiſting of them-

ſelves, and Objects without thoſe that dreamed. For to ſome men, as well ſleeping as waking, but eſpecially to guilty men, and in the night, and in hallowed places, Feare alone, helped a little with the ſtories of ſuch Apparitions, hath raiſed in their minds terrible Phantaſmes, which have been, and are ſtill deceiptfully received for things really true, under the names of *Ghoſts* and *Incorporeal Subſtances*.

10 In moſt living Creatures there are obſerved five kinds of Senſes, which are diſtinguiſhed by their Organs, and by their different kinds of Phantaſmes; namely, *Sight*, *Hearing*, *Smell*, *Taſt* and *Touch*; and theſe have their Organs partly peculiar to each of them ſeverally, and partly common to them all. The Organ of Sight is partly *animate*, and partly *inanimate*. The inanimate parts, are the three *Humours*; namely, the *Watry Humour*, which by the interpoſition of the Membrane called *Uvea*, (the perforation whereof is called the *Apple* of the *Eye*) is contained on one ſide by the firſt concave ſuperficies of the Eye, and on the other ſide by the *Ciliary proceſſes* and the Coat of the *Criſtalline humour*; the *Criſtalline*, which (hanging in the midſt between the Ciliary proceſſes, and being almoſt of Spherical figure, and of a thick conſiſtence) is incloſed on all ſides with its own tranſparent Coat; and the *Vitreous* or *Glaſſie Humour*, which filleth all the reſt of the Cavity of the Eye, and is ſomewhat thicker then the Watry Humour, but thinner then the Criſtalline. The *animate* part of the Organ is, firſt, the Membrane *Choroeides*, which is a part of the *Pia Mater*, ſaving that it is covered with a Coat derived from the marrow of the Optique Nerve, which is called the *Retina*; and this *Choroeides*, ſeeing it is part of the *Pia Mater*, is continued to the beginning of the *Medulla Spinalis* within the Scull, in which all the Nerves which are within the Head have their roots. Wherefore all the *Animal Spirits* that the Nerves receive, enter into them there; for it is not imaginable that they can enter into them any where elſe. Seeing therefore Senſe is nothing elſe but the action of Objects propagated to the furtheſt part of the Organ; and ſeeing alſo that *Animal Spirits* are nothing but *Vital Spirits* purified by the Hart and carried from it by the Arteries; it follows neceſſarily, that the action is derived from the Heart by ſome of the Arteries to the roots of the Nerves

which

which are in the Head, whether thoſe Arteries be the *Plexus Retiformis*, or whether they be other Arteries which are inſerted into the ſubſtance of the Brain. And therefore thoſe Arteries are the Complement, or the remaying part of the whole Organ of Sight. And this laſt part is a common Organ to all the Senſes; wheras that which reacheth from the Eie to the roots of the Nerves is proper onely to Sight. The proper Organ of Hearing is the *Tympanum* of the Eare, and its own Nerve; from which to the Heart the Organ is Common. So the proper Organs of Smel & Taſt are Nervous Membranes, in the Palate and Tongue for the Taſte, and in the Noſtrils for the Smell; and from the roots of thoſe Nerves to the Heart all is common. Laſtly, rhe proper Organ of Touch are Nerves and Membranes diſperſed through the whole Body; which Membranes are derived from the root of the Nerves. And all things elſe belonging alike to all the Senſes ſeem to be adminiſtred by the Arteries, and not by the Nerves.

The proper Phantaſme of Sight is *Light*; and under this name of Light, *Colour* alſo (which is nothing but perturbed Light) is comprehended. Wherefore the Phantaſme of a Lucid Body, is Light; and of a coloured Body, Colour. But the Object of Sight, properly ſo called is neither Light nor Colour, but the Body it ſelf which is lucid, or enlightned, or coloured. For Light and Colour being Phantaſmes of the Sentient, cannot be Accidents of the Object. Which is manifeſt enough from this, that Viſible things appear oftentimes in places, in which we know aſſuredly they are not, and that in different places they are of different colours, and may at one and the ſame time appear in divers places. Motion, Reſt, Magnitude and Figure are common both to the Sight and Touch; and the whole appearance together of *Figure, and Light or Colour*, is by the Greeks commonly called εἶδος, & εἴδωλον, & ἰδέα; and by the Latines, *Species* and *Imago*; all which names ſignifie no more but *Appearance*.

The phantaſme which is made by Hearing, is *Sound*; by Smell, *Odour*; by Taſt, *Savour*; and by Touch, *Hardneſs* and *Softneſs*, *Heat* and *Cold*, *Wetneſs*, *Oilineſs*, and many more, which are eaſier to be diſtinguiſhed by ſenſe then words. *Smoothneſs*, *Roughneſs*, *Rarity* and *Denſity* refer to Figure, and are therefore common both to

Touch

Touch and Sight. And as for the Objects of Hearing, Smel, Taſt, and Touch, they are not Sound, Odour, Savour, Hardneſs, &c. but the Bodies themſelves from which Sound, Odour, Savour, Hardneſs, &c. proceed; Of the cauſes of which, and of the manner how they are produced, I ſhall ſpeak hereafter.

But theſe Phantaſmes, though they be effects in the Sentient, (as Subject) produced by Objects working upon the Organs; yet there are alſo other effects beſides theſe, produced by the ſame Objects, in the ſame Organs; namely, certain Motions proceeding from Senſe, which are called *Animal Motions*. For ſeeing in all Senſe of external things, there is mutual Action and Reaction, that is, two Endeavours oppoſing one another, it is manifeſt, that the motion of both of them together will be continued every way, eſpecially to the confines of both the Bodies. And when this happens in the internal Organ, the Endeavour outwards will proceed in a ſolid Angle, which will be greater, and conſequently the Idea greater, then it would have been if the impreſſion had been weaker.

11 From hence the Natural cauſe is manifeſt, Firſt, why thoſe things ſeem to be greater, which *(cæteris paribus)* are ſeen in a greater Angle. Secondly, why in a ſerene cold night when the Moon doth not ſhine, more of the fixed Stars appear, then at another time. For their action is leſs hindred by the ſerenity of the Aire, and not obſcured by the greater Light of the Moon, which is then abſent; and the Cold making the Air more preſſing, helpeth or ſtrengtheneth the action of the Stars upon our Eies, in ſo much as Stars may then be ſeen which are ſeen at no other time. And this may ſuffice to be ſaid in general concerning Senſe made by the Reaction of the Organ. For, as for the place of the Image, the deceptions of Sight, and other things of which we have experience in our ſelves by Senſe, being they depend for the moſt part upon the Fabrick it ſelf of the Eie of Man, I ſhall ſpeak of them then when I come to ſpeak of Man.

12 But, there is another kind of Senſe, of which I will ſay ſomthing in this place, namely, the Senſe of *Pleaſure* and *Pain*, proceeding, not from the Reaction of the Heart outwards, but from continual action from the outermoſt part of the Organ towards the Heart.

Heart. For the original of Life being in the Heart, that motion in the Sentient which is propagated to the Heart, muſt neceſſarily make ſome alteration or diverſion of Vital Motion, namely, by quickning or ſlackening, helping or hindering the ſame. Now when it helpeth, it is Pleaſure, and when it hindereth, it is Pain, Trouble, Grief, &c. And as Phantaſmes ſeem to be without, by reaſon of the Endeavour outwards; ſo Pleaſure and Pain by reaſon of the Endeavour of the Organ inwards ſeem to be within, namely, there where the firſt Cauſe of the Pleaſure or Pain is; as when the Pain proceeds from a Wound, we think the Pain and the Wound are both in the ſame place.

Now Vital Motion, is the Motion of the Bloud, perpetually circulating (as hath been ſhewn from many infallible ſignes and marks by Doctor *Harvey*, the firſt Obſerver of it) in the Veins and Arteries. Which Motion, when it is hindered by ſome other Motion made by the action of ſenſible Objects, may be reſtored again either by bending or ſetting ſtraight the parts of the Body; which is done when the Spirits are carried now into theſe, now into other Nerves, till the Pain (as farre as is poſſible) be quite taken away. But if Vital Motion be helped by Motion made by Senſe, then the parts of the Organ will be diſpoſed to guide the Spirits in ſuch manner, as conduceth moſt to the preſervation and augmentation of that motion by the help of the Nerves. And in animal motion this is the very firſt Endeavour, and found even in the *Embrio*; which while it is in the wombe, moveth its limbes with voluntary motion, for the avoiding of whatſoever troubleth it, or for the purſuing of what pleaſeth it. And this firſt Endeavour, when it tends towards ſuch things as are known by experience to be pleaſant, is called APPETITE, (that is, an Approaching;) and when it ſhuns what is troubleſome, AVERSION, or Flying from it. And little Infants, at the beginning, and as ſoon as they are born, have appetite to very few things, as alſo they avoid very few, by reaſon of their want of Experience and Memory; & therefore they have not ſo great a variety of animal Motion as we ſee in thoſe that are more grown. For it is not poſſible, without ſuch knowledge as is derived from Senſe, that is, without Experience and Memory, to know what will prove pleaſant, or hurtful; onely there is ſome

place.

place for conjecture from the looks or aſpects of things. And hence it is, that though they do not know what may do them good or harm, yet ſometimes they approach, and ſometimes retire from the ſame thing as their doubt prompts them. But afterwards by accuſtoming themſelves by little and little, they come to know readily what is to be purſued, and what to be avoided; and alſo to have a ready uſe of their Nerves and other Organs in the purſuing and avoiding of good and bad. Wherefore *Appetite* and *Averſion* are the firſt Endeavours of *Animal Motion*.

Conſequent to this firſt Endeavour, is the Impulſion into the Nerves, and Retraction again of Animal Spirits, of which it is neceſſary there be ſome Receptacle or place neer the original of the Nerves; and this Motion or Endeavour is followed by a ſwelling and Relaxation of the Muſcles; and laſtly, theſe are followed by Contraction and Extenſion of the limbes, which is Animal Motion.

13 The Conſiderations of Appetites and Averſions are divers. For ſeeing Living Creatures have ſometimes Appetite, and ſometimes Averſion to the ſame thing, as they think it will either be for their good or their hurt, while that viciſſitude of Appetites and Averſions remains in them, they have that ſeries of Thoughts which is called DELIBERATION; which laſteth as long as they have it in their power to obtain that which pleaſeth, or to avoid that which diſpleaſeth them. *Appetite* therefore and *Averſion* are ſimply ſo called as long as they follow not Deliberation. But if Deliberation have gone before, then the laſt act of it, if it be Appetite, is called WILL; if Averſion, UNWILLINGNESSE; ſo that the ſame thing is called both *Will* and *Appetite*; but the conſideration of them (namely, before and after Deliberation) is divers. Nor is that which is done within a Man whileſt he Willeth any thing, different from that which is done in other living Creatures, whileſt (Deliberation having preceded) they have Appetite.

Neither is the freedome of Willing, or not willing, greater in Man, then in other living Creatures. For where there is Appetite, the entire cauſe of Appetite hath preceded; and conſequently the act of Appetite could not chooſe but follow, that is, hath of neceſſity followed, (as is ſhewn Chapt. 9th Article 5.) And therefore

fore ſuch a *Liberty* as is free from *Neceſſity*, is not to be found in the *Will* either of Men or Beaſts. But if by *Liberty* we underſtand the faculty or power, not of *Willing*, but of *Doing* what they Will, then certainly that Liberty is to be allowed to both; and both may equally have it, whenſover it is to be had.

Again, when Appetite and Averſion do with celerity ſucceed one another, the whole ſeries made by them, hath its name ſometimes from one, ſometimes from the other. For the ſame Deliberation (whileſt it inclines ſometimes to one, ſometimes to the other) is from Appetite called HOPE, and from Averſion FEAR. For where there is no Hope, it is not to be called Fear, but HATE; and where no Fear, not Hope, but DESIRE. To conclude; all the Paſſions, called *Paſſions of the Minde*, conſiſt of Appetite and Averſion (except pure Pleaſure and Pain, which are a certain *Fruition* of good or Evil;) as *Anger*, is Averſion from ſome imminent evil, but ſuch as is joyned with Appetite of avoiding that evil by force. But becauſe the Paſſions and Perturbations of the Minde are innumerable; and many of them not to be diſcerned in any Creatures beſides Men; I will ſpeak of them more at large in that Section which is concerning *Man*. As for thoſe Objects (if there be any ſuch) which do not at all ſtir the Mind, we are ſaid to *Contemn* them.

And thus much of Senſe in general. In the next place I ſhall ſpeak of Senſible Objects.

CHAP. XXVI.

Of the World, *and of the* Starres.

1 *The Magnitude and Duration of the World, inſcrutable.* 2 *No place in the World Empty.* 3 *The arguments of Lucretius for Vacuum, invalid.* 4 *Other arguments for the eſtabliſhing of Vacuum, invalid.* 5 *Six ſuppoſitions for the ſalving of the Phænomena of Nature.* 6 *Poſſible cauſes of the Motions Annual and Diurnal; and of the apparent Direction, Station and Retrogradation of the Planets.* 7 *The ſuppoſition of Simple Motion, why likely.* 8 *The cauſe of the Excentricity of the annual motion of the Earth.* 9 *The cauſe why the Moon hath alwayes one and the ſame face turned towards the Earth.* 10 *The cauſe of the Tides of the Ocean.* 11 *The cauſe of the Præceſſion of the Equinoxes.*

1 Conſequent to the Contemplation of *Senſe* is the contemplation of *Bodies*, which are the efficient cauſes or Objects of Senſe. Now every *Object* is either a part of the whole World, or an Aggregate of parts. The greateſt of all Bodies, or ſenſible Objects, is the *World* it ſelf, which we behold when we look round about us from this point of the ſame which we call the Earth. Concerning the World, as it is one Aggregate of many parts, the things that fall under inquiry are but few; and thoſe we can determine, none. Of the whole World we may inquire what is its *Magnitude*, what its *Duration*, and how many there be; but nothing elſe. For as for *Place* and *Time*, that is to ſay, Magnitude and Duration, they are only our own fancie of a Body ſimply ſo called, that is to ſay of a Body indefinitely taken, as I have ſhewne before in the 7 chapter. All other Phantasmes, are of Bodies or Objects as they are diſtinguiſhed from one another; as Colour, the Phantaſme of coloured Bodies; Sound, of Bodies that move the Senſe of Hearing, &c. The queſtions concerning the Magnitude of the World, are, whether it be *Finite* or *Infinite*, *Full* or *not Full*; Concerning its Duration, whether it had a *Beginning*, or be *Eternall*; and concerning the num-

number, whether there be *One* or *Many* (though as concerning the Number, if it were of infinite Magnitude, there could be no controversy at all.) Also if it had a beginning, then by what *Cause* and of what *Matter* it was made; and againe, from whence that Cause and that Matter had their being, will be new questions; till at last we come to one or many eternall Cause or Causes. And the determination of all these things belongeth to him that professeth the universal doctrine of Philosophy, in case as much could be known as can be sought. But the knowledge of what is Infinite can never be attained by a finite Inquirer. Whatsoever we know that are Men, we learn it from our Phantasmes; and of *Infinite* (whether Magnitude or Time) there is no Phantasme at all; so that it is impossible either for a man, or any other creature to have any conception of *Infinite*. And though a man may from some Effect proceed to the immediate Cause thereof, & frō that to a more remote Cause, and so ascend continually by right ratiocination from Cause to Cause; yet he will not be able to proceed eternally; but wearied, will at last give over, without knowing whether it were possible for him to proceed to an end, or not. But whether we suppose the World to be Finite, or Infinite, no absurdity will follow. For the same things which now appear, might appear, whether the Creator had pleased it should be Finite or Infinite. Besides, though from this, that nothing can move it self, it may rightly be inferred that there was some first eternal Movent; yet it can never be inferred (though some use to make such inference) that that Movent was eternally Immoveable, but rather eternally Moved. For as it is true, that nothing is moved by it self; so it is true also that nothing is moved but by that which is already moved. The questions therefore about the *Magnitude* and *Beginning* of the World, are not to be determined by Philosophers, but by those that are lawfully authorised to order the Worship of God. For as Almighty God, when he had brought his People into *Judæa*, allowed the Priests the first fruits reserved to himself; so when he had delivered up the World to the disputations of Men, it was his pleasure, that all Opinions concerning the nature of *Infinite* and *Eternal*, known onely to himself, should (as the first-fruits of Wisdom) be judged by those whose Ministery he meant to use in the ordering of Religion. I cannot therefore

commend those that boast they have demonstrated by reasons drawn from natural things, that the World had a Beginning. They are contemned by Idiots, because they understand them not; and by the Learned, because they understand them; by both deservedly. For who can commend him that demonstrates thus? If the World be Eternal, then an infinite number of dayes (or other measures of Time) preceded the birth of *Abraham*. But the birth of *Abraham* preceded the birth of *Isaac*; and therefore one Infinite is greater then another Infinite, or one Eternal then another Eternal; which (he sayes) is absurd. This Demonstration is like his, who from this, that the number of even Numbers is infinite, would conclude, that there are as many even Numbers, as there are Numbers simply, that is to say, the even Numbers are as many, as all the even and od together. They which in this manner take away Eternity from the World, do they not by the same means take away Eternity from the Creator of the World? From this absurdity therefore they run into another, being forced to call Eternity *Nunc stans*, a standing still of the present Time, or *an abiding Now*; and (which is much more absurd) to give to the infinite number of Numbers, the name of *Unity*. But why should Eternity be called an *Abiding Now*, rather then an *Abiding Then*? Wherefore there must either be many Eternities, or *Now* and *Then* must signifie the same. With such Demonstrators as these, that speak in another language, it is impossible to enter into disputation. And the men that reason thus absurdly, are not Idiots, but (which makes the absurdity unpardonable) Geometricians, and such as take upon them to be Judges (impertinent, but severe Judges) of other mens Demonstrations. The reason is this, that as soon as they are entangled in the Words *Infinite* and *Eternal*, (of which we have in our mind no Idea, but that of our own insufficiency to comprehend them) they are forced either to speak something absurd, or (which they love worse) to hold their peace. For Geometry hath in it somewhat like Wine; which when new, is windy; but afterwards though less pleasant, yet more wholsome. Whatsoever therefore is true, young Geometricians think Demonstrable; but elder not. Wherefore I purposely pass over the Questions of Infinite and Eternal; contenting my self with that Doctrine con-

concerning the Beginning and Magnitude of the World, which I have been perſwaded to by the holy Scriptures, and fame of the Miracles which confirm them; and by the Cuſtome of my Country, and reverence due to the Laws. And ſo I paſs on to ſuch things as it is not unlawful to diſpute of.

2 Concerning the World it is further queſtioned, whether the parts thereof be contiguous to one another, in ſuch manner, as not to admit of the leaſt empty ſpace between; and the diſputation both for & againſt it, is carried on with probability enough. For the taking away of *Vacuum*, I will inſtance in onely one experiment, a common one, but (I think) unanſwerable.

Let A B (in the firſt fig.) repreſent a veſſel (ſuch as Gardiners uſe to water their Gardens withal;) whoſe bottom B is ful of litle holes; & whoſe mouth A may be ſtopt with ones finger when there ſhall be need. If now this veſſel be filled with water, the hole at the top A being ſtopt, the water will not flow out at any of the holes in the bottom B. But if the finger be removed to let in the air above, it will run out at them all; and as ſoon as the finger is applyed to it again, the water wil ſuddenly & totally be ſtayed again from running out. The cauſe whereof ſeems to be no other but this, that the Water cannot by its natural endeavour to deſcend, drive down the aire below it, becauſe there is no place for it to go into, unleſs either by thruſting away the next contiguous aire it proceed by continual endeavour to the hole A, where it may enter and ſucceed into the place of the water that floweth out; or elſe by reſiſting the endeavour of the water Downwards, penetrate the ſame, and paſs up through it. By the firſt of theſe wayes (while the hole at A remains ſtopped) there is no poſſible paſſage; nor by the ſecond, unleſs the holes be ſo great, that the water flowing out at them, can by its own waight force the Air at the ſame time to aſcend into the veſſel by the ſame holes; as we ſee it does in a veſſel whoſe mouth is wide enough, when we turn ſuddenly the bottom upwards to poure out the water; for then the Aire being forced by the waight of the water, enters (as is evident by the ſobbing and reſiſtance of the water) at the ſides or circumference of the orifice. And this I take for a ſign that all Space is full; for without this the naturall motion of the water (which is a heavy Body) downwards, would not be hindered.

3 On

3 On the contrary, for the eſtabliſhing of *Vacuum*, many & ſpecious arguments and experiments have been brought. Nevertheleſſe there ſeemes to be ſomething wanting in all of them to conclude it firmely. Theſe arguments for *Vacuum* are partly made by the followers of the doctrine of *Epicurus*; who taught that the World conſiſts of very ſmall Spaces not filled by any Bodie, and of very ſmall Bodies that have within them no empty Space, (which by reaſon of their hardneſſe he calls *Atomes*); and that theſe ſmall Bodies and Spaces are every where intermingled. Their arguments are thus delivered by *Lucretius*.

And firſt he ſayes that unleſſe it were ſo, there could be no motion; For the office and property of Bodies is to withſtand and hinder motion. If therfore the Univerſe were filled with Body, motion would every where be hindered, ſo, as to have no beginning any where; & conſequently there would be no motion at all. It is true that in whatſoever is full, and at reſt in all its parts, it is not poſſible motion ſhould have beginning. But nothing is drawn from hence for the proving of *Vacuum*. For though it ſhould be granted that there is *Vacuum*; yet if the Bodies which are intermingled with it, ſhould all at once and together be at reſt, they would never be moved again. For it has been demonſtrated above (in the 9th Chapter 7th Article) that nothing can be moved but by that which is contiguous and already moved. But ſuppoſing that all things are at reſt together, there can be nothing contiguous and moved; and therefore no beginning of motion. Now the denying of the beginning of motion, doth not take away preſent motion, unleſs beginning be taken away from Body alſo. For motion may be either coeternal, or concreated with Body. Nor doth it ſeem more neceſſary that Bodies were firſt at reſt, and afterwards moved, then that they were firſt moved, and reſted (if ever they reſted at all) afterwards. Neither doth there appear any cauſe, why the matter of the World ſhould for the admiſſion of motion, be intermingled with empty ſpaces, rather then full; I ſay full, but withall fluid. Nor laſtly, is there any reaſon why thoſe hard Atomes may not alſo by the motion of intermingled fluid matter be congregated & brought together into compounded Bodies of ſuch bigneſs as we ſee. Wherefore nothing can by this argument be con-

concluded, but that motion was either coeternal, or of the ſame duration with that which is moved; neither of which concluſions conſiſteth with the doctrine of *Epicurus*, who allows neither to the World nor to Motion any Beginning at all. The neceſſity therefore of *Vacuum* is not hitherto demonſtrated. And the cauſe (as far as I underſtand from them that have diſcourſed with me of *Vacuum*) is this, that whileſt they contemplate the nature of *Fluid*, they conceive it to conſiſt as it were of ſmall grains of hard matter, in ſuch manner as meal is fluid, made ſo by grinding of the Corn; when nevertheleſs it is poſſible to conceive *Fluid* to be of its own nature as homogeneous, as either an Atome, or as *Vacuum*, it ſelf.

The ſecond of their arguments is taken from waight, and is contained in theſe Verſes of *Lucretius*,

Corporis officium eſt quoniam premere omnia deorſum;
Contrà autem natura manet ſine Pondere Inanis;
Ergo quod magnum eſt æque, Leviuſque videtur,
Nimirum plus eſſe ſibi declarat Inanis.

That is to ſay, *Seeing the office and property of Body is to preſs all things downwards; and on the contrary, ſeeing the nature of* Vacuum *is to have no waight at all; Therefore when of two Bodies of equal magnitude, one is lighter then the other, it is manifeſt that the lighter Body hath in it more* Vacuum *then the other.*

To ſay nothing of the Aſſumption concerning the endeavour of Bodies downwards; which is not rightly aſſumed, becauſe the World hath nothing to do with *Downwards*, which is a mere fiction of ours; Nor of this, that if al things tended to the ſame loweſt part of the World, either there would be no coaleſcence at all of Bodies, or they would all be gathered together into the ſame place. This onely is ſufficient to take away the force of the argument, that Aire intermingled with thoſe his Atomes, had ſerved as well for his purpoſe, as his intermingled *Vacuum*.

The third argument is drawn from this, That Lightning, Sound, Heat and Cold do penetrate all Bodies (except Atomes) how ſolid ſoever they be. But this reaſon, except it be firſt demonſtrated that the ſame things cannot happen (without *Vacuum*) by perpetual

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generation of Motion, is altogether invalid. But that all the ſame things may ſo happen, ſhall in due place be demonſtrated.

Laſtly, the fourth argument is ſet down by the ſame *Lucretius* in theſe Verſes.

Duo de concurſu corpora lata
Si citò diſſiliant, nempe aer omne neceſſe eſt
Inter corpora quod fuerat, poſſidat Inane.
Is porro quamvis circum celerantibus auris
Confluat, haud poterit tamen uno tempore totum
Compleri ſpatium; nam primum quemque neceſſe eſt
Occupet ille locum, deinde omnia poſſideantur.

That is, *If two flat Bodies be ſuddenly pulled aſunder, of neceſſity the Air muſt come between them to fill all the ſpace they left empty. But with what celerity ſoever the Air flow in, yet it cannot in one inſtant of time fill the whole ſpace, but firſt one part of it, then ſucceſſively all.*

Which nevertheleſs is more repugnant to the opinion of *Epicurus*, then of thoſe that deny *Vacuum*. For though it be true, that if two Bodies were of infinite hardneſs, and were joyned together by their Superficies which were moſt exactly plain, it would be impoſſible to pull them aſunder, in regard it could not be done but by Motion in an inſtant; yet, if as the greateſt of all Magnitudes cannot be given, nor the ſwifteſt of all Motions, ſo neither the hardeſt of all hard Bodies; it might be, that by the application of very great force, there might be place made for a ſucceſſive flowing in of the Aire, namely by ſeparating the parts of the joyned Bodies by ſucceſſion, beginning at the outermoſt and ending at the innermoſt part. He ought therefore firſt to have proved, that there are ſome Bodies extreamly hard, not relatively, as compared with ſofter Bodies, but abſolutely, that is to ſay, infinitely hard; which is not true. But if we ſuppoſe (as *Epicurus* doth) that Atomes are indiviſible, and yet have ſmall ſuperficies of their own; then if two Bodies ſhould be joyned together by many, or but one onely ſmall ſuperficies of either of them, then I ſay this argument of *Lucretius* would be a firme demonſtration, that no two Bodies made up of Atomes (as he ſuppoſes) could ever poſſibly be pulled aſunder by any

any force whatsoever. But this is repugnant to daily experience.

And thus much of the arguments of *Lucretius*. Let us now consider the arguments which are drawn from the experiments of later Writers.

4 The first experiment is this, That if a hollow vessel be thrust into water with the bottom upwards, the water will ascend into it; which they say it could not do, unless the Aire within were thrust together into a narrower place; and that this were also impossible except there were little empty places in the Aire. Also that when the Aire is compressed to a certain degree, it can receive no further compression, its small particles not suffering themselves to be pent into less room. This reason, if the Aire could not pass through the Water as it ascends within the vessel, might seem valid. But it is sufficiently known, that Aire will penetrate Water by the application of a force equal to the gravity of the Water. If therefore the force by which the Vessel is thrust down, be greater, or equal to the endeavour by which the water naturally tendeth downwards, the Aire will go out that way where the resistance is made, namely, towards the edges of the Vessel. For, by how much the deeper is the water which is to be penetrated, so much greater must be the depressing force. But after the Vessel is quite under water, the force by which it is depressed, that is to say, the force by which the water riseth up, is no longer encreased. There is therefore such an equilibration between them, as that the natural endeavour of the water downwards, is equal to the endeavour by which the same water is to be penetrated to the encreased depth.

The second experiment is, That if a concave Cylinder of sufficient length (made of Glass, that the experiment may be the better seen) having one end open, and the other close shut, be filled with Quicksilver, and the open end being stopped with ones finger, be together with the finger dipped into a dish or other vessel in which also there is Quicksilver, and the Cylinder be set upright, we shall, the finger being taken away (to make way for the descent of the Quicksilver) see it descend into the Vessel under it, till there be onely so much remayning within the Cylinder as may fill about 26 Inches of the same; and thus it will alwayes happen whatsoever be the Cylinder, provided that the length be

not leſs then 26 Inches. From whence they conclude that the cavity of the Cylinder above the Quickſilver remayns empty of all Body. But in this experiment I finde no neceſſity at all of *Vacuum*. For when the Quickſilver which is in the Cylinder deſcends; the Veſſel under it muſt needs be filled to a greater height, and conſequently ſo much of the contiguous Air muſt be thruſt away as may make place for the Quickſilver which is deſcended. Now if it be asked whether that Aire goes, what can be anſwered but this, that it thruſteth away the next Aire, & that the next, & ſo ſucceſſively, till there be a return to the place where the propulſion firſt began? and there, the laſt Aire thus thruſt on will preſs the Quickſilver in the Veſſel with the ſame forcewith which the firſt Aire was thruſt away; and if the force with which the Quickſilver deſcends be great enough (which is greater or leſs, as it deſcends from a place of greater or leſs height) it will make the Aire penetrate the Quickſilver in the veſſel, and go up into the Cylinder to fill the place which they thought was left empty. But becauſe the Quickſilver hath not in every degree of height force enough to cauſe ſuch penetration, therefore in deſcending it muſt of neceſſity ſtay ſomewhere, namely there, where its endeavour downwards, and the reſiſtance of the ſame to the penetration of the Aire come to an *æquilibrium*. And by this experiment it is manifeſt, that this *æquilibrium* will be at the height of 26 Inches, or thereabouts.

The third experiment is, That when a Veſſel hath as much Air in it as it can naturally contain, there may nevertheleſs be forced into it as much water as will fill three quarters of the ſame Veſſel. And the experiment is made in this manner. Into the glaſs bottle, repreſented (in the 2d figure) by the Sphere F G, whoſe center is A, let the pipe B A C be ſo fitted, that it may preciſely fill the mouth of the bottle; and let the end B, be ſo neer the bottom, that there may be onely ſpace enough left for the free paſſage of the water which is thruſt in above. Let the upper end of this pipe have a Cover at D, with a ſpout at E, by which the water (when it aſcends in the pipe) may run out. Alſo let H C be a Cock, for the opening or ſhutting of the paſſage of the water between B and D, as there ſhall be occaſion. Let the Cover D E be taken off; and the Cock H C being opened, let a Syringe full of water be forced

ced in; and before the Syringe be taken away, let the Cock be turned to hinder the going out of the Aire. And in this manner let the injection of water be repeated as often as it ſhall be requiſite, till the water riſe within the bottle, for example, to G F. Laſtly, the Cover being faſtened on again, and the Cock H C opened, the water will run ſwiftly out at E, and ſink by little and little from G F to the bottom of the pipe B.

From this *Phænomenon* they argue for the neceſſity of *Vacuum* in this manner. The Bottle from the beginning was full of Aire; which Aire could neither go out by penetrating ſo great a length of water as was injected by the pipe, nor by any other way. Of neceſſity therefore all the water as high as F G, as alſo all the Aire that was in the bottle before the water was forced in, muſt now be in the ſame place, which at firſt was filled by the Aire alone; which were impoſſible, if all the ſpace within the bottle were formerly filled with Aire preciſely, that is, without any *Vacuum*. Beſides, though ſome man perhaps may think the Air, being a thinne Body, may paſs through the Body of the water contained in the pipe, yet from that other *Phænomenon*, (namely, that all the water which is in the ſpace B F G, is caſt out again by the ſpout at E, for which it ſeems impoſſible that any other reaſon can be given beſides the force by which the Aire frees it ſelf from compreſſion) it follows, that either there was in the bottle ſome ſpace empty, or that many Bodies may be together in the ſame place. But this laſt is abſurd; and therefore the former is true, namely, that there was *Vacuum*.

This argument is infirm in two places. For firſt that is aſſumed which is not to be granted; and in the ſecond place an experiment is brought, which I think is repugnant to *Vacuum*. That which is aſſumed is, that the Aire can have no paſſage out through the pipe. Neverthelefs we ſee daily that Aire eaſily aſcends from the bottom to the Superficies of a River (as is manifeſt by the bubbles that riſe); nor doth it need any other cauſe to give it this motion, then the natural endeavour downwards of the Water. Why therefore may not the endeavour upwards of the ſame Water, acquired by the injection (which endeavour upwards is greater then the natural endeavour of the water downwards) cauſe the aire in the bottle to penetrate in like manner the water that preſſeth it

it downwards; especially seeing the water as it riseth in the bottle, doth so press the Aire that is above it, as that it generateth in every part thereof an endeavour towards the external superficies of the pipe, and consequently maketh all the parts of the enclosed aire to tend directly towards the passage at B? I say this is no less manifest, then that he aire which riseth up from the bottom of a River should penetrate the water, how deep soever it be. Werefore I do not yet see any cause why the force by which the water is injected, should not at the same time eject the aire.

And as for their arguing the necessity of *Vacuum* from the rejection of the water; In the first place, supposing there is *Vacuum*, I demand by what principle of motion that ejection is made. Certainly, seeing this motion is from within outwards, it must needs be caused by some Agent within the bottle; that is to say, by the aire it self. Now the motion of that aire, being caused by the rising of the water, begins at the bottom, and tends upwards; whereas the motion by which it ejecteth the water ought to begin above, and tend downwards. From whence therefore hath the enclosed aire this endeavour towards the bottom. To this question I know not what answer can be given, unless it be said, that the aire descends of its own accord to expel the water. Which because it is absurd, and that the aire after the water is forced in, hath as much room as its magnitude requires, there will remain no cause at all why the water should be forced out. Wherefore the assertion of *Vacuum* is repugnant to the very experiment which is here brought to establish it,

Many other *Phænomena* are usually brought for *Vacuum*, as those of *Weather-glasses*, *Æolipiles*, *Wind-guns*, &c. Which would all be very hard to be salved, unless water be penetrable by aire, without the intermixture of empty space. But now, seeing aire may with no great endeavour pass through, not onely water, but any other fluid Body, though never so stubborn, as Quicksilver, these *Phænomena* prove nothing. Nevertheless, it might in reason be expected, that he that would take away *Vacuum*, should without *Vacuum* shew us such causes of these *Phænomena*, as should be at least of equal, if not greater probability. This therefore shall be done in the following discourse, when I come to speak of these *Phænomena* in their

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proper places. But first the most general *Hypotheses* of natural Philosophy are to be premised.

And seeing that Suppositions are put for the true Causes of apparent Effects, every Supposition, except such as be absurd, must of necessity consist of some supposed possible Motion (for Rest can never be the Efficient Cause of any thing),& Motion supposeth Bodies Moveable; of which there are three kinds, *Fluid*, *Consistent*, and *mixt of both*. *Fluid* are those, whose parts may by very weak endeavour be separated from one another; and *Consistent* those, for the separation of whose parts greater force is to be applyed. There are therefore degrees of Consistency; which degrees, by comparison with more or less Consistent, have the names of *Hardness*, or *Softness*. Wherefore a Fluid Body is alwayes divisible into Bodies equally Fluid, as Quantity into Quantities; and Soft Bodies, of whatsoever degree of Softness, into Soft Bodies of the same degree. And though many men seem to conceive no other difference of *Fluidity*, but such as ariseth from the different magnitudes of the parts (in which Sense, Dust, though of Diamonds, may be called Fluid); Yet I understand by *Fluidity*, that which is made such by Nature equally in every part of the Fluid Body; not as Dust is Fluid, for so a House which is falling in pieces may be called Fluid; but in such manner as Water seems Fluid, and to divide it self into parts perpetually Fluid. And this being well understood, I come to my Suppositions.

5 First, therefore I suppose, That the Immense Space which we call the World, is the Aggregate of all Bodies; which are either Consistent & Visible, as the Earth and the Starres; or Invsible, as the small Atomes which are disseminated through the whole space between the Earth and the Stars; and lastly, that most Fluid *Æther*, which so fils all the rest of the Universe, as that it leaves in it no empty place at all.

Secondly, I suppose with *Copernicus*, That the greater Bodies of the World, which are both consistent and permanent, have such order amongst themselves, as that the *Sunne* hath the first place, *Mercury* the second, *Venus* the third, *The Earth* (with the *Moon* going about it) the fourth, *Mars* the fifth, *Jupiter* (with his Attendants) the sixth, *Saturne* the seventh, and after these the *Fixed Starres* have their several distances from the Sunne.

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Thirdly, I ſuppoſe, That in the Sunne & the reſt of the Planets, there is and alwayes has been a Simple Circular Motion.

Fourthly, I ſuppoſe, That in the Body of the Aire there are certain other Bodies intermingled, which are not Fluid; but withal that they are ſo ſmall, that they are not preceptible by Senſe, and that theſe alſo have their proper Simple Motion; and are ſome of them more, ſome leſs hard or conſiſtent.

Fifthly, I ſuppoſe with *Kepler*, That as the diſtance between the Sunne and the Earth, is to the diſtance between the Moon and the Earth; ſo the diſtance between the Moon and the Earth, is to the Semidiameter of the Earth.

As for the Magnitude of the Circles, and the Times in which they are deſcribed by the Bodies which are in them, I will ſuppoſe them to be ſuch as ſhall ſeem moſt agreeable to the *Phænomena* in queſtion,

6 The cauſes of the different Seaſons of the Year, and of the ſeveral variations of Dayes and Nights in all the parts of the ſuperficies of the Earth, have been demonſtrated firſt by *Copernicus*, and ſince by *Kepler*, *Galilæw* and others, from the ſuppoſition of the Earths diurnal revolution about its own Axis, together with its Annual motion about the Sunne, in the Ecliptick according to the order of the Signes, and thirdly, by the annual revolution of the ſame Earth about its own center contrary to the order of the Signs. I ſuppoſe with *Copernicus*, That the diurnal revolution is from the motion of the Earth, by which the Æquinoctial Circle is deſcribed about it. And as for the other two annual motions, they are the efficient cauſe of the Earths being carried about in the Ecliptick in ſuch manner, as that its Axis is alwayes kept parallel to it ſelf. Which paralleliſme was for this reaſon introduced, leſt by the Earths annual revolution, its Poles ſhould ſeem to be neceſſarily carried about the Sunne, contrary to experience. I have (in the 10th Artic. of the 20th Chap.) demonſtrated from the ſuppoſition of Simple Circular Motion in the Sun, that the Earth is ſo carried about the Sunne, as that its Axis is thereby kept always parallel to it ſelf. Wherefore, from theſe two ſuppoſed motions in the Sunne, the one Simple Circular Motion; the other Circular Motion about its owns Center, it may be demonſtrated, that the Year hath both the ſame variations of Dayes and Nights, as have been de-

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For if the Circle *a b c d* (in the 3d Figure) be the Ecliptick, whoſe Center is *e*, and Diameter *a e c*; and the Earth be placed in *a*, & the Sunne be moved in the little Circle *f g h i*, namely, according to the order *f*, *g*, *h* & *i*, it hath been demonſtrated, that a Body placed in *a*, will be moved in the ſame order through the points of the Ecliptick *a*, *b*, *c* & *d*, and will alwayes keep its Axis parallel to its ſelf.

But if (as I have ſuppoſed) the Earth alſo be moved with Simple Circular Motion in a plain that paſſeth through *a*, cutting the plain of the Ecliptick ſo, as that the common ſection of both the plains be in *a c*, thus alſo the Axis of the Earth will be kept alwayes parallel to it ſelf. For let the Center of the Earth be moved about in the Circumference of the Epicycle whoſe Diameter is *l a k*, which is a part of the ſtraight line *l a c*. Therefore *l a k* the Diameter of the Epicycle, paſſing through the Center of the Earth, will be in the plain of the Ecliptick. Wherefore ſeeing that by reaſon of the Earths Simple Motion both in the Ecliptick and in its Epicycle, the ſtraight line *l a k* is kept alwayes parallel to it ſelf, every other ſtraight line alſo taken in the Body of the Earth; and conſequently its Axis, will in like manner be kept alwayes parallel to it ſelf; ſo that in what part ſoever of the Ecliptick the Center of the Epicycle be found, and in what part ſoever of the Epicycle the Center of the Earth be found at the ſame time, the Axis of the Earth will be parallel to the place where the ſame Axis would have been, if the Center of the Earth had never gone out of the Ecliptick.

Now as I have demonſtrated the ſimple annual motion of the Earth from the ſuppoſition of Simple Motion in the Sunne; ſo from the ſuppoſition of Simple Motion in the Earth may be demonſtrated the monethly Simple Motion of the Moon. For if the names be but changed the Demonſtration will be the ſame, and therefore need not be repeated.

7 That which makes this ſuppoſition of the Sunnes Simple Motion in the Epicycle *f g h i* probable, is Firſt, that the Periods of all the Planets are not onely deſcribed about the Sunne, but ſo deſcribed, as that they are all contained within the Zodiack; that is to ſay, within the latitude of about 16 degrees; for the cauſe of this

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ſeems to depend upon ſome power in the Sunne, eſpecially in that part of the Sunne which reſpects the Zodiack. Secondly, that in the whole compaſſe of the heavens there appears no other Body, from which the cauſe of this *Phænomenon* can in probability be derived. Beſides, I could not imagine, that ſo many and ſuch various motions of the Planets ſhould have no dependance at all upon one another. But by ſuppoſing motive power in the Sunne, we ſuppoſe motion alſo; for power to move, without motion, is no power at all. I have therefore ſuppoſed that there is in the Sunne for the governing of the primary Planets, and in the Earth for the governing of the Moon, ſuch motion, as being received by the primary Planets and by the Moon, makes them neceſſarily appear to us in ſuch manner as we ſee them; Whereas, that circular motion (which is commonly attributed to them) about a fixed Axis, (which is called Converſion) being a motion of their parts onely, and not of their whole Bodies, is inſufficient to ſalve their Appearances. For ſeeing whatſoever is ſo moved, hath no endeavour at all towards thoſe parts which are without the circle, they have no power to propagate any endeavour to ſuch Bodies as are placed without it. And as for them that ſuppoſe this may be done by Magnetical Virtue, or by incorporeall and immateriall Species, they ſuppoſe no naturall cauſe; nay no cauſe at all. For there is no ſuch thing as an Incorporeal Movent; and Magnetical Virtue is a thing altogether unknown; and whenſoevr it ſhall be known, it will be found to be a motion of Body. It remaines therefore, that if the primary Planets be carried about by the Sunne, and the Moon by the Earth, they have the ſimple circular motions of the Sunne and the Earth for the cauſes of their circulations. Otherwiſe, if they be not carried about by the Sunne and the Earth, but that every Planet hath been moved as it is now moved ever ſince it was made, there will be of their motions no cauſe naturall. For either theſe motions were concreated with their Bodies, and their cauſe is ſupernatural; or they are coeternal with them, and ſo they have no cauſe at all. For whatſoever is Eternall was never generated.

I may add beſides, to confirme the probability of this ſimple motion, that allmoſt all learned men are now of the ſame opinion with

with *Copernicus* concerning the paralleliſme of the Axis of the Earth, it ſeemed to me to be more agreeable to truth, or at leaſt more handſome, that it ſhould be cauſed by ſimple Circular Motion alone, than by two motions, one in the Ecliptick, and the other about the Earths own Axis the contrary way, neither of them Simple, nor either of them ſuch as might be produced by any motion of the Sunne. I thought beſt therefore to retain this Hypotheſis of Simple Motion; and from it to derive the cauſes of as many of the *Phænomena* as I could, and to let ſuch alone as I could not deduce frm thence.

It will perhaps be objected, that although by this ſuppoſition the reaſon may be given of the Paralleliſme of the Axis of the Earth, and of many other Appearances; nevertheleſs, ſeeing it is done by placing the Body of the Sunne in the Center of that Orbe which the Earth deſcribes with its annual motion, the ſuppoſition it ſelf is falſe, becauſe this annual Orbe is excentrique to the Sunne. In the firſt place therefore let us examine what that Excentricity is, and whence it proceeds.

8 Let the annual Circle of the Earth *a b c d* (in the ſame 3d figure) be divided into four equal parts by the ſtraight lines *a c* & *b d*, cutting one another in the Center *e*; and let *a* be the beginning of *Libra*, *b* of *Capricorn*, *c* of *Aries*, and *d* of *Cancer*; and let the whole Orbe *a b c d* be underſtood (according to *Copernicus*) to have every way ſo great diſtance from the Zodiack of the fixed Starres, that it be in compariſon with it but as a point. Let the Earth be now ſuppoſed to be in the beginning of *Libra* at *a*. The Sunne therefore will appear in the beginning of *Aries* at *c*. Wherefore if the Earth be moved from *a* to *b*, the apparent motion of the Snne will be from *c* to the beginning of *Cancer* in *d*; and the Earth being moved forwards from *b* to *c*, the Sunne alſo will appear to be moved forwards to the beginning of *Libra* in *a*; Wherefore *c d a* will be the Summer Arch, and the Winter Arch will be *a b c*. Now in the time of the Suns apparent motion in the Summer Arch, there are numbred $186\frac{3}{4}$ dayes; and conſequently the Earth makes in the ſame time the ſame number of diurnal converſions in the Arch *a b c*; and therefore the Earth in its motion through the Arch *c d a* will make onely $178\frac{1}{2}$ diurnal converſions, Wherefore

the Arch *a b c* ought to be greater then the Arch *c d a* by $8\frac{1}{4}$ dayes, that is to say, by almost so many degrees. Let the Arch *a r*, as also *c s*, be each of them an Arch of two degrees and $\frac{1}{16}$. Wherefore the Arch *r b s* will be greater then the Semicircle *a b c* by 4 degrees and $\frac{1}{8}$, and greater then the Arch *s d r* by 8 degrees and $\frac{1}{4}$. The Equinoxes therefore will be in the points *r* & *s*; and therefore also when the Earth is in *r*, the Sunne will appear in *s*. Wherefore the true place of the Sunne will be in *t*, that is to say, without the Center of the Earths annual motion by the quantity of the Sine of the Arch *a r*, or the Sine of two degrees and 16 minutes. Now this Sine, putting 100000 for the Radius, will be neer 3580 parts thereof. And so munh is the Excentricity of the Earths annual motion, provided that that motion be in a perfect circle; and *s* & *r* are the Equinoctial points; and the straight lines *ſr* & *c a* produced both wayes till they reach the Zodiack of the fixed Starres, wil fall stil upon the same fixed Starres, because the whole Orbe *a b c d* is supposed to have no magnitude at all in respect of the great distance of the fixed Starres.

Supposing now the Sun to be in *c*, it remains that I shew the cause why the Earth is neerer to the Sunne, when in its annual motion it is found to be in *d*, then when it is in *b*. And I take the cause to be this. When the Earth is in the beginning of *Capricorn* at *b*, the Sunne appears in the beginning of *Cancer* at *d*; & then is the midst of Summer. But in the midst of Summer, the Northern parts of the Earth are towards the Sunne, which is almost all dry land, containing all *Europe*, and much the greatest part of *Asia* and *America*. But when the Earth is in the beginning of *Cancer* at *d*, it is the midst of Winter, and that part of the Earth is towards the Sunne, which contains those great Seas called the South Sea and the Indian Sea, which are of farre greater extent then all the dry Land in that Hemisphere. Wherefore (by the last Article of the 21 Chapter) when the Earth is in *d*, it will come neerer to its first Movent, that is, to the Sunne which is in *t*; that is to say, the Earth is neerer to the Sunne in the midst of Winter when it is in *d*, then in the midst of Summer when it is *b*; and therefore during the Winter the Sunne is in its *Perigæum*, and in its *Apogæum* during the Summer. And thus I have shewn a possible cause of the

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Excentricity of the Earth; which was to be done.

I am therefore of *Keplers* opinion, in this, that he attributes the Excentricity of the Earth to the difference of the parts thereof, and ſuppoſes one part to be affected, and another diſaffected to the Sunne. And I diſſent from him in this, that he thinks it to be by Magnetick virtue, and that this Magnetick virtue, or attraction and thruſting back of the Earth is wrought by immateriate Species; which cannot be; becauſe nothing can give motion, but a Body moved and contiguous. For if thoſe Bodies be not moved which are contiguous to a Body unmoved, how this Body ſhould begin to be moved is not imaginable; as has been demonſtrated in the 7th Article of the 9th Chapter, and often inculcated in other places, to the end that Philoſophers might at laſt abſtain from the uſe of ſuch unconceiveable connexions of words. I diſſent alſo from him in this, that he ſays the ſimilitude of Bodies is the cauſe of their mutual attraction, For if it were ſo, I ſee no reaſon why one Egg ſhould not be attracted by another. If therefore one part of the Earth be more affected by the Sunne, then another part, it proceeds from this, that one part hath more water, the other more dry land. And from hence it is, (as I ſhewed above) that the Earth comes neerer to the Sunne when it ſhines upon that part where there is more water, then when it ſhines upon that where there is more dry Land.

9 This Excentricity of the Earth is the cauſe why the way of its annual motion is not a perfect Circle, but either an Elliptical, or almoſt an Elliptical line; as alſo why the Axis of the Earth is not kept exactly parallel to it ſelf in all places, but onely in the Equinoctial points.

Now ſeeing I have ſaid that the Moon is carried about by the Earth, in the ſame manner that the Earth is by the Sunne; and that the Earth goeth about the Sunne in ſuch manner as that it ſhews ſometimes one Hemiſphere, ſometimes the other to the Sunne; it remains to be enquired, why the Moon has alwayes one and the ſame face turned towards the Earth.

Suppoſe therefore the Sunne to be moved with Simple Motion in the little Circle *f g h i*, (in the fourth figure) whoſe Center is *t*; and let ♈ ♋ ♎ ♑ be the annuall Circle of the Earth; and ♈ the be-

ginning of *Libra*. About the point *a* let the little Circle *l k* be described; and in it let the Center of the Earth be understood to be moved with Simple motion; and both the Sunne & the Earth to be moved according to the order of the Signes. Upon the Center *a* let the way of the Moon *m n o p* be described; and let *q r* be the Diameter of a Circle cutting the Globe of the Moon into two Hemispheres, whereof one is seen by us when the Moon is at the full, and the other is turned from us.

The Diameter therefore of the Moon *q o r* will be perpendicular to the Straight Line *t a*. Wherefore the Moon is carried by reason of the Motion of the Earth from *o* towards *p*. But by reason of the motion of the Sunne, if it were in *p* it would at the same time be carried from *p* towards *o*; and by these two contrary Movents the straight line *q r* will be turned about; and in a Quadrant of the Circle *m n o p* it will be turned so much as makes the fourth part of its whole conversion. Wherefore when the Moon is in *p*, *q r* will be parallel to the straight line *m o*. Secondly, when the Moon is in *m*, the straight line *q r* will by reason of the motion of the Earth be in *m o*. But by the working of the Suns motion upon it in the quadrant *p m*, the same *q r* will be turned so much as makes another quarter of its whole conversion. When therefore the Moon is in *m*, *q r* will be perpendicular to the straight line *o m*. By the same reason, when the Moon is in *n*, *q r* will be parallel to the straight line *m o*; and the Moon returning to *o*, the same *q r* will return to its first place; and the Body of the Moon will in one entire period make also one entire conversion upon her own Axis. In the making of which, it is manifest, that one and the same face of the Moon is always turned towards the Earth. And if any Diameter were taken in that little Circle, in which the Moon were supposed to be carried about with simple motion, the same effect would follow; for if there were no action from the Sun, every Diameter of the Moon would be carried about always parallel to it self. Wherefore I have given a possible cause, why one and the same face of the Moon is alwayes turned towards the Earth.

But it is to be noted, that when the Moon is without the Eclipick, we do not alwayes see the same face precisely. For we see onely that part which is illuminated. But when the Moon is without

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the Ecliptick, that part which is towards us, is not exactly the same with that which is illuminated.

10 To these three simple motions, one of the Sunne, another of the Moon, and the third of the Earth, in their own little Circles *f g h, i, l k*, & *q r*, together with the Diurnal conversion of the Earth (by which conversion all things that adhere to its superficies are necessarily carried about with it) may be referred the three *Phænomena* concerning the Tides of the Ocean. Whereof the first, is the alternate elevation and depression of the Water at the Shores, twice in the space of 24 houres and neer upon 52 minutes; for so it has constantly continued in all ages. The second, that at the New and Full Moons the elevations of the Water are greater, then at other times between. And the third, that when the Sunne is in the Equinoctial, they are yet greater then at any other time. For the salving of which *Phænomena*, we have already the foure above-mentioned Motions; to which I assume also this, that the part of the Earth which is called *America*, being higher then the Water, and extended almost the space of a whole Semicircle from North to South, gives a stop to the motion of the Water. This being granted,

In the same 4th figure, where *l b k c* is supposed to be in the plain of the Moons monethly motion, let the little Circle *l d k e* be described about the same Center *a* in the plain of the Equinoctial. This Circle therefore will decline from the Circle *l b k c* in an angle of almost 28 degrees and $\frac{1}{2}$ (for the greatest declination of the Ecliptick is $23\frac{1}{2}$, to which adding 5 for the greatest declination of the Moon from the Ecliptick, the summe wil be 28 degrees and $\frac{1}{2}$). Seeing now the Waters which are under the Circle of the Moons course, are (by reason of the Earths Simple Motion in the plain of the same Circle) moved together with the Earth, (that is to say, together with their own bottoms) neither out-going nor out-gone; if we add the Diurnal motion, by which the other Waters which are under the Equinoctial are moved in the same order; and consider withall that the Circles of the Moon and of the Equinoctial intersect one another; it will be manifest, that both those Waters, which are under the Circle of the Moon, and under the Equinoctiall, will runne together under the Equinoctial; and consequently,

ly, that their Motion will not onely be ſwifter then the ground that carries them; but alſo that the waters themſelves will have greater elevation whenſoever the Earth is in the Equinoctial. Wherefore, whatſoever the cauſe of the Tides may be, this may be the cauſe of their augmentation at that time.

Againe, ſeeing I have ſuppoſed the Moon to be carried about by the ſimple motion of the Earth in the little circle *l b k c*; and demonſtrated (at the 4 article of the 21 chapter) that whatſoever is moved by a Movent that hath ſimple motion, will be moved allwayes with the ſame velocity; it follows, that the center of the Earth will be carried in the circumference *l b k c* with the ſame velocity, with which the Moon is carried in the circumference *m n o p*. Wherefore the time in which the Moon is carried about in *m n o p*, is to the time in which the Earth is carried about in *l b k c*, as one circumference to the other, that is, as *a o* to *a k*. But *a o* is obſerved to be to the Semidiameter of the Earth as 59 to 1; and therefore the Earth (if *a k* be put for its Semidiameter) will make 59 revolutions in *l b k c*, in the time that the Moon makes one monthly circuit in *m n o p*. But the Moon makes her monthly circuit in little more then 29 dayes. Wherefore the Earth ſhal make its circuit in the circumference *l b k c* in 12 hours and a little more, namely about 26 minutes more; that is to ſay, it ſhall make two circuits in 24 hours and allmoſt 52 minutes; which is obſerved to be the time between the high water of one day and the high water of the day following. Now the courſe of the waters being hindered by the ſouthern part of *America*, their motion will be interrupted there; and conſequently, they will be elevated in thoſe places, and ſink down again by their own waight, twice in the ſpace of 24 hours and 52 minutes. And thus I have given a poſſible cauſe of the diurnall reciprocation of the Ocean.

Now from this ſwelling of the Ocean in thoſe parts of the Earth, proceed the Flowings and Ebbings in the Atlantick, Spaniſh, Brittiſh and German Seas; which though they have their ſet times, yet upon ſeverall Shores they happen at ſeverall hours of the day; and they receive ſome augmentation from the North, by reaſon that the ſhores of *China* and *Tartaria*, hinder-

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ing the generall courſe of the waters, makes them ſwell there, and diſcharge themſelves in part through the ſtraight of *Anian* into the Northern Ocean, and ſo into the German Sea.

As for the Spring Tides which happen at the time of the New & Full Moons, they are cauſed by that ſimple motion which at the beginning I ſuppoſed to be allwayes in the Moone. For as, when I ſhewed the cauſe of the Excentricity of the Earth, I derived the elevation of the waters from the ſimple motion of the Sunne; ſo the ſame may here be derived from the ſimple motion of the Moon. For though from the generation of Clouds, there appeare in the Sunne a more manifeſt power of elevating the waters, then in the Moon; yet the power of encreaſing moiſture in Vegetables and living creatures appears more manifeſtly in the Moon then in the Sunne; which may perhaps proceed from this, that the Sunne raiſeth up greater, and the Moon leſſer drops of water. Nevertheleſſe, it is more likely, and more agreeable to common obſervation, that Raine is raiſed not only by the Sunne but alſo by the Moon; for allmoſt all men expect change of weather at the time of the Conjunctions of the Sunne and Moon with one another, and with the Earth, more then in the time of their Quarters.

In the laſt place, the cauſe why the Spring Tides are greater at the time of the Æquinoxes, hath been already ſufficiently declared in this Article, where I have demonſtrated, that the two Motions of the Earth, namely, its Simple Motion in the little Circle *l b k c*, and its Diurnal motion in *l d k e*, cauſe neceſſarily a greater elevation of Waters when the Sunne is about the Æquinoxes, then when he is in other places. I have therefore given poſſible cauſes of the *Phænomenon* of the Flowing and Ebbing of the Ocean.

11 As for the explication of the yearly *Præceſſion of the Æquinoctial points*, we muſt remember, that (as I have already ſhewn) the annual motion of the Earth is not in the Circumference of a Circle, but of an Ellipſis, or a line not conſiderably different from that of an Ellipſis. In the firſt place therefore this Elliptical line is to be deſcribed.

Let the Ecliptick ♎ ♑ ♈ ♋ (in the 5th figure) be divided into four equal parts by the two ſtraight lines *a b* and ♑ ♋, cutting one

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another at right angles in the Center *c*; and taking the Arch *b d* of 2 degrees and 16 minutes, let the ſtraight line *d e* be drawn parallel to *a b* and cutting ♈ ♋ in *f*; which being done, the Excentricity of the Earth will be *c f*. Seeing therefore the annual motion of the Earth is in the Circumference of an Ellipſis, of which ♈ ♋ is the greater Axis, *a b* cannot be the leſſer Axis; for *a b* and ♈ ♋ are equal. Wherefore the Earth paſſing through *a* & *b*, will either paſs above ♈, as through *g*, or paſſing through ♈, will fall between *c* and *a*; it is no matter which. Let it paſs therefore through *g*; and let *g l* be taken equal to the ſtraight line ♈ ♋; and dividing *g l* equally in *i*, *g i* will be equal to ♈ ♋, & *i l* equal to *f* ♋; and conſequently the point *i* will cut the Excentricity *c f* into two equal parts; and taking *i h* equal to *i f*, *h i* will be the whole Excentricity. If now a ſtraight line (namely, the line ♎ *i* ♈) be drawn through *i* parallel to the ſtraight lines *a b* and *e d*, the way of the Sunne in Summer (namely, the Arch ♎ *g* ♈) will be greater then his way in Winter by 8 degrees and ¼. Wherefore the true Æquinoxes wil be in the ſtraight line ♎ *i* ♈; and therefore the Ellipſis of the Earths annual motion will not paſs through *a*, *g*, *b* & *l*; but through ♎, *g*, ♈ & *l*. Wherfore the annual motion of the Earth is in the Ellipſis ♎ *g* ♈ *l*; and cannot be (the Excentricity being ſalved) in any other line. And this perhaps is the reaſon, why *Kepler*, againſt the opinion of all the Aſtronomers of former time, thought fit to biſect the Excentricity of the Earth, or (according to the Ancients) of the Sunne, not by diminiſhing the quantity of the ſame Excentricity, becauſe the true meaſure of that quantity, is the difference by which the Summer Arch exceeds the Winter Arch, but by taking for the Center of the Ecliptick of the great Orbe the point *c* neerer to *f*, & ſo placing the whole great Orbe as much neerer to the Ecliptick of the fixed Stars towards ♋, as is the diſtance between *c* & *i*. For ſeeing the whole great Orbe is but as a point in reſpect of the immenſe diſtance of the fixed Starres, the two ſtraight lines ♎ ♈ and *a b* being produced both wayes to the beginnings of *Aries* and *Libra*, will fall upon the ſame points of the Sphere of the fixed Stars. Let therefore the Diameter of the Earth *m n* be in the plain of the Earths annual motion. If now the Earth be moved by the Sunnes ſimple motion in the Circumference of the

the Ecliptick about the Center *i*, this Diameter will bee kept alwayes parallel to it ſelf and to the ſtraight line *g l*. But ſeeing the Earth is moved in the Circumference of an Ellipſis without the Ecliptick, the point *n*, whilſt it paſſeth through ♎ ♑ ♈ will go in a leſſer Circumference then the point *m*; and conſequently as ſoon as ever it begins to be moved, it will loſe its paralleliſme with the ſtraight line ♑ ♋; ſo that *m n* produced will at laſt cut the ſtraight line *g l* produced. And contrarily, as ſoon as *m n* is paſt ♈, (the Earth making its way in the internal Ellipticall line ♈ *l* ♎) the ſame *m n* produced towards *m*, will cut *l g* produced. And when the Earth hath allmoſt finiſhed its whole circumference, the ſame *m n* ſhall againe make a right angle with a line drawn from the center *i*, a little ſhort of the point from which the Earth began its motion. And there the next yeare ſhall be one of the Æquinoctial points, namely, neer the end of ♓; the other ſhall be oppoſite to it neer the end of ♍. And thus the points in which the Days and Nights are made equall, doe every year fall back; but with ſo ſlow a motion, that in a whole year, it makes but 51 firſt minutes. And this relapſe being contrary to the order of the Signes, is commonly called the *Præceſſion of the Æquinoxes*. Of which I have from my former Suppoſitions deduced a poſſible cauſe; which was to be done.

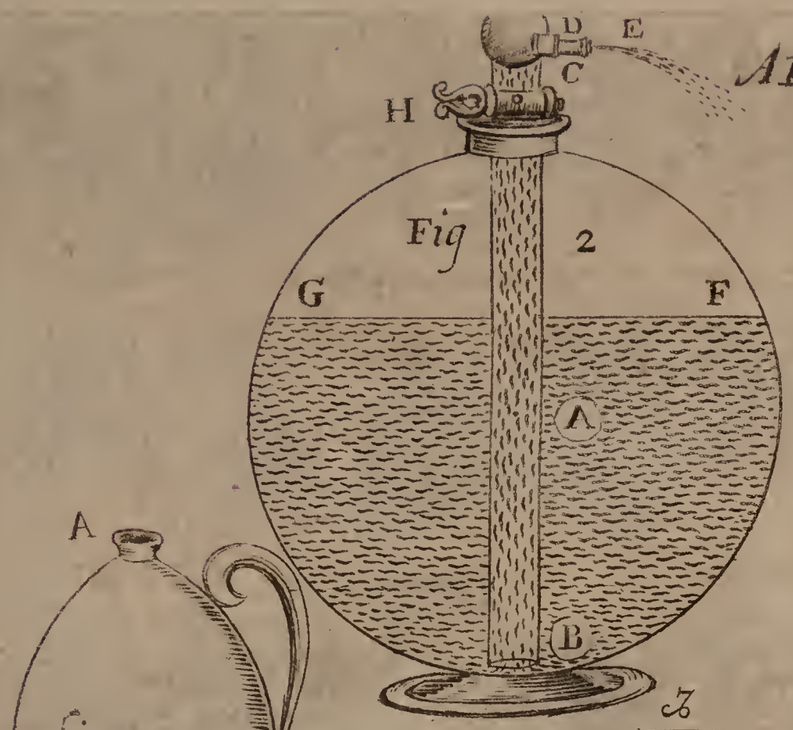
According to what I have ſaid concerning the cauſe of the Excentricity of the Earth; and according to *Kepler*, who for the cauſe thereof ſuppoſeth one part of the Earth to be affected to the Sunne the other part to be diſaffected, the *Apogæum* & *Perigæum* of the Sunne ſhould be moved every year in the ſame order, and with the ſame velocity, with which the Æquinoctiall points are moved; and their diſtance from them ſhould allwayes be the quadrant of a circle; which ſeems to be otherwiſe. For Aſtronomers ſay, that the Æquinoxes are now, the one about 28 degrees gone back from the firſt Star of *Aries*, the other as much from the beginning of *Libra*. So that the *Apogæum* of the Sunne, or the *Aphelium* of the Earth ought to be about the 28th degree of *Cancer*; but it is reckoned to be in the 7th degree. Seeing therefore we have not ſufficient evidence of the ὅτι (*that ſo it is*), it is in vaine to ſeek for the διότι (*why it is ſo*). Wherefore, as long as the motion of the

Apogæum is not obſervable by reaſon of the ſlowneſſe thereof; and as long as it remaiues doubtful whether their diſtance from the Æquinoctiall points be more or leſſe then a quadrant preciſely; ſo long it may be lawfull for me to thinke they proceed both of them with equall velocity.

Alſo, I doe not at all meddle with the cauſes of the Excentricities of *Saturne*, *Jupiter*, *Mars*, and *Mercury*. Nevertheleſſe, ſeeing the Excentricity of the Earth may (as I have ſhewne) be cauſed by the unlike conſtitution of the ſeveral parts of the Earth which are alternately turned towards the Sunne, it is credible alſo that like effects may be produced in theſe other Planets from their having their Superficies of unlike parts.

And this is all I ſhall ſay concerning *Sidereal Philoſophy*. And though the cauſes I have here ſuppoſed be not the true cauſes of theſe *Phænomena*, yet I have demonſtrated that they are ſufficient to produce them, according to what I at firſt propounded.

Ad Cap XXVI



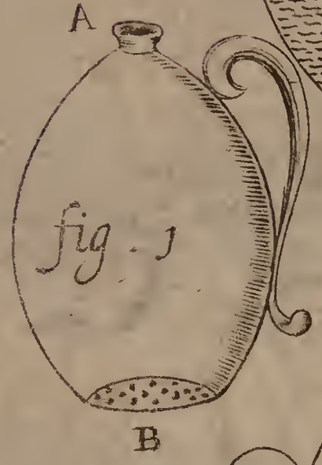


fig. 1

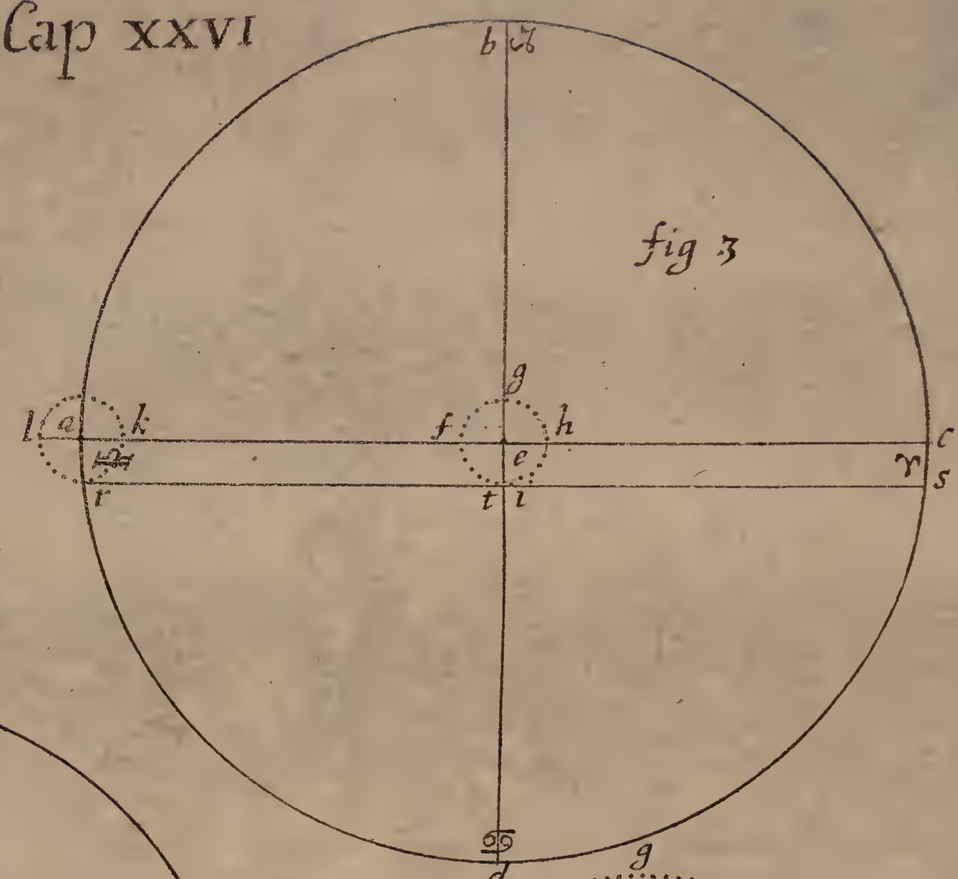


fig 3

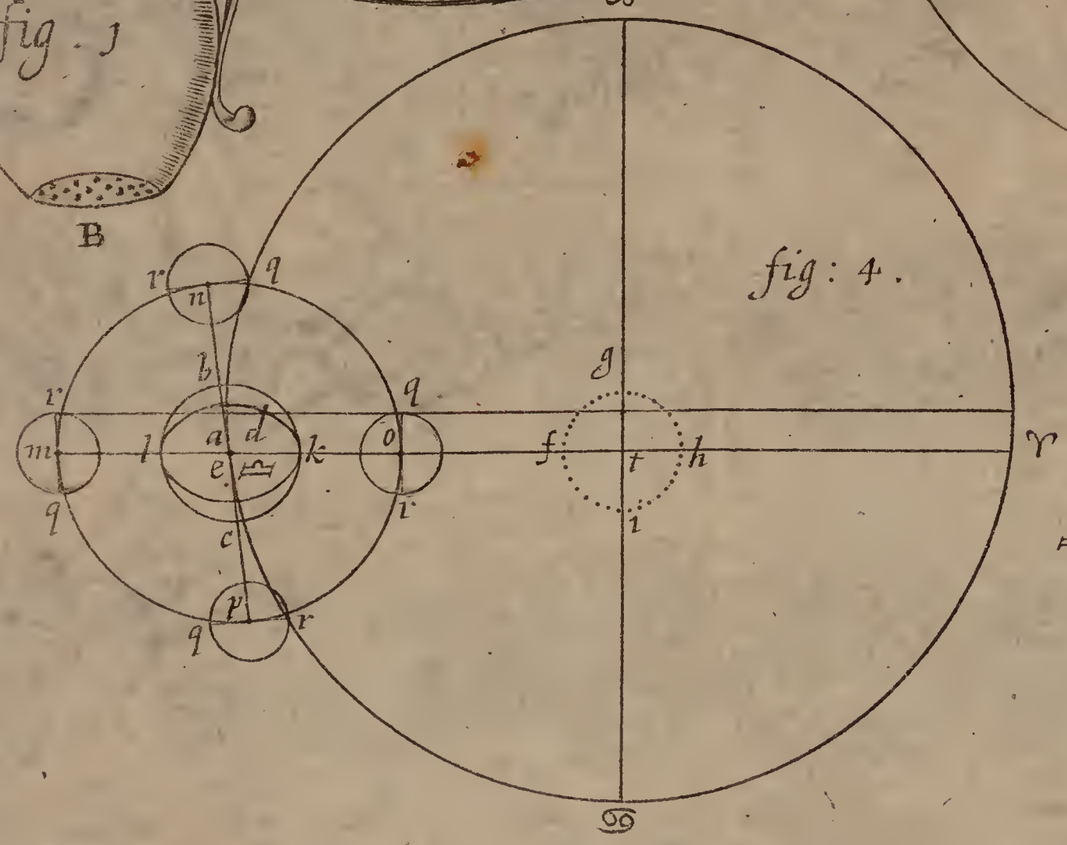


fig: 4.

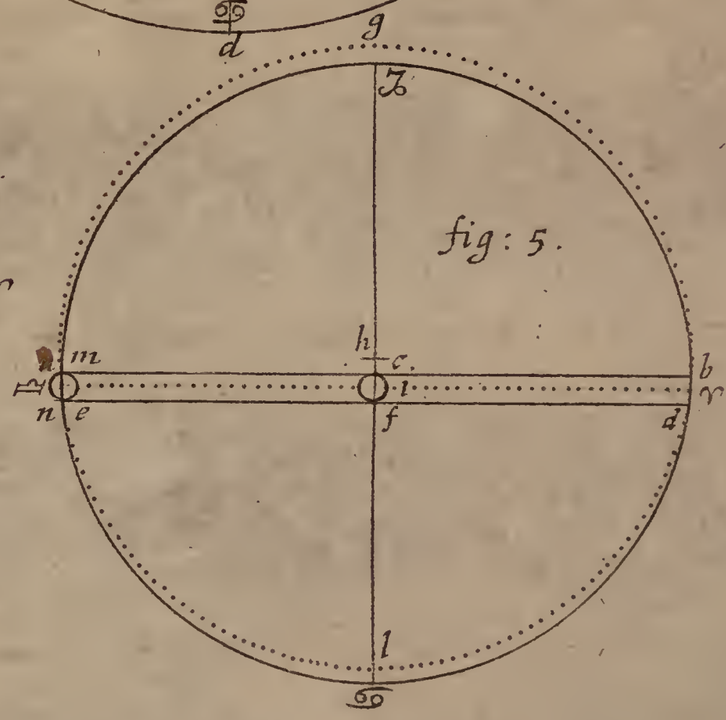


fig: 5.

CHAP. XXVII.

Of Light, Heat, *and of* Colours.

1 *Of the immenſe Magnitude of ſome Bodies, and the unſpeakable Littleneſs of others.* 2 *Of the cauſe of the Light of the Sun.* 3 *How Light heateth.* 4 *The generation of Fire from the Sunne.* 5 *The generation of Fire from Colliſion.* 6 *The cauſe of Light in Glow-wormes, Rotten Wood, and the* Bolonian *Stone.* 7 *The cauſe of Light in the concuſſion of Sea-water.* 8 *The cauſe of Flame, Sparks, and Colliquation.* 9 *The cauſe why wet Hay ſometimes burns of its own accord. Alſo the cauſe of Lightning.* 10 *The cauſe of the force of Gunpowder; and what is to be aſcribed to the Coals, what to the Brimſtone, and what to the Nitre.* 11 *How Heat is cauſed by Attrition.* 12 *The diſtinction of Light into Firſt, Second, &c.* 13 *The cauſes of the Colours we ſee in looking through a* Priſma *of Glaſs, namely, of Red, Yellow, Blue and Violet Colour.* 14 *Why the Moon and the Starres appear redder in the Horizon then in the midſt of the Heaven.* 15 *The cauſe of Whiteneſs.* 16 *The cauſe of Blackneſs.*

1 BEſides the Starres (of which I have ſpoken in the laſt Chapt.), whatſoever other Bodies there be in the World, they may be all comprehended under the name of *Interſidereal* Bodies. And theſe I have already ſuppoſed to be either the moſt fluid *Æther*, or ſuch Bodies whoſe parts have ſome degree of cohæſion. Now theſe differ from one another in their ſeveral *Conſiſtencies*, *Magnitudes*, *Motions* and *Figures*. In Conſiſtency, I ſuppoſe ſome Bodies to be Harder, others Softer through all the ſeveral degrees of *Tenacity*. In Magnitude, ſome to be Greater, others Leſs, and many unſpeakably Little. For we muſt remember that by the Underſtanding, Quantity is diviſible into diviſibles perpetually. And therefore if a man could do as much with his hands as he can with his Underſtanding, he would be able to take from any given magnitude, a part which ſhould be leſs then any other magnitude given. But the Omnipotent Creator of the World can actually from a part of any thing take another part, as farre as we

by our Understanding can conceive the same to be divisible. Wherefore there is no impossible Smalness of Bodies. And what hinders but that we may think this likely? For we know there are some living Creatures so small, that we can scarce see their whole Bodies. Yet even these have their young ones, their little Veins, and other Vessels, and their Eyes so smal, as that no *Microscope* can make them visible. So that we cannot suppose any magnitude so little, but that our very supposition is actually exceeded by Nature. Besides, there are now such *Microscopes* commonly made, that the things we see with them appear a hundred thousand times bigger, then they would do if we looked upon them with our bare Eyes. Nor is there any doubt but that by augmenting the power of these *Microscopes* (for it may be augmented as long as neither Matter nor the hands of Workmen are wanting) every one of those hundred thousandth parts might yet appear a hundred thousand times geater then they did before. Neither is the Smalness of some Bodies to be more admired, then the vast Greatness of others. For it belongs to the same infinite Power, as well to augment infinitely, as infinitely to diminish; To make the great Orbe, (namely, that whose Radius reacheth from the Earth to the Sunne) but as a point in respect of the distance between the Sunne and the fixed Starres; and on the contrary, to make a Body so little, as to be in the same proportion less then any other visible Body, proceeds equally from one and the same Authour of Nature. But this of the immense distance of the fixed Starres (which for a long time was accounted an incredible thing) is now believed by almost all the Learned. Why then should not that other of the smalness of some Bodies, become credible at some time or other? For the Majesty of God appears no less in small things then in great; and as it exceedeth humane sense in the immense greatness of the Universe; so also it doth in the smalness of the parts thereof. Nor are the first Elements of Compositions, nor the first Beginnings of Actions, nor the first Moments of Times more credible, then that which is now believed of the vast distance of the fixed Starres.

Some things are acknowledged by mortal men to be very Great, though Finite, as seeing them to be such. They acknowledge also, that some things which they do not see, may be of infinite magnitude.

nitude. But they are not presently, nor without great study perswaded that there is any Mean between Infinite & the Greatest of those things which either they see or imagine. Nevertheless, when after meditation & contemplation, many things which we wondred at before are now grown more familiar to us, we then believe them, and transferre our admiration from the Creatures to the Creator. But how little soever some Bodies may be; yet I will not suppose their quantity to be less, then is requisite for the salving of the *Phænomena*. And in like manner I shall suppose their motion, namely, their Velocity and Slowness, and the Variety of their Figures, to be onely such as the explication of their natural causes requires. And lastly, I suppose, that the parts of the pure *Æther* (as if it were the First Matter) have no motion at all, but what they receive from Bodies which float in them, and are not themselves fluid.

2 Having laid these Grounds, let us come to speake of Causes, and in the first place let us inquire what may be the cause of the Light of the Sunne. Seeing therefore the Body of the Sunne doth by its simple circular motion thrust away the ambient æthereall substance sometimes one way sometimes another, so that those parts which are next the Sunne being moved by it, doe propagate that motion to the next remote parts, and these to the next, and so on continually; it must needs be, that notwithstanding any distance, the foremost part of the Eie will at last be pressed; and by the pressure of that part, the motion will be propagated to the innermost part of the Organ of Sight, namely to the Heart; and from the reaction of the Heart, there will proceed an endeavour back by the same way, ending in the endeavour outwards of the Coat of the Eie called the *Retina*. But this endeavour outwards (as has been defined in the 25 chapter) is the thing which is called Light, or the Phantasme of a Lucid Body. For it is by reason of this Phantasme that an Object is called Lucid. Wherefore we have a possible cause of the Light of the Sunne; which I undertook to find.

3 The generation of the Light of the Sunne is accompanied with the generation of Heat. Now every man knowes what Heat is in himselfe, by feeling it when he growes Hot; but what it is in other things he knowes onely by ratiocination. For it is one thing to

grow Hot, and another thing to Heat, or make Hot. And therefore though we perceive that the Fire or the Sunne Heateth, yet we doe not perceive that it is it ſelfe Hot. That other living creatures whileſt they make other things Hot, are Hot themſelves, we inferre by reaſoning from the like ſenſe in our ſelves. But this is not a neceſſary inference. For though it may truly be ſaid of living Creatures, that *They Heat, therefore they are themſelves Hot*; yet it cannot from hence be truly inferred, that *Fire Heateth, therefore it is it ſelfe Hot*; no more then this, *Fire cauſeth Pain, therefore it is it ſelf in Pain.* Wherefore that is onely and properly called Hot, which when we feel, we are neceſſarily Hot.

Now when we grow Hot, we find that our Spirits and Blood, and whatſoever is fluid within us, is called out from the internall to the externall parts of our Bodies, more or leſſe, according to the degree of the Heat; and that our Skin ſwelleth. He therefore that can give a poſſible cauſe of this Evocation and Swelling, and ſuch as agreeth with the reſt of the *Phænomena* of Heat; may be thought to have given the cauſe of the Heat of the Sunne.

It hath been ſhewn (in the 5 article of the 21 chapter) that the fluid Medium which we call the Aire, is ſo moved by the ſimple circular motion of the Sunne, as that all its parts, even the leaſt, do perpetually change places with one another; which change of places is that which there I called Fermentation. From this Fermentation of the Aire, I have (in the 8 article of the laſt chapter) demonſtrated, that the water may be drawn up into the clouds.

And I ſhall now ſhew, that the fluid parts may in like manner by the ſame Fermentation be drawn out from the internall to the externall parts of our Bodies. For ſeeing that whereſoever the fluid Medium is contiguous to the Body of any living creature, there the parts of that Medium are by perpetuall change of place ſeparated from one another; the contiguous parts of the living creature muſt of neceſſity endeavour to enter into the ſpaces of the ſeparated parts. For otherwiſe thoſe parts (ſuppoſing there is no *Vacuum*) would have no place to go into. And therefore that which is moſt fluid and ſeparable in the parts of the living creature which are contiguous to the Medium, will go firſt out; and into the place thereof will ſucceed ſuch other parts as can moſt

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easily transpire through the pores of the skin. And from hence it is necessary, that the rest of the parts which are not separated, must all together be moved outwards, for the keeping of all places full. But this motion outwards of all parts together must of necessity press those parts of the ambient Aire which are ready to leave their places; and therefore all the parts of the Body endeavouring at once that way, makes the Body swell. Wherefore a possible cause is given of Heat from the Sunne; which was to be done.

4 We have now seen how Light and Heat are generated; Heat by the simple motion of the Medium, making the parts perpetually change places with one another; and Light by this, that by the same simple motion Action is propagated in a straight line. But when a Body hath its parts so moved, that it sensibly both Heats and Shines at the same time, then it is, that we say Fire is generated.

Now by Fire I do not understand a Body distinct from matter combustible or glowing, as Wood or Iron, but the matter it self, not simply and always, but then onely when it shineth and heateth. He therefore that renders a cause possible, and agreeable to the rest of the *Phænomena*, namely, whence and from what action both the *Shining* and *Heating* proceed, may be thought to have given a possible cause of the generation of *Fire*.

Let therefore A B C (in the first Figure) be a Sphere, or the portion of a Sphere, whose Center is D; and let it be transparent and homogeneous, as Cristal, Glass, or Water, and objected to the Sunne. Wherefore the foremost part A B C, will by the simple motion of the Sunne, by which it thrusts forwards the Medium, be wrought upon by the Sun-beams in the straight lines E A, F B and G C; which straight lines may in respect of the great distance of the Sunne be taken for parallels. And seeing the Medium within the Sphere is thicker then the Medium without it, those Beams will be refracted towards their perpendiculars. Let the straight lines E A and G C be produced till they cut the Sphere in H and I; and drawing the perpendiculars A D and C D, the refracted Beams E A and G C will of necessity fall, the one between A H and A D, the other between C I and C D. Let those refracted

Beams

Beams be A K and C L. And again, let the lines D K M & D L N be drawn perdendicular to the Sphere; and let A K and C L be produced till they meet with the ſtraight line B D produced in O. Seeing therefore the Medium within the Sphere is thicker then that without it, the refracted line A K will recede further from the perpendicular K M, then K O will recede from the ſame. Wherefore K O will fall between the refracted line and the perpendicular. Let therefore the refracted line be K P, cutting F O in P; and for the ſame reaſon the ſtraight line L P will be the refracted line of the ſtraight line C L. Wherfore, ſeeing the Beams are nothing elſe but the Wayes in which the motion is propagated, the motion about P will be ſo much more vehement then the motion about A B C, by how much the baſe of the portion A B C is greater then the baſe of a like portion in the Sphere whoſe Center is P, and whoſe magnitude is equal to that of the little Circle about P, which comprehendeth all the Beams that are propagated from A B C; and this Sphere being much leſs then the Sphere A B C, the parts of the Medium, that is, of the Aire about P, will change places with one another with much greater celerity, then thoſe about A B C. If therefore any matter Combuſtible, that is to ſay, ſuch as may be eaſily diſſipated, be placed in P, the parts of that matter (if the proportion be great enough between A C and a like portion of the little circle about P) wil be freed from their mutual cohæſion, and being ſeparated will acquire ſimple motion. But vehement ſimple motion generates in the beholder a Phantaſm of Lucid and Hot, as I have before demonſtrated of the ſimple motion of the Sunne; and therefore the combuſtible matter which is placed in P will be made Lucid and Hot, that is to ſay, will be Fire. Wherefore I have rendered a poſſible cauſe of Fire; which was to be done.

5 From the manner by which the Sunne generateth Fire, it is eaſy to explaine the manner by which Fire may be generated by the colliſion of two Flints. For by that Colliſion, ſome of thoſe particles of which the ſtone is compacted, are violently ſeparated and thrown off; and being withall ſwiftly turned round, the Eie is moved by them, as it is in the generation of Light by the Sunne. Wherefore they ſhine; and falling upon matter which

is

is already halfe diſſipated, ſuch as is Tinder, they throughly diſſipate the parts thereof,and make them turn round. From whence (as I have newly ſhewn) Light and Heat, that is to ſay, Fire is generated.

6 The ſhining of Glow-worms,ſome kinds of Rotten Wood, and of a kinde of ſtone made at *Bologna*, may have one common cauſe, namely the expoſing of them to the hot Sunne. We finde by experience that the *Bolonian* ſtone ſhines not unleſs it be ſo expoſed; and after it has been expoſed it ſhines but for a little time, namely, as long as it retains a certain degree of heat. And the cauſe may be, that the parts of which it is made,may together with heat have Simple Motion imprinted in them by the Sunne. Which if it be ſo,it is neceſſary,that it ſhine in the dark,as long as there is ſufficient heat in it; but this ceaſing, it will ſhine no longer. Alſo we find by expetience, that in the Glow-worm there is a certain thick humour,like theCriſtalline humour of the Eie;which if it be taken out, and held long enough in ones fingers,and then be carried into the dark, it will ſhine by reaſon of the warmth it received from the fingers; but as ſoon as it is cold, it will ceaſe ſhining. From whence therefore can theſe creatures have their Light, but from lying all day in the Sun-ſhine, in the hotteſt time of Summer? In the ſame manner, Rotten Wood, except it grow rotten in the Sunſhine, or be afterwards long enough expoſed to the Sunne, will not ſhine. That this doth not happen in every Worm, nor in all kinds of RottenWood,nor in all Calcined Stones,the cauſe may be, that the parts of which thoſe Bodies are made, are different both in motion and figure from the parts of Bodies of other kinds.

7 Alſo the Sea-water ſhineth when it is either daſhed with the ſtrokes of Oares, or when a Ship in its courſe breaks ſtrongly through it; but more or leſs according as the Winde blows from different points. The cauſe whereof may be this, that the particles of ſalt (though they never ſhine in the Salt-pits, where they are but ſlowly drawn up by the Sunne) being here beaten up into the aire in greater quantities, and with more force, are withall made to turn round, and conſequently to ſhine, though weakly. I have therefore given a poſſible cauſe of this *Phænomenon*.

8 If ſuch matter as is compounded of hard little Bodies, be ſet

on fire, it muſt needs be, that as they flye out in greater or leſſe quantities, the Flame which is made by them, will be greater or leſs. And if the æthereal or fluid part of that matter fly out together with them, their motion will be the Swifter, as it is in Wood, and other things which flame with a manifeſt mixture of Winde. When therefore theſe hard particles by their flying out, move the Eye ſtrongly, they ſhine bright; and a great quantity of them flying out together, they make a great ſhining Body. For Flame being nothing but an aggregate of ſhining particles, the greater the aggregate is, the greater and more manifeſt will be the Flame. I have therefore ſhewn a poſſible cauſe of Flame. And from hence the cauſe appears evidently, why Glaſs is ſo eaſily and quickly melted by the ſmall Flame of a candle blown, which will not be melted without blowing, but by a very ſtrong Fire.

Now if from the ſame matter, there be a part broken off (namely ſuch a part as conſiſteth of many of the ſmall particles), of this is made a Spark. For from the breaking off, it hath a violent turning round; and from hence it ſhines. But though from this matter, there fly neither Flame nor Sparks; yet ſome of the ſmalleſt parts of it may be carried out as farre as to the Superficies, and remain there, as Aſhes; the parts whereof are ſo extremely ſmall, that it cannot any longer be doubted how farre Nature may proceed in Dividing.

Laſtly, though by the application of fire to this matter, there fly little or nothing from it, yet there will be in the parts an endeavour to Simple motion, by which the whole Body will either be Melted, or (which is a degree of Melting) Softned. For all Motion has ſome effect upon all Matter whatſoever (as has been ſhewn at the 3d Article of the 16th Chapter). Now if it be ſoftned to ſuch a degree, as that the ſtubborneſs of the parts be exceeded by their gravity, then we ſay it is Melted; otherwiſe, Softned, and made Pliant and *Ductile*.

Again, the matter having in it ſome particles hard, others æthereal or watery; if by the application of fire theſe later be called out, the former will thereby come to a more full contact with one another; and conſequently will not be ſo eaſily ſeparated; that is to ſay, the whole Body will be made Harder. And this may be

be the cauſe why the ſame Fire makes ſome things Soft, others Hard.

9 It is known by experience, that if Hay be laid wet together in a heap, it will after a time begin to ſmoke, and then burn as it were of it ſelf. The cauſe whereof ſeems to be this, that in the Aire which is encloſed within the Hay, there are thoſe little Bodies, which (as I have ſuppoſed) are moved freely with ſimple Motion. But this Motion being by degrees hindred more and more by the deſcending moiſture, which at the laſt fils and ſtops all the paſſages, the thinner parts of the Aire aſcend by penetrating the water; and thoſe hard little Bodies being ſo thruſt together that they touch and preſs one another, acquire ſtronger motion, till at laſt by the increaſed ſtrength of this motion the watery parts are firſt driven outwards, from whence appears Vapour; and by the continued increaſe of this motion, the ſmalleſt particles of the dryed Hay are forced out, and recovering their natural ſimple Motion, they grow Hot and Shine, that is to ſay, they are ſet on Fire.

The ſame alſo may be the cauſe of Lightning; which happens in the hotteſt time of the yeare, when the water is raiſed up in greateſt quantity, and carried higheſt. For after the firſt Clouds are raiſed, others after others follow them; and being congeled above, they happen (whileſt ſome of them aſcend and others deſcend) to fall upon another in ſuch manner, as that in ſome places all their parts are joyned together, in others they leave hollow Spaces between them; aud into theſe ſpaces (the æthereall parts being forced out by the compreſſure of the Clouds) many of the harder little Bodies are ſo pent together, as that they have not the liberty of ſuch motion as is naturall to the Aire. Wherefore their endeavour growes more vehement, till at laſt they force their way through the Clouds, ſometimes in one place, ſometimes in another; and breaking through with great noiſe, they move the aire violently, & ſtriking our Eies generate Light, that is to ſay, they Shine. And this Shining is that we call Lightning.

10 The moſt common *Phænomenon* proceeding from Fire, and yet the moſt admirable of all others, is the force of Gunpowder fired; which being compounded of Niter, Brimſtone and Coles beaten ſmall, hath from the Coles its firſt taking fire; from the

Brimſtone its nouriſhment and flame, that is to ſay, Light and motion; and from the Niter the vehemence of both. Now if a piece of Niter, (before it is beaten) be laid upon a burning Cole, firſt it melts, and like water quencheth that part of the Cole it toucheth. Then Vapor or Aire flying out where the Cole and Niter joyne, bloweth the Cole with great ſwiftneſſe and vehemence on all ſides. And from hence it comes to paſſe, that by two contrary motions (the one, of the particles which go out of the burning Cole, the other, of thoſe of the æthereall and watery ſubſtance of the Niter) is generated that vehement Motion and Inflammation. And laſtly, when there is no more action from the Niter (that is to ſay, when the volatile parts of the Niter are flown out) there is found about the ſides a certain white ſubſtance, which being thrown again into the fire, will grow red hot again, but will not be diſſipated, at leaſt, unleſſe the fire be augmented. If now a poſſible cauſe of this be found out, the ſame will alſo be a poſſible cauſe why a grain of Gunpowder ſet on fire doth expand it ſelfe with ſuch vehement motion, and Shine. And it may be cauſed in this manner.

Let the particles of which Niter conſiſteth, be ſuppoſed to be ſome of them hard, others watery, and the reſt æthereall. Alſo let the hard particles be ſuppoſed to be ſpherically hollow, like ſmall bubbles, ſo that many of them growing together may conſtitute a Body whoſe little cavernes are filled with a ſubſtance which is either watery, or æthereal, or both. As ſoon therefore as the hard particles are diſſipated, the watery and æthereal particles will neceſſarily fly out; and as they fly, of neceſſity blow ſtrongly the burning Coles and Brimſtone which are mingled together; whereupon there will follow a great expanſion of Light, with vehement flame, and a violent diſſipation of the particles of the Niter, the Brimſtone and the Coles. Wherefore I have given a poſſible cauſe of the force of fired Gunpowder.

It is manifeſt from hence, that for the rendering of the cauſe why a bullet of lead or iron ſhot from a peece of Ordnance flies with ſo great velocity, there is no neceſſity to introduce ſuch Rarefaction, as (by the common definition of it) makes the ſame Matter to have ſometimes more, ſometimes leſſe Quantity; which is un-

unconceiveable. For every thing is ſaid to be greater or leſſe, as it hath more or leſſe Quantity. The violence with which a bullet is thruſt out of a Gun, proceeds from the ſwiftneſſe of the ſmall particles of the fired Powder; at leaſt it may proceed from that cauſe, without the ſuppoſition of any Empty Space.

11 Beſides, by the attrition or rubbing of one Body againſt another, as of Wood againſt Wood, we find that not only a certaine degree of Heat, but Fire it ſelfe is ſometimes generated. For ſuch motion, is the reciprocation of preſſure, ſometimes one way ſometimes the other; and by this reciprocation, whatſoever is fluid in both the peeces of Wood, is forced hither and thither; and conſequently, to an endeavour of getting out; and at laſt by breaking out makes Fire.

12 Now Light is diſtinguiſhedinto, Firſt, Second, Third, and ſo on infinitely. And we call that Firſt Light, which is in the firſt Lucid Bodie; as the Sunne, Fire, &c. Second, that which is in ſuch Bodies as being not tranſparent are illuminated by the Sunne; as the Moon, a Wall, &c. and Third, that which is in Bodies not tranſparent but illuminated by Second Light, &c.

13 *Colour* is Light, but troubled Light; namely, ſuch as is generated by perturbed motion; as ſhall be made manifeſt by the Red, Yellow, Blew and Purple which are generated by the interpoſition of a *Diaphanous Priſma* (whoſe oppoſite baſes are triangular) between the Light and that which is enlightened.

For let there be a Priſma of Glaſſe, or of any other tranſparent matter which is of greater denſity then Aire; and let the triangle A B C be the baſe of this *Priſma*. Alſo let the ſtraight line D E be the diameter of the Sunnes Body, having oblique poſition to the ſtraight line A B; and let the Sunne-beames paſſe in the lines D A and E B C. And laſtly let the ſtraight lines D A and E C be produced indefinitely to F and G. Seeing therefore the ſtraight line D A, by reaſon of the denſity of the Glaſſe is refracted towards the perpendicular; let the line refracted at the point A be the ſtraight line A H. And againe, ſeeing the Medium below A C is thinner then that above it, the other refraction which will be made there, will diverge from the perpendicular. Let therefore this ſecond refracted line be A I. Alſo let the ſame

be

be done at the point C, by making the firſt refracted line to be C K, and the ſecond C L. Seeing therefore the cauſe of the refraction in the point A of the ſtraight line of A B, is the exceſs of the reſiſtance of the Medium in A B above the reſiſtance of the Aire, there muſt of neceſſity be reaction from the point A towards the point B; and conſequently the Medium at A within the triangle A B C, will have its motion troubled; that is to ſay, the ſtraight motion in A F and A H, will be mixed with the tranſverſe motion between the ſame A F and A H, repreſented by the ſhort tranſverſe lines in the triangle A F H. Againe, ſeeing at the point A of the ſtraight line A C, there is a ſecond refraction from A H in A I, the motion of the Medium will againe be perturbed by reaſon of the tranſverſe reaction from A towards C, repreſented likewiſe by the ſhort tranſverſe lines in the triangle A H I. And in the ſame manner there is a double perturbation repreſented by the tranſverſe lines in the triangles C G K and C K L. But as for the light between A I and C G, it will not be perturbed; becauſe if there were in all the points of the ſtraight lines A B and A C, the ſame action which is in the points A and C, then the plaine of the triangle C G K would be every where coincident with the plaine of the triangle A F H; by which meanes all would appear alike between A and C. Beſides, it is to be obſerved, that all the reaction at A, tends towards the illuminated parts which are between A and C, and conſequently perturbeth the Firſt Light. And on the contrary, that all the reaction at C tends towards the parts without the triangle, or without the *Priſma* A B C, where there is none but Second Light; and that the triangle A F H ſhewes that perturbation of Light which is made in the Glaſſe it ſelfe; as the triangle A H I ſhewes that perturbation of Light which is made below the Glaſſe. In like manner, that C G K ſhewes the perturbation of Light within the Glaſſe; and C K L that which is below the Glaſſe. From whence there are four divers motions, or four different illuminations or Cólours; whoſe differences appear moſt manifeſtly to the Senſe in a *Priſma* (whoſe baſe is an equilaterall triangle) when the Sunne-beames that paſſe through it are received upon a white paper. For the triangle A F H appears Red to the Senſe; the triangle A H I Yellow; the tri-

triangle C G K Green, and approaching to Blew; and laſtly the triangle C K L appears Purple. It is therefore evident, that when weak but Firſt Light paſſeth through a more reſiſting diaphanous Body, as Glaſſe, the beames which fall upon it tranverſly, make Redneſſe; and when the ſame Firſt Light is ſtronger, as it is in the thinner Medium below the ſtraight line A C, the tranſverſe beames make Yellowneſſe. Alſo when Second Light is ſtrong, as it is in the triangle C G K (which is neereſt to the Firſt Light) the tranſverſe beames make Greeneſſe; and when the ſame Second Light is weaker, as in the triangle C K L, they make a Purple colour.

14 From hence may be deduced a cauſe why the Moon and Starres appear bigger and redder neer the Horizon then in the Mid-heaven. For between the Eie and the apparent Horizon, there is more impure aire, ſuch as is mingled with Watery and Earthy little Bodies, then is between the ſame Eie and the more elevated part of Heaven. But Viſion is made by Beames which conſtitute a Cone, whoſe baſe, if we look upon the Moon, is the Moons Face, and whoſe vertex is in the Eie; and therefore many beams from the Moon muſt needs fall upon little Bodies that are without the Viſual Cone, and be by them reflected to the Eie. But theſe reflected beams tend all in lines which are tranſverſe to the Viſual Cone, and make at the Eie an angle which is greater then the angle of the Cone. Wherefore the Moon appeares greater in the Horizon, then when ſhe is more elevated. And becauſe thoſe reflected beames go tranſverſely, there will be generated (by the laſt article) Redneſſe. A poſſible cauſe therefore is ſhewne, why the Moon, as alſo the Starres appear Greater and Redder in the Horizon, then in the midſt of heaven. The ſame alſo may be the cauſe why the Sunne appears in the Horizon, Greater, and of a colour more degenerating to Yellow, then when he is higher elevated. For the reflection from the little Bodies between, and the tranſverſe motion of the Medium are ſtill the ſame. But the Light of the Sunne is much ſtronger then that of the Moon; and therefore (by the laſt article) his Splendor muſt needs by this perturbation degenerate into Yellowneſſe.

But for the generation of theſe four colours, it is not neceſſary that

that the figure of the Glaſs be a *Priſma*; for if it were Spherical it would doe the ſame. For in a Sphere the Sunne-beames are twice refracted and twice reflected. And this being obſerved by *Des Cartes*; and with all that a Rainebow never appeares but when it rains; as alſo, that the drops of raine have their figures almoſt Spherical; he hath ſhewne from thence the cauſe of the colours in the Rainbow; which therefore need not be repeated.

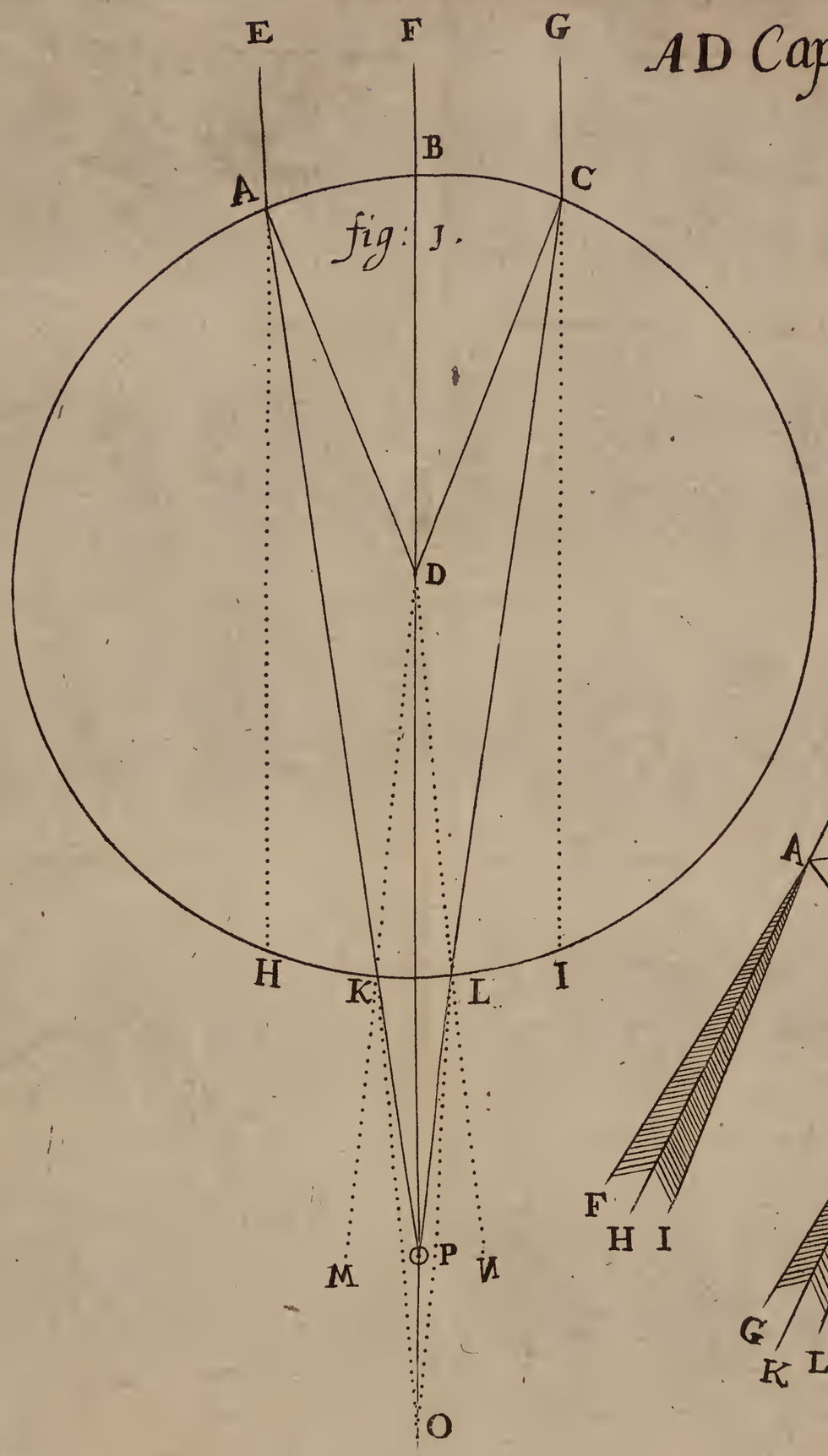
15 *Whiteneſs* is Light; but Light perturbed by the reflexions of many beams of Light comming to the Eye together within a little ſpace. For if Glaſs, or any other *Diaphanous* Body be reduced to very ſmall parts by contuſion or concuſſion, every one of thoſe parts (if the Beams of a lucid Body be from any one point of the ſame reflected to the Eye) will repreſent to the beholder an Idea or Image of the whole lucid Body, that is to ſay, a Phantaſme of White. For the ſtrongeſt Light is the moſt White; and therefore many ſuch parts will make many ſuch Images. Wherefore if thoſe parts lie thick and cloſe together, thoſe many Images will appear confuſedly, and will by reaſon of the confuſed Light repreſent a White Colour. So that from hence may be deduced a poſſible cauſe, why Glaſs beaten, that is, reduced to powder, looks White. Alſo why Water and Snow are White; they being nothing but a heap of very ſmall *Diaphanous* Bodies, namely, of little Bubbles, from whoſe ſeveral convex Superficies, there are by reflexion made ſeveral confuſed Phantaſmes of the whole lucid Body; that is to ſay, *Whiteneſs*. For the ſame reaſon, Salt and Nitre are White; as conſiſting of ſmall Bubbles which contain within them Water and Aire; as is manifeſt in Nitre, from this, that being thrown into the fire, it violently blowes the ſame; which Salt alſo doth, but with leſs violence. But if a White Body be expoſed, not to the Light of the Day, but to that of the Fire, or of a Candle, it will not at the firſt ſight be eaſily judged whether it be White or Yellow; the cauſe whereof may be this, that the light of thoſe things which burn and flame, is almoſt of a middle Colour between Whiteneſs and Yellowneſs.

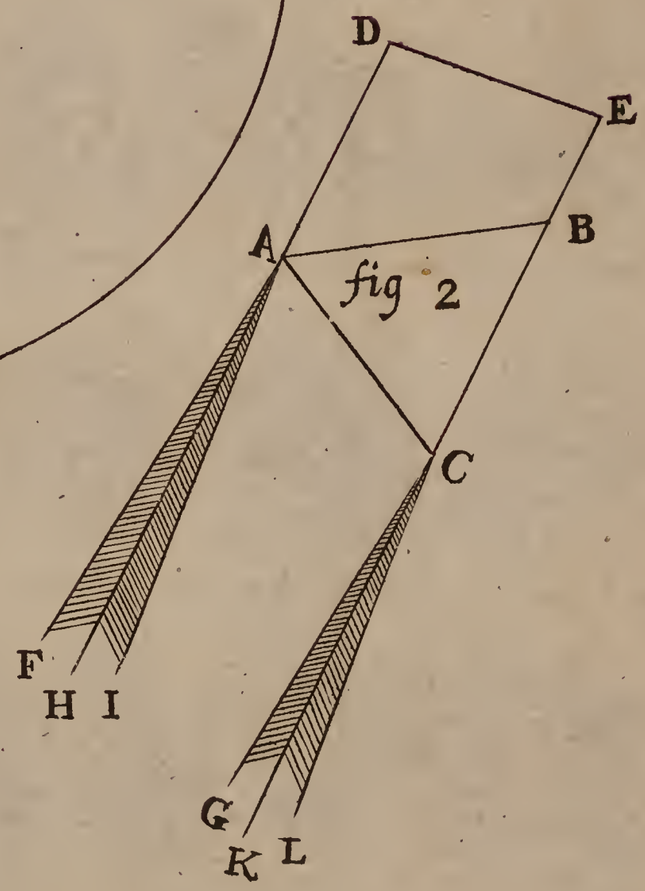
16 As Whiteneſs is Light, ſo *Blackneſs* is the privation of Light, or Darkneſs. And from hence it is; Firſt, that all Holes, from which no light can be reflected to the Eie, appear Black.

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Secondly, that when a Body hath little eminent particles erected ſtraight up from the Superficies(ſo that the Beams of Light which fall upon them are reflected, not to the Eie, but to the Body it ſelf) that Superficies appears Black, in the ſame manner as the Sea appears Black, when ruffled by the Wind. Thirdly, that any combuſtible matter is by the fire made to look Black before it ſhines. For the endeavour of the fire being to diſſipate the ſmalleſt parts of ſuch Bodies as are thrown into it, it muſt firſt raiſe and erect thoſe parts, before it can work their diſſipation. If therefore the fire be put out before the parts be totally diſſipated, the Cole will appear Black; for the parts being onely erected, the Beams of Light falling upon them will not be reflected to the Eie, but to the Cole it ſelf. Fourthly, that Burning Glaſſes do more eaſily burn Black things then White. For in a White Superficies, the eminent parts are convex, like little bubbles;and therefore the Beams of Light which fall upon them are reflected every way from the reflecting Body. But in a Black Superficies, where the eminent particles are more erected,the Beams of Light falling upon them, are all neceſſarily reflected towards the Body it ſelf; and therefore Bodies that are Black are more eaſily ſet on fire by the Sun-beams,then thoſe that areWhite.Fifthly,that all Colours that are made of the mixture of White and Black, proceed from the different poſition of the particles that riſe above the Superficies, and their different forms of aſperity. For according to theſe differences, more or fewer Beams of Light are reflected from ſeveral Bodies to the Eie.But in regard thoſe differences are innumerable, and the Bodies themſelves ſo ſmall,that we cannot perceive them, the explication and preciſe determination of the Cauſes of all Colours is a thing of ſo great difficulty, that I dare not undertake it.

AD Cap XXVII





CHAP. XXVIII.

Of Cold, Wind, Hard, Ice, Restitution *of* *Bodies bent*, Diaphanous, Lightning *and* Thunder; *and of the* Heads *of Rivers.*

1 *Why Breath from the same mouth sometimes heats, and sometimes cools.* 2 *Wind, and the Inconstancy of Winds, whence.* 3 *Why there is a constant, though not a great Wind from East to West neer the Equator.* 4 *What is the effect of Aire pent in between the Clouds.* 5 *No change from Soft to Hard, but by motion.* 6 *What is the cause of Cold neer the Poles.* 7 *The cause of Ice; and why the Cold is more remiss in rainy then in clear weather. Why water doth not freeze in deep Wells, as it doth neer the Superficies of the Earth. Why Ice is not so heavy as Water; and why Wine is not so easily frozen as Water.* 8 *Another cause of Hardness from the fuller contact of Atomes. Also how Hard things are broken.* 9 *A third cause of Hardness from Heat.* 10 *A fourth cause of Hardness from the motion of Atomes enclosed in a narrow space.* 11 *How Hard things are Softned.* 12 *Whence proceeds the spontaneous Restitution of things Bent.* 13 *Diaphanous, and Opacous, what they are, and whence.* 14 *The cause of Lightning and Thunder.* 15 *Whence it proceeds that Clouds can fall again, after they are once elevated and frozen.* 16 *How it could be that the Moon was eclipsed, when she was not diametrally opposite to the Sunne.* 17 *By what means many Sunnes may appear at once.* 18 *Of the Heads of Rivers.*

1 AS, when the motion of the ambient æthereal substance makes the Spirits and fluid parts of our Bodies tend outwards, we acknowledge Heat; so, by the endeavour inwards of the same spirits and humours, we feel Cold. So that to *Cool*, is to make the exterior parts of the Body endeavour inwards, by a motion contrary to that of *Calefaction*, by which the internal parts are called out-

wards. He therefore that would know the cauſe of Cold muſt find by what motion or motions, the exterior parts of any Body endeavour to retire inwards. To begin with thoſe *Phænomena* which are the moſt familiar; There is almoſt no man but knows, that breath blown ſtrongly, and which comes from the mouth with violence, that is to ſay, the paſſage being ſtraight, will Cool the hand; and that the ſame breath blown gently, that is to ſay, through a greater aperture, wil warm the ſame. The cauſe of which *Phænomenon* may be this. The breath going out, hath two motiõs; the one, of the whole and direct, by which the formoſt parts of the hand are driven inwards; the other, ſimple motion of the ſmall particles of the ſame breath, which (as I have ſhewn in the 3d Article of the laſt Chapter) cauſeth Heat. According therefore as either of theſe Motions is predominant, ſo there is the ſenſe ſometimes of Cold, ſometimes of Heat. Wherefore, when the breath is ſoftly breathed out at a large paſſage, that ſimple Motion which cauſeth Heat prevaileth, and conſequently Heat is felt; and when by compreſſing the lips the breath is more ſtrongly blown out, then is the direct motion prevalent, which makes us feel Cold. For the direct motion of the breath or aire, is Wind; and all Wind Cools, or diminiſheth former heat.

2 And ſeeing not onely great Wind, but almoſt any Ventilation and ſtirring of the Aire, doth refrigerate; the reaſon of many experiments concerning Cold cannot well be given, without finding firſt what are the cauſes of Wind. Now Wind is nothing elſe but the direct motion of the Aire thruſt forwards; which nevertheleſs, when many Winds concurre may be circular, or otherwiſe indirect, as it is in Whirle-winds. Wherefore in the firſt place we are to enquire into the Cauſes of Winds. *Wind* is Aire moved in a conſiderable quantity, and that either in the manner of Waves, which is both forwards & alſo up & down; or elſe forwards onely.

Suppoſing therefore the Aire both cleer and calm, for any time how little ſoever; yet the greater Bodies of the World, being ſo diſpoſed and ordered as has been ſaid, it will be neceſſary that a Wind preſently ariſe ſome where. For ſeeing that motion of the parts of the Aire which is made by the Simple Motion of the Sunne in his own Epicycle, cauſeth an exhalation of the particles

of

of water from the Seas and all other moiſt Bodies, and thoſe particles make Clouds; it muſt needs follow, that whileſt the particles of water paſs upwards, the particles of Aire (for the keeping of all Spaces full) be juſtled out on every ſide, and urge the next particles, and theſe the next, till having made their circuit, there comes continually ſo much Aire to the hinder parts of the Earth, as there went water from before it. Wherefore the aſcending Vapours move the Aire towards the ſides every way; and all direct motion of the Aire being Wind, they make a Wind. And if this Wind meet often with other Vapours which ariſe in other places, it is manifeſt that the force thereof will be augmented, & the way or courſe of it changed. Beſides, according as the Earth by its diurnal motion turns ſometimes the drier, ſometimes the moiſter part towards the Sunne, ſo ſometimes a greater, ſometimes a leſs quantity of Vapours will be raiſed, that is to ſay, ſometimes there will be a leſs, ſometimes a greater Wind. Wherefore I have rendred a poſſible cauſe of ſuch Winds, as are generated by Vapours; and alſo of their Inconſtancy.

From hence it follows, that theſe Winds cannot be made in any place which is higher then that to which Vapours may aſcend. Nor is that incredible which is reported of the higheſt Mountains, as the *Pique of Tenariffe* and the *Andes of Peru*, namely, that they are not at all troubled with theſe inconſtant Winds. And if it were certain, that neither Rain nor Snow were ever ſeen in the higheſt tops of thoſe Mountains, it could not be doubted but that they are higher then any place to which Vapours uſe to aſcend.

3. Nevertheleſs, there may be Wind there, though not that which is made by the aſcent of Vapours, yet a leſs & more conſtant Wind (like the continued blaſt of a pair of bellows) blowing from the Eaſt. And this may have a double cauſe; the one, the diurnal motion of the Earth; the other, its ſimple motion in its own Epicycle. For theſe Mountains being (by reaſon of their height) more eminent then all the reſt of the parts of the Earth, do by both theſe Motions drive the Aire from the Weſt Eaſtwards. To which though the diurnal Motion contribute but little; yet ſeeing I have ſuppoſed that the ſimple Motion of the Earth in its own Epicycle, makes two revolutions in the ſame time in which the diurnal Motion

tion makes but one; and that the Semidiameter of the Epicycle is double to the Semidiameter of the diurnal Converſion, the Motion of every point of the Earth in its own Epicycle will have its velocity quadruple to that of the diurnal Motion; ſo that by both theſe Motions together, the tops of thoſe Hils will ſenſibly be moved againſt the Aire; and conſequently a Wind will be felt. For whether the Air ſtrike the Sentient, or the Sentient the Air, the perception of Motion will be the ſame. But this Wind, ſeeing it is not cauſed by the aſcent of Vapours, muſt neceſſarily be very Conſtant.

4 When one Cloud is already aſcended into the Aire, if another Cloud aſcend towards it, that part of the Aire which is intercepted between them both, muſt of neceſſity be preſſed out every way. Alſo when both of them, whileſt the one aſcends, and the other either ſtayes, or deſcends, come to be joyned in ſuch manner as that the æthereal ſubſtance be ſhut within them on every ſide, it will by this compreſſion alſo go out by penetrating the Water. But in the mean time, the hard particles which are mingled with the Aire, and are agitated (as I have ſuppoſed) with Simple Motion, wil not paſs through the water of the clouds, but be more ſtraightly compreſſed within their cavities. And this I have demonſtrated at the 4th and 5th Articles of the 22th Chapter. Beſides, ſeeing the Globe of the Earth floateth in the Aire which is agitated by the Sunnes Motion, the parts of the Aire reſiſted by the Earth, will ſpread themſelves every way upon the Earths Superficies; as I have ſhewn at the 8th Article of the 21th Chapter.

5 We perceive a Body to be Hard, from this, that when touching it we would thruſt forwards that part of the ſame which we touch, we cannot do it otherwiſe then by thruſting forwards the whole Body. We may indeed eaſily and ſenſibly thruſt forwards any particle of the Aire or Water which we touch, whilſt yet the reſt of its parts remain (to ſenſe) unmoved. But we cannot do ſo to any part of a ſtone. Wherfore I define a *Hard* Body to be that, whereof no part can be ſenſibly moved, unleſs the whole be moved. Whatſoever therefore is Soft or Fluid, the ſame can never be made Hard but by ſuch motion, as makes many of the parts together ſtop the motion of ſome one part, by reſiſting the ſame.

6 Theſe things premiſed, I ſhall ſhew a poſſible cauſe why there

is greater Cold neer the Poles of the Earth, then further from them. The motion of the Sunne between the Tropicks, driving the Aire towards that part of the Earths Superficies which is perpendicularly under it, makes it spread it self every way; and the velocity of this expansion of the Aire grows greater and greater, as the Superficies of the Earth comes to be more and more straightned, that is to say, as the Circles which are parallel to the Æquator come to be less and less. Wherefore this expansive motion of the Aire, drives before it the parts of the Aire which are in its way continually towards the Poles more and more strongly, as its force comes to be more and more united, that is to say, as the Circles which are parallel to the Æquator are less and less; that is, so much the more, by how much they are neerer to the Poles of the Earth. In those places therefore which are neerer to the Poles, there is greater Cold, then in those which are more remote from them. Now this expansion of the Aire upon the Superficies of the Earth from East to West, doth by reason of the Sunnes perpetual accession to the places which are successively under it, make it Cold at the time of the Sunnes Rising and Setting; but as the Sunne comes to be continually more and more perpendicular to those cooled places, so by the Heat which is generated by the supervening Simple Motion of the Sunn, that Cold is again remitted; and can never be great, because the action by which it was generated, is not permanent. Wherefore I have rendred a possible cause of Cold in those places that are neer the Poles, or where the obliquity of the Sunne is great.

7 How Water may be congeled by Cold, may be explained in this manner. Let A (in the first figure) represent the Sunne, and B the Earth. A will therefore be much greater then B. Let E F be in the plain of the Equinoctial; to which let G H, I K and L C be parallel. Lastly, let C and D be the Poles of the Earth. The Aire therefore by its action in those parallels will rake the Superficies of the Earth; and that with motion so much the stronger, by how much the parallel Circles towards the Poles grow less and less. From whence must arise a Wind, which will force together the uppermost parts of the water, and withal raise them a little, weakning their endeavour towards the Center of the Earth. And from

their

their endeavour towards the center of the Earth, joyned with the endeavour of the ſaid Wind, the uppermoſt parts of the water will be preſſed together, and coagulated, that is to ſay, the top of the water will be skinned over and hardned. And ſo againe the Water next the top will be hardned in the ſame manner, till at length the Ice be thick. And this Ice being now compacted of little hard Bodies, muſt alſo containe many particles of ayre received into it.

As Rivers and Seas, ſo alſo in the ſame manner may the Clouds be frozen. For when by the aſcending and deſcending of ſeverall Clouds at the ſame time, the Air intercepted between them is by compreſſion forced out, it rakes, & by little & little hardens them. And though thoſe ſmal drops (which uſually make Clouds) be not yet united into greater Bodies, yet the ſame Wind will be made; & by it, as water is congeled into Ice, ſo will Vapours in the ſame manner be congeled into Snow. From the ſame cauſe it is that Ice may be made by art, and that not farre from the fire. For it is done by the mingling of Snow and Salt together, and by burying in it a ſmall veſſell full of Water. Now while the Snow and Salt (which have in them a great deale of aire) are melting, the aire which is preſſed out every way in Wind, rakes the ſides of the Veſſel; and as the Wind by its motion rakes the Veſſell, ſo the Veſſell by the ſame motion and action congeles the Water within it.

We find by experience, that Cold is allwayes more Remiſſe in places where it raynes, or where the weather is cloudy (things being alike in all other reſpects) then where the aire is cleare. And this agreeth very well with what I have ſayd before. For in cleare weather, the courſe of the Wind which (as I ſayd even now) rakes the Superficies of the Earth, as it is free from all interruption, ſo alſo it is very ſtrong. But when ſmall drops of water are either riſing or falling, that Wind is repelled, broken and diſſipated by them; and the leſſe the Wind is, the leſſe is the Cold.

We find alſo by experience, that in deep Wells the Water freezeth not ſo much, as it doth upon the Superficies of the Earth. For the Wind by which Ice is made, entring into the Earth (by reaſon of the laxity of its parts) more or leſſe, loſeth ſome of its force,

force, though not much. So that if the Well be not deep, it will freeze; whereas if it be ſo deep, as that the Wind which cauſeth cold cannot reach it, it will not freeze.

We find moreover by experience, that Ice is lighter then Water. The cauſe whereof is manifeſt from that which I have already ſhewn, namely, that Aire is received in and mingled with the particles of the Water whileſt it is in congeling.

8 We have ſeen one way of making things Hard, namely, by Congelation. Another way is thus. Having already ſuppoſed, that innumerable Atomes, ſome harder then others, and that have ſeveral ſimple motions of their own, are intermingled with the æthereal ſubſtance; it follows neceſſarily from hence, that by reaſon of the fermentation of the whole Aire (of which I have ſpoken in the 21 Chap.) ſome of thoſe Atomes meeting with others, will cleave together, by applying themſelvs to one another in ſuch manner as is agreeable to their motions and mutual contacts; and (ſeeing there is no *Vacuum*) cannot be pulled aſunder, but by ſo much force as is ſufficient to overcome their Hardneſs.

Now there are innnmerable degrees of Hardneſs. As (for example) there is a degree of it in Water, as is manifeſt from this, that upon a plain it may be drawn any way at pleaſure by ones finger. There is a greater degree of it in clammy liquors; which when they are poured out, doe in falling downwards diſpoſe themſelves into one continued thred; which thred before it be broken will by little and little diminiſh its thickneſs, till at laſt it be ſo ſmall, as that it ſeems to break onely in a point; and in their ſeparation the external parts break firſt from one another, and then the more internal parts ſucceſſively one after another. In Wax there is yet a greater degree of Hardneſs. For when we would pull one part of it from another, we firſt make the whole maſs ſlenderer, before we can pull it aſunder. And how much the harder any thing is which we would break, ſo much the more force we muſt apply to it. Wherefore, if we go on to harder things, as Ropes, Wood, Metals, Stones, &c. reaſon prompteth us to believe that the ſame (though not alwayes ſenſibly) will neceſſarily happen; and that even the hardeſt things are broken aſunder in the ſame manner, namely, by Solution of their continuity, begun in the outermoſt Superficies,

and proceeding ſucceſſively to the innermoſt parts. In like manner, when the parts of Bodies are to be ſeparated, not by pulling them aſunder, but by breaking them, the firſt ſeparation will neceſſarily be in the convex Superficies of the bowed part of the Body, and afterwards in the concave Superficies. For in all bowing, there is in the convex Superficies an endeavour in the parts to go one from another, and in the concave Superficies to penetrate one another.

This being well underſtood, a reaſon may be given, how two Bodies which are contiguous in one common Superficies, may by force be ſeparated without the introduction of *Vacuum*; though *Lucretius* thought otherwiſe, believing that ſuch ſeparation was a ſtrong eſtabliſhment of *Vacuum*. For a Marble Pillar being made to hang by one of its baſes, if it be long enough it will by its own weight be broken aſunder, and yet it will not neceſſarily follow that there ſhould be *Vacuum*, ſeeing the ſolution of its continuity may begin in the Circumference, and proceed ſucceſſively to the midſt thereof.

Laſtly, Wine is not ſo eaſily congeled as Water, becauſe in Wine there are particles which being not fluid, are moved very ſwiftly, and by their motion congelation is retarded; but if the Cold prevail againſt this motion, then the outermoſt parts of the Wine will be firſt frozen, and afterwards the inner parts; whereof this is a ſigne, that the Wine which remains unfrozen in the midſt will be very ſtrong.

9 Another cauſe of Hardneſs in ſome things may be in this manner. If a ſoft Body conſiſt of many hard particles, which by the intermixture of many other fluid particles cohære but looſely together, thoſe fluid parts (as hath been ſhewn in the laſt Article of the 21 Chapter) will be exhaled; by which means each hard particle will apply it ſelf to the next to it according to a greater Superficies; and conſequently they will cohære more cloſely to one another; that is to ſay, the whole maſs will be made Harder.

10 Again, in ſome things Hardneſs may be made to a certain degree, in this manner. When any fluid ſubſtance hath in it certain very ſmall Bodies intermingled, which being moved with ſimple motion of their own, contribute like motion to the parts of

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fluid ſubſtance, and this be done in a ſmall encloſed ſpace (as in the hollow of a little Sphere, or a very ſlender Pipe) if the motion be vehement, and there be a great nnmber of theſe ſmall encloſed Bodies, two things will happen; the one, that the fluid ſubſtance will have an endeavour of dilating it ſelf at once every way; the other, that if thoſe ſmal Bodies can no where get out, then from their reflexion it will follow, that the motion of the parts of the encloſed fluid ſubſtance, which was vehement before, will now be much more vehement. Wherefore if any one particle of that fluid ſubſtance ſhould be touched & preſſed by ſome external Movent, it could not yeild but by the application of very ſenſible force. Wherefore the fluid ſubſtance which is encloſed, and ſo moved, hath ſome degree of Hardneſs. Now greater and leſs degree of Hardneſs depends upon the quantity and velocity of thoſe ſmall Bodies and upon the narrowneſs of the place both together.

11 Such things as are made Hard by ſudden heat, namely, ſuch as are hardned by fire, are commonly reduced to their former ſoft form by Maceration. For fire hardens by Evaporation, and therefore if the evaporated moiſture be reſtored again, the former nature and form is reſtored together with it. And ſuch things as are frozen with Cold, if the Wind by which they were frozen change into the oppoſite quarter, they will be unfrozen again (unleſs they have gotten a habit of new motion or endeavour by long continuance in that hardneſs). Nor is it enough to cauſe thawing, that there be a ceſſation of the freezing Wind (for the taking away of the Cauſe doth not deſtroy a produced effect); but the thawing alſo muſt have its proper cauſe, namely, a contrary Wind, or at leaſt a Wind oppoſite in ſome degree. And this we finde to be true by experience. For if Ice be laid in a place ſo well encloſed that the motion of the Aire cannot get to it, that Ice will remain unchanged, though the place be not ſenſibly cold.

12 Of Hard Bodies, ſome may manifeſtly be bowed; others not, but are broken in the very firſt moment of their bending. And of ſuch Bodies as may manifeſtly be bended, ſome being bent, do as ſoon as ever they are ſet at liberty, Reſtore themſelves to their former poſture; others remain ſtill bent. Now if the cauſe of this Reſtitution be asked, I ſay it may be in this manner; namely, that the particles of the bended Body, whileſt it is held bent, do neverthe-

leſs retain their motion; and by this motion they reſtore it as ſoon as the force is removed by which it was bent. For when any thing is bent (as a plate of ſteel), and as ſoon as the force is removed reſtores it ſelf again, it is evident that the cauſe of its reſtitution cannot be referred to the ambient aire; nor can it be referred to the removal of the force by which it was bent; for in things that are at reſt, the taking away of impediments, is not a ſufficient cauſe of their future Motion; there being no other cauſe of Motion, but Motion. The cauſe therefore of ſuch Reſtitution is in the parts of the Steel it ſelf. Wherefore whileſt it remains bent, there is in the parts of which it conſiſteth, ſome motion, though inviſible, that is to ſay, ſome endeavour at leaſt that way by which the reſtitution is to be made; and therefore this endeavour of all the parts together is the firſt beginning of Reſtitution; ſo that the impediment being removed, that is to ſay, the force by which it was held bent, it will be reſtored again. Now the motion of the parts by which this is done, is that which I called Simple Motion, or Motion returning into it ſelf. When therefore in the bending of a plate, the ends are drawn together, there is on one ſide a mutual compreſſion of the parts; which compreſſion is one endeavour oppoſite to another endeavour; and on the other ſide a divulſion of the parts. The endeavour therefore of the parts on one ſide tends to the reſtitution of the plate from the middle towards the ends; and on the other ſide, from the ends towards the middle. Wherefore the impediment being taken away, this endeavour (which is the beginning of reſtitution) will reſtore the plate to its former poſture. And thus I have given a poſſible cauſe why ſome Bodies when they are bent Reſtore themſelves again; which was to be done.

As for Stones, ſeeing they are made by the accretion of many very hard particles within the Earth; which particles have no great coherence, that is to ſay, touch one another in ſmall latitude, and conſequently admit many particles of aire, it muſt needs be that in bending of them, their internal parts will not eaſily be compreſſed by reaſon of their hardneſs. And becauſe their coherence is not firm, as ſoon as the external hard particles are disjoyned, the æthereal parts will neceſſarily break out, and ſo the Body will ſuddenly be broken.

13 Thoſe Bodies are called *Diaphanous*, upon which whileſt the Beams of a lucid Body do work, the action of every one of thoſe Beams is propagated in them in ſuch manner as that they ſtill retain the ſame order amongſt themſelves, or the inverſion of that order; and therefore Bodies which are perfectly Diaphanous, are alſo perfectly homogeneous. On the contrary, an *Opacous* Body is that, which by reaſon of its heterogeneous nature, doth by innumerable reflexions and refractions in particles of different figures and unequal hardneſs, weaken the Beams that fall upon it before they reach the Eie. And of Diaphanous Bodies, ſome are made ſuch by Nature from the beginning; as the ſubſtance of the Aire, and of the Water, and perhaps alſo ſome parts of Stones, unleſs theſe alſo be Water that has been long congeled. Others are made ſo by the power of Heat, which congregates homogeneons Bodies. But ſuch as are made Diaphanous in this manner, conſiſt of parts which were formerly Diaphanous.

14 In what manner Clouds are made by the motion of the Sunne, elevating the particles of Water from the Sea and other moiſt places, hath been explained in the 26th Chapter. Alſo how Clouds come to be frozen, hath been ſhewn above at the 7th Article. Now from this, that Aire may be encloſed, as it were in Caverns, and pent together more and more by the meeting of aſcending and deſcending Clouds, may be deduced a poſſible Cauſe of *Thunder* and *Lightening*. For ſeeing the Aire conſiſts of two parts, the one Æthereal, which has no proper motion of its own, as being a thing diviſible into the leaſt parts; the other Hard, namely, conſiſting of many hard Atomes which have every one of them a very ſwift ſimple motion of its own; whileſt the Clouds by their meeting do more and more ſtraighten ſuch Cavities as they intercept, the Æthereal parts will penetrate and paſs through their watry ſubſtance; but the hard parts will in the mean time be the more thruſt together, and preſs one another; and conſequently (by reaſon of their vehement motions) they will have an endeavour to rebound from each other. Whenſoever therefore the compreſſion is great enough, and the concave parts of the Clouds are (for the cauſe I have already given) congeled into Ice, the Cloud wil neceſſarily be broken; & this breaking of the Cloud produceth

the

the first clap of Thunder. Afterwards, the Aire which was pent in, having now broken through, makes a concussion of the Aire without; and from hence proceeds the roaring and murmur which follows; and both the first Clap and the Murmur that follows it, make that noise which is called *Thunder*. Also from the same Aire breaking through the Clouds, and with concussion falling upon the Eie, proceeds that action upon our Eie, which causeth in us a perception of that Light which we call *Lightening*. Wherefore I have given a possible cause of *Thunder* and *Lightening*.

15 But if the Vapours which are raised into Clouds, do run together again into Water, or be congeled into Ice, from whence is it (seeing both Ice and Water are heavy) that they are sustained in the Aire? Or rather, what may the cause be, that being once elevated, they fall down again? For there is no doubt but the same force which could carry up that Water, could also sustain it there. Why therefore being once carried up, doth it fall again? I say it proceeds from the same Simple Motion of the Sunne, both that Vapours are forced to ascend, and that Water gathered into Clouds is forced to descend. For in the 21th Chapter and 11th Article I have shewn how Vapours are elevated; and in the same Chapter and 5th Article I have also shewn how by the same motion Homogeneous Bodies are congregated, & Heterogeneous dissipated; that is to say, how such things as have a like nature to that of the Earth, are driven towards the Earth; that is to say, what is the cause of the descent of Heavy Bodies. Now if the action of the Sun be hindered in the raising of vapours, and be not at all hindered in the casting of them down, the Water will descend. But a Cloud cannot hinder the action of the Sunne in making things of an earthly nature descend to the Earth, though it may hinder it in making Vapours ascend For the lower part of a thick Cloud is so covered by its upper part, as that it cannot receive that action of the Sunne by which Vapours are carried up, because Vapours are raised by the perpetual fermentation of the Aire, or by the separating of its smallest parts from one another, which is much weaker when a thick Cloud is interposed, then when the Skie is cleere. And therefore whensoever a Cloud is made thick enough, the water which would

would not descend before, will then descend, unless it be kept up by the agitation of the Winde. Wherefore I have rendred a possible cause, both why the Clouds may be sustained in the Aire, and also why they may fall down again to the Earth; which was propounded to be done.

16 Granting that the Clouds may be frozen, it is no wonder if the Moon have been seen eclipsed at such time as she hath been almost two degrees above the Horizon, the Sunne at the same time appearing in the Horizon; for such an Eclipse was observed by *Mestline* at *Tubing* in the year 1590. For it might happen that a frozen Cloud was then interposed between the Sunne and the Eie of the Observer. And if it were so, the Sunne which was then almost two Degrees below the Horizon, might appear to be in it, by reason of the passing of his Beams through the Ice. And it is to be noted, that those that attribute such refractions to the *Atmosphere*, cannot attribute to it so great a refraction as this. Wherefore not the *Atmosphere*, but either Water in a continued Body, or else Ice must be the cause of that refraction.

17 Again, granting that there may be Ice in the Clouds, it will be no longer a wonder that many Sunnes have sometimes appeared at once. For Looking-glasses may be so placed, as by reflections to shew the same object in many places. And may not so many frozen Clouds serve for so many Looking-glasses? and may they not be fitly disposed for that purpose? Besides, the number of Appearances may be encreased by refractions also; and therefore it would be a greater wonder to me, if such *Phænomena* as these should never happen.

And were it not for that one *Phænomenon* of the new Starre which was seen in *Cassiopæa*, I should think *Comets* were made in the same manner, namely, by Vapours drawn not onely from the Earth, but from the rest of the Planets also, and congeled into one continued Body. For I could very well from hence give a reason both of their Haire, and of their motions. But seeing that Starre remained sixteen whole moneths in the same place amongst the fixed Starres, I cannot believe the matter of it was Ice. Wherefore I leave to others, the disquisition of the cause of Comets; concerning which, nothing that hath hitherto been published, (besides

the bare Hiſtories of them) is worth conſidering.

18 The Heads of Rivers may be deduced from Rain-water, or from melted Snowes, very eaſily; but from other cauſes, very hardly, or not at all. For both Rain-water, and melted Snowes run down the deſcents of Mountains; and if they deſcend onely by the outward Superficies, the Showres or Snowes themſelves may be accounted the Springs or Fountains; but if they enter the Earth & deſcend within it, then whereſoever they break out, there are their Springs. And as theſe Spings make ſmall ſtreams, ſo, many ſmall ſtreams running together make Rivers. Now there was never any Spring foũd, but where the Water w^ch flowed to it, was either further, or at leaſt as farre from the center of the Earth, as the Spring it ſelf. And whereas it has bin objected by a great Philoſopher, that in the top of *Mount-Cenis* (which parts Savoy from Piemont) there Springs a River which runs down by *Suſa*; it is not true. For there are above that River, for two miles length, very high hils on both ſides, which are almoſt perpetually covered with Snow; from which, innumerable little ſtreams running down do manifeſtly ſupply that River with water ſufficient for its magnitude.

CHAP. XXIX.

Of Sound, Odour, Savour, *and* Touch.

1 *The definition of Sound, and the distinctions of Sounds.* 2 *The cause of the degrees of Sounds.* 3 *The difference between Sounds Acute and Grave.* 4 *The difference between Clear and Hoarse Sounds, whence.* 5 *The Sound of Thunder and of a Gunne, whence it proceeds.* 6 *Whence it is, that Pipes by blowing into them have a clear Sound.* 7 *Of Reflected Sound.* 8 *From whence it is that Sound is Uniform and Lasting.* 9 *How Sound may be helped aud hindered by the Wind.* 10 *Not onely Aire, but other Bodies how hard soever they be, conveigh Sound.* 11 *The causes of Grave and Acute Sounds, and of Concent.* 12 *Phænomena for Smelling.* 13 *The first Organ and the generation of Smelling.* 14 *How it is helped by Heat and by Wind.* 15 *Why such Bodies are least smelt, which have least intermixture of Aire in them.* 16 *Why Odorous things become more Odorous by being bruised.* 17 *The first Organ of Tasting; and why some Savours cause Nauseousnes.* 18 *The first Organ of Feeling; and how we come to the knowledge of such Objects as are common to the Touch and other Senses.*

SOUND *is Sense generated by the action of the Medium, when its motion reacheth the Eare and the rest of the Organs of Sense.* Now the motion of the Medium is not the Sound it self, but the cause of it. For the Phantasme which is made in us, that is to say, the Reaction of the Organ is properly that which we call *Sound*.

The principal distinctions of Sounds are these; First, that one Sound is stronger, another Weaker. Secondly, that one is more Grave, another more Acute. Thirdly, that one is Clear, another Hoarse. Fourthly, that one is Primary, another Derivative. Fifthly, that one is Uniform, another not. Sixthly, that one is more Durable, another less Durable. Of all which distinctions the members may be subdistinguished into parts distinguishable almost infinitely. For the variety of Sounds seems to be not much less then that of Colours.

As Viſion, ſo Hearing is generated by the motion of the Medium, but not in the ſame manner. For Sight is from Preſſure, that is, from an Endeavour; in which there is no perceptible progreſſion of any of the parts of the Medium; but one part urging or thruſting on an other, propagateth that action ſucceſſively to any diſtance whatſoever; whereas the motion of the Medium by which Sound is made, is a Stroke. For when we Hear, the Drumme of the Eare (which is the firſt Organ of Hearing) is ſtricken; and the Drumme being ſtricken, the *Pia Mater* is alſo ſhaken, and with it the Arteries which are inſerted into it; by which the action being propagated to the Heart it ſelf, by the reaction of the Heart a Phantaſm is made which we call Sound; and (becauſe the reaction tendeth outwards) we think it is without.

2 And ſeeing the effects produced by Motion, are greater or leſſe, not onely when the Velocity is greater or leſs, but alſo when the Body hath greater or leſs Magnitude though the Velocity be the ſame; a Sound may be greater or leſſe both theſe wayes. And becauſe neither the greateſt nor the leaſt Magnitude or Velocity can be given, it may happen, that either the motion may be of ſo ſmall velocity, or the Body it ſelf of ſo ſmall magnitude, as to produce no Sound at all; or either of them may be ſo great, as to take away the Faculty of Senſe by hurting the Organ.

From hence may be deduced poſſible cauſes of the ſtrength and weakneſs of Sounds in the following *Phænomena*.

The firſt whereof is this, That if a man ſpeak through a Trunk which hath on end applyed to the mouth of the Speaker, and the other to the eare of the Hearer, the Sound will come ſtronger then it would do through the open Aire. And the cauſe (not onely the poſſible, but the certain and manifeſt cauſe) is this, that the Aire which is moved by the firſt breath, and carried forwards in the Trunk, is not diffuſed, as it would be in the open Aire, and is conſequently brought to the eare almoſt with the ſame velocity with which it was firſt breathed out; Whereas in the open Aire, the firſt motion diffuſeth it ſelf every way into Circles, ſuch as are made by the throwing of a Stone into a ſtanding water, where the velocity grows leſs and leſs as the Undulation proceeds further and further from the beginning of its motion.

The ſecond is this, That if the Trunk be ſhort, and the end

end which is applyed to the mouth be wider then that which is applyed to the eare, thus alſo the Sound will be ſtronger then if it were made in the open aire. And the cauſe is the ſame, namely, that by how much the wider end of the Trunk is leſs diſtant from the beginning of the Sound, by ſo much the leſs is the diffuſion.

The third, That it is eaſier for one that is within a Chamber, to heare what is ſpoken without, then for him that ſtands without, to hear what is ſpoken within. For the Windows and other inlets of the moved Aire, are as the wide end of the Trunk. And for this reaſon ſome creatures ſeem to hear the better, becauſe Nature has beſtowed upon them wide and capacious Ears.

The fourth is this, That though he which ſtandeth upon the Sea ſhore, cannot heare the Colliſion of the two neereſt waves, yet nevertheſs he hears the roaring of the whole Sea. And the cauſe ſeems to be this, that though the ſeveral colliſions move the Organ, yet they are not ſeverally great enough to cauſe Senſe; whereas nothing hinders but that all of them together may make Sound.

3 That Bodies when they are ſtricken do yeild ſome a more Grave, others a more Acute Sound, the cauſe may conſiſt in the difference of the times in which the parts ſtricken and forced out of their places, return to the ſame places again. For in ſome Bodies, the reſtitution of the moved parts is quick, in others ſlow. And this alſo may be the cauſe why the parts of the Organ which are moved by the Medium, return to their reſt again, ſometimes ſooner, ſometimes later. Now by how much the Vibrations, or the reciprocal motions of the parts are more frequent, by ſo much doth the whole Sound made (at the ſame time) by one ſtroke, conſiſt of more, and conſequently of ſmaller parts. For what is Acute in Sound, the ſame is Subtle in Matter; and both of them, namely, Acute Sound, and Subtle Matter conſiſt of very ſmall parts, that of Time, and this of the Matter it ſelf.

The third diſtinction of Sounds cannot be conceived clearly enough by the names I have uſed of *Clear* and *Hoarſe*, nor by any other that I know; and therefore it is needful to explain them by examples. When I ſay *Hoarſe*, I underſtand Whiſpering and Hiſſing, and whatſoever is like to theſe, by what appellation ſoever it be expreſſed. And Sounds of this kind ſeem to be made by the force of ſome ſtrong Wind, raking rather then ſtriking ſuch hard Bo-

Bodies, as it falls upon. On the contrary, when I uſe the word *Clear*, I do not underſtaud ſuch a Sound as may be eaſily and diſtinctly heard, for ſo Whiſpers would be Clear, but ſuch as is made by ſomewhat that is Broken; and ſuch as is Clamor, Tinkling, the Sound of a Trumpet, &c. and (to expreſs it ſignificantly in one word), Noiſe. And ſeeing no Sound is made but by the concourſe of two Bodies at the leaſt, by which conconrſe it is neceſſary that there be as well Reaction as Action, that is to ſay, one motion oppoſite to another; it follows, that according as the proportion between thoſe two oppoſite motions is diverſified, ſo the Sounds which are made will be different from one another. And whenſoever the proportion between them is ſo great, as that the motion of one of the Bodies be inſenſible if compared with the motion of the other, then the Sound will not be of the ſame kind; as when the Wind falls very obliquely upon a hard Body, or when a hard Body is carried ſwiftly through the Aire; for then there is made that Sound which I call a Hoarſe Sound, in Greek *οὐριγμος*. Therefore the breath blown with violence from the mouth makes a Hiſſing, becauſe in going out it rakes the Superficies of the Lips, whoſe reaction againſt the force of the breath is not Senſible. And this is the cauſe why the Winds have that Hoarſe Sound. Alſo if two Bodies how hard ſoever, be rubbed together with no great preſſure, they make a Hoarſe Sound. And this Hoarſe Sound when it is made (as I have ſaid) by the Aire raking the Superficies of a hard Body, ſeemeth to be nothing but the dividing of the Aire into innumerable and very ſmall Files. For the aſperity of the Superficies doth by the eminencies of its innumerable parts divide or cut in pieces the Aire that ſlides upon it.

4. *Noiſe*, or that which I call Clear Sound, is made two wayes; one, by two Hoarſe Sounds, made by oppoſite motions; the other, by Colliſion, or by the ſuddain pulling aſunder of two Bodies, whereby their ſmall particles are put into commotion, or being already in commotion, ſuddenly reſtore themſelves again; which motion making impreſſion upon the Medium, is propagated to the Organ of Hearing. And ſeeing there is in this Colliſion, or divulſion, an endeavour in the particles of one Body, oppoſite to the endeavour of the particles of the other Body, there will alſo be made in the Organ of Hearing a like oppoſition of endeavours, that is to

ſay,

ſay, of motions; and conſequently the Sound ariſing from thence, will be made by two oppoſite motions, that is to ſay, by two oppoſite Hoarſe Sounds in one and the ſame part of the Organ. For (as I have already ſaid) a Hoarſe Sound ſuppoſeth the ſenſible motion of but one of the Bodies. And this oppoſition of motions in the Organ is the cauſe why two Bodies make a Noyſe, when they are either ſuddenly ſtricken againſt one another, or ſuddenly broken aſunder.

5 This being granted; and ſeeing withall, that Thunder is made by the vehement eruption of the Aire out of the cavities of congeled Clouds; the cauſe of the great Noyſe or Clap, may be the ſuddain breaking aſunder of the Ice. For in this action it is neceſſary, that there be not onely a great concuſſion of the ſmall particles of the broken parts, but alſo that this Concuſſion (by being communicated to the Aire) be carried to the Organ of Hearing, & make impreſſion upon it. And then, from the firſt reaction of the Organ proceeds that firſt and greateſt Sound, which is made by the colliſion of the parts whilſt they reſtore themſelvs. And ſeeing there is in all Concuſſion a reciprocation of Motion forwards and backwards in the parts ſtricken, (for oppoſite motions cannot extinguiſh one another in an inſtant, as I have ſhewn in the 11th Art. of the 8th Chap.) it follows neceſſarily, that the Sound will both continue, and grow weaker and weaker, till at laſt the action of the reciprocating aire grow ſo weak, as to be unperceptible. Wherefore a poſſible cauſe is given both of the firſt fierce Noyſe of the Thunder; and alſo of the Murmur that follows it.

The cauſe of the great Sound from a diſcharged piece of Ordnance, is like that of a Clap of Thunder. For the Gunpowder being fired, doth in its endeavour to go out, attempt every way the ſides of the metal in ſuch manner, as that it enlargeth the Circumference all along, and withall ſhortneth the axis; ſo that whileſt the peece of Ordnance is in diſcharging, it is made both wider and ſhorter then it was before; and therefore alſo preſently after it is diſcharged its wideneſs will be diminiſhed, and its length encreaſed again by the reſtitution of all the particles of the matter of which it conſiſteth to their former poſition. And this is done with ſuch motion of the parts, as are not onely very vehement, but alſo oppoſite to one another; which motions being;

being communicated to the Aire, make impreſſion upon the Organ, and by the reaction of the Organ create a Sound; which laſteth for ſome time, as I have already ſhewn in this Article.

I note by the way (as not belonging to this place) that the poſſible cauſe why a Gun recoyles when it is ſhot off, may be this; That being firſt ſwoln by the force of the fire, and afterwards reſtoring it ſelf; from this reſtitution there proceeds an endeavour from all the ſides towards the cavity; and conſequently this endeavour is in thoſe parts which are next the breech; which being not hollow, but ſolid, the effect of the reſtitution is by it hindered and diverted into the length; and by this means both the breech and the whole Gun is thruſt backwards; and the more forcibly by how much the force is greater by which the part next the breech is reſtored to its former poſture; that is to ſay, by how much the thiner is that part. The cauſe therefore why Gunnes recoyle, ſome more, ſome leſs, is the difference of their thickneſs towards the breech; & the greater that thickneſs is, the leſs they recoyl; and contrarily.

6 Alſo the cauſe why the Sound of a Pipe, which is made by blowing into it, is nevertheleſs Clear, is the ſame with that of the Sound which is made by colliſion. For if the breath when it is blown into a Pipe doe onely rake its concave Superficies, or fall upon it with a very ſharp angle of incidence, the Sound will not be Clear, but Hoarſe. But if the angle be great enough, the percuſſion which is made againſt one of the hollow ſides, will be reverberated to the oppoſite ſide; and ſo ſucceſſive repercuſſions will be made from ſide to ſide, till at laſt the whole concave Superficies of the Pipe be put into motion; which motion will be reciprocated, as it is in Colliſion; and this reciprocation being propagated to the Organ, from the reaction of the Organ will ariſe a Cleare Sound, ſuch as is made by Colliſion, or by breaking aſunder of hard Bodies.

In the ſame manner it is with the Sound of a Mans voice. For when the breath paſſeth out, without interruption, and doth but lightly touch the cavities through which it is ſent, the Sound it maketh is a Hoarſe Sound. But if in going out it ſtrike ſtrongly upon the *Larinx*, then a Clear Sound is made, as in a Pipe. And the ſame breath, as it comes in divers manners to the Palate, the Tongue, the Lips, the Teeth, and other Organs of Speech, ſo the Sounds,

Sounds into which it is articulated become different from one another.

7 I call that *Primary Sound*, which is generated by motion from the ſounding Body to the Organ in a ſtraight line without reflexion; and I call that *Reflected Sound*, which is generated by one or more reflexions; being the ſame with that we call *Echo*; and is iterated as often as there are reflexions made from the Object to the Eare. And theſe reflexions are made by Hils, Wals, and other reſiſting Bodies, ſo placed, as that they make more or fewer reflexions of the motion, according as they are themſelves more or fewer in number; and they make them more or leſs frequently, according as they are more or leſs diſtant from one another. Now the cauſe of both theſe things is to be ſought for in the ſituation of the reflecting Bodies, as is uſually done in Sight. For the Lawes of Reflexion are the ſame in both, namely, that the Angles of Incidence and Reflexion be equal to one another. If therefore in a hollow Elliptique Body whoſe inſide is well poliſhed, or in two right Paraboli-cal Solids which are joyned together by one common baſe, there be placed a Sounding Body in one of the Burning Points, & the Ear in the other, there will be heard a Sound by many degrees greater then in the open Aire; and both this, and the burning of ſuch combuſtible things, as being put in the ſame places are ſet on fire by the Sun-beams, are effects of one and the ſame cauſe. But as when the viſible Object is placed in one of the Burning Points, it is not diſtinctly ſeen in the other, becauſe every part of the Object being ſeen in every line which is reflected from the Concave Superficies to the Eie, makes a confuſion in the Sight; ſo neither is Sound heard articulately and diſtinctly when it comes to the Eare in all thoſe reflected lines. And this may be the reaſon, why in Churches which have arched rooffs, though they be neither Elliptical nor Parabolical; yet becauſe their figure is not much different from theſe, the voice from the Pulpit will not be heard ſo articulately as it would be if there were no vaulting at all.

8 Concerning the *Uniformity* and *Duration* of Sounds, both which have one common cauſe, we may obſerve, that ſuch Bodies as being ſtricken yeild an unequal or harſh Sound, are very heterogeous, that is to ſay, they conſiſt of parts which are very unlike both in figure and hardneſs, ſuch as are Wood, Stones, and others not a

few

few. When theſe are ſtricken, there follows a concuſſion of their internal particles, and a reſtitution of them again. But they are neither moved alike, nor have they the ſame action upon one another; ſome of them recoyling from the ſtroke whileſt others which have already finiſhed their recoylings are now returning; by which means they hinder and ſtop on another. And from hence it is that their motions are not only unequal and harſh, but alſo that their reciprocations come to be quickly extinguiſhed. Whenſoever therfore this motion is propagated to the Eare, the Sound it makes is Unequal and of ſmall Duration. On the contrary, if a Body that is ſtricken, be not onely ſufficiently hard, but have alſo the particles of which it conſiſteth like to one another both in hardneſs and figure, (ſuch as are the particles of Glaſs and Metals, which being firſt melted do afterwards ſettle and harden) the Sound it yeildeth, will (becauſe the motions of its parts and their reciprocations are like and Uniform) be Uniform and pleaſant, and be more or leſs Laſting according as the Body ſtricken hath gteater or leſs magnitude. The poſſible cauſe therefore of Sounds Uniform, and Harſh, and of their longer or ſhorter Duration, may be one and the ſame likeneſs and unlikeneſs of the internal parts of the Sounding Body, in reſpect both of their figure and hardneſs.

Beſides, if two plain Bodies of the ſame matter, and of equal thickneſs, do both yeild an Uniform Sound, the Sound of that Body which hath the greateſt extent of length will be the longeſt heard. For the motion which in both of them hath its beginning from the point of percuſſion, is to be propagated in the greater Body through a greater Space, and conſequently that propagation will require more time; and therefore alſo the parts which are moved will require more time for their return. Wherefore all the reciprocations cannot be finiſhed but in longer time; and being carried to the Eare, will make the Sound laſt the longer. And from hence it is manifeſt, that of hard Bodies which yeild an Uniform Sound, the Sound laſteth longer which comes from thoſe that are round and hollow, then from thoſe that are plain, if they be like in all other reſpects. For in circular lines the action which begins at any point, hath not frō the figure any end of its propagation, becauſe the line in which it is propagated returns again to its beginning; ſo that the figure hinders not but that the motion may

have

have infinite progreſſion; whereas in a plain, every line hath its magnitude finite, beyond which the action cannot proceed. If therefore the matter be the ſame, the motion of the parts of that Body whoſe figure is round and hollow, wil laſt longer, then of that which is plain.

Alſo, if a ſtring which is ſtretched, be faſtned at both ends to a hollow Body, and be ſtricken, the Sound will laſt longer then if it were not ſo faſtned; becauſe the trembling or reciprocation which it receives from the ſtroke, is by reaſon of the connexion communicated to the hollow Body; and this trembling, if the hollow Body be great, will laſt the longer by reaſon of that greatneſs. Wherefore alſo (for the reaſon above mentioned) the Sound will laſt the longer.

9 In Hearing, it happens (otherwiſe then in Seeing) that the action of the Medium is made ſtronger by the Wind when it blows the ſame way; and weaker when it blows the contrary way. The cauſe whereof cannot proceed from any thing but the different generation of Sound and Light. For in the generation of Light, none of the parts of the Medium between the object and the Eie are moved from their own places to other places ſenſibly diſtant; but the action is propagated in ſpaces imperceptible; ſo that no contrary Wind can diminiſh, nor favourable Winde encreaſe the Light, unleſs it be ſo ſtrong as to remove the Object further off, or bring it nearer to the Eie. For the Wind, that is to ſay, the aire moved, doth not by its interpoſition between the object and the eie, worke otherwiſe then it would doe if it were ſtil and calme. For where the preſſure is perpetuall, one part of the aire is no ſooner carried away, but another by ſucceeding it receives the ſame impreſſion which the part carried away had received before. But in the generation of Sound, the firſt colliſion or breaking aſunder, beateth away & driveth out of its place the neareſt part of the aire, and that to a conſiderable diſtance, and with conſiderable velocity; and as the circles grow (by their remoteneſſe) wider and wider, ſo the aire being more & more diſſipated, hath alſo its motion more & more weakned. Whenſoever therfore the air is ſo ſtricken as to cauſe Sound, if the Wind fall upon it, it will move it all, neerer to the Eare if it blow that way, and further from it if it blow the contrary way; ſo that

ding as it blowes from or towards the Object, ſo the Sound which is heard will ſeeme to come from a neerer or remoter place; and the action by reaſon of the unequall diſtances be ſtrengthened or debilitated.

From hence may be underſtood the reaſon, why the voice of ſuch as are ſaid to ſpeake in their bellies, though it be uttered neer hand, is nevertheleſſe heard by thoſe that ſuſpect nothing, as if it were a great way off. For having no former thought of any determined place from which the voice ſhould proceed, and judging according to the greateſſe, if it be weake they thinke it a great way off, if ſtrong neer. Theſe *Ventriloqui* therefore by forming their voice, not (as others) by the emiſſion of their breath, but by drawing it inwards, doe make the ſame appear ſmall and weake; which weakneſſe of the voice deceives thoſe that neither ſuſpect the artifice, nor obſerve the endeavour which they uſe in ſpeaking; and ſo inſtead of thinking it weake they thinke it farre off.

10 As for the Medium which conveighs Sound, it is not Aire onely. For Water, or any other Body how hard ſoever may be that Medium. For the Motion may be propagated perpetually in any hard continuous Body; but by reaſon of the difficulty with which the parts of hard Bodies are moved, the motion in going out of hard matter makes but a weak impreſſion upon the Aire. Nevertheleſs if one end of a very long and hard beam be ſtricken, & the eare be applyed at the ſame time to the other end, ſo that when the action goeth out of the beam, the aire which it ſtriketh may immediately be received by the eare, and be carried to the *Tympanum*, the Sound will be conſiderably ſtrong.

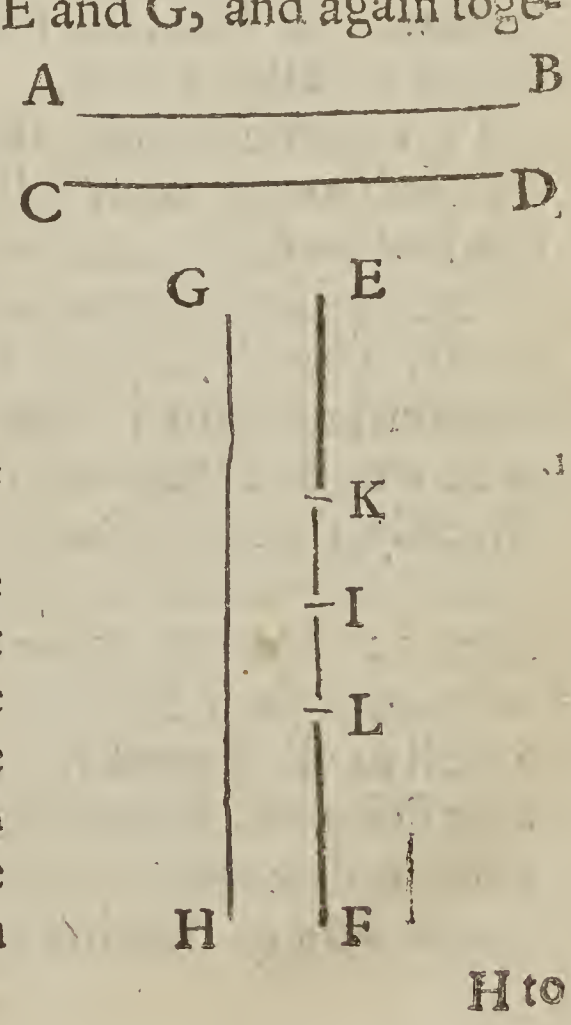
In like manner, if in the night (when all other noyſe, which may hinder Sound, ceaſeth) a man lay his eare to the ground, he will hear the Sound of the ſteps of Paſſengers, though at a great diſtance; becauſe the motion which by their treading they communicate to the earth, is propagated to the eare by the uppermoſt parts of the earth which receiveth it from their feet.

11 I have ſhewn above, that the difference between Grave and Acute Sounds conſiſteth in this, that by how much the ſhorter the time is, in which the reciprocations of the parts of a Body ſtricken are made, by ſo much the more Acute will be the Sound.

Now

Now by how much a Body of the ſame bigneſs, is either more heavy, or leſs ſtretched, by ſo much the longer will the reciprocations laſt; and therefore heavier, and leſs ſtretched Bodies (if they be like in all other reſpects) will yeild a Graver Sound then ſuch as be lighter and more ſtretched.

As for the *Concent* of Sounds, it is to be conſidered, that the reciprocation or Vibration of the Aire (by which Sound is made) after it hath reached the Drumme of the eare, imprinteth a like Vibration upon the air that is encloſed within it; by which means the ſides of the Drumme within, are ſtricken alternately. Now the Concent of two Sounds conſiſts in this, that the *Tympanum* receive its ſtroke from both the Sounding Bodies in equal, & equally frequēt ſpaces of time; So that when two ſtrings make their Vibrations in the ſame times, the Concent they produce, is the moſt exquiſite of all other. For the ſides of the *Tympanum*, (that is to ſay, of the Organ of Hearing) will be ſtricken by both thoſe Vibrations together at once, on one ſide or other. For example, if the two equal ſtrings A B and C D be ſtricken together, and the latitudes of their Vibrations E F and G H be alſo equal, and the points E, G, F & H be in the concave ſuperficies of the *Tympanum*, ſo that it receive ſtrokes from both the ſtrings together in E and G, and again together in F and H, the Sound which is made by the Vibrations of each ſtring, will be ſo like, that it may be taken for the ſame Sound, and is called *Uniſon*; which is the greateſt Concord. Again, the ſtring A B retaining ſtill its former Vibration E F, let the ſtring C D be ſtretched, till its Vibration have double the ſwiftneſs it had before, and let E F be divided equally in I. In what time therefore the ſtring C D makes one part of its Vibration from G to H, in the ſame time the ſtring A B will make one part of its Vibration from E to I; and in what time the ſtring C D hath made the other part of its Vibration back from



H to

H to G, in the ſame time another part of the Vibration of the ſtring A B will be made from I to F. But the points F and G are both in the ſides of the Organ, and therefore they will ſtrike the Organ both togerher, not at every ſtroke, but at every other ſtroke. And this is the neereſt Concord to Uniſon, and makes that Sound which is called an *Eighth*. Again, the Vibration of the ſtring A B remayning ſtill the ſame it was, let C D be ſtretched till its Vibration be ſwifter then that of the ſtring A B, in the proportion of 3 to 2, and let E F be divided into three equal parts in K and L. In what time therefore the ſtring C D makes one third part of its Vibration, which third part is from G to H, the String A B will make one third part of its Vibration, that is to ſay, two thirds of E F, namely, E L. And in what time the ſtring C D makes another third part of its Vibration, namely H G, the ſtring A B will make another third part of its Vibration, namely, from L to F, and back again from F to L. Laſtly, whileſt the ſtring C D makes the laſt third part of its Vibration. that is, from G to H, the ſtring A B will make the laſt third part of its Vibration from I to E. But the poi ntsE and H are both in the ſides of the Organ. Wherefore at every third time the Organ will be ſtricken by the Vibration of both the ſtrings together, and make that Concord which is called a *Fifth*.

12. For the finding out of the cauſe of *Smels*, I ſhall make uſe of the evidence of theſe following *Phænomena*. Firſt, that Smelling is hindred by Cold, and helped by Heat. Secondly, that when the Wind bloweth from the Object, the Smel is the ſtronger; and contrarily when it bloweth from the Sentient towards the Object, the weaker; both which *Phænomena* are by experience manifeſtly found to be true in Doggs which follow the track of Beaſts by the Sent. Thirdly, that ſuch Bodies as are leſs pervious to the fluid Medium, yeild leſs Smell then ſuch as are more pervious; as may be ſeen in Stones and Metals, which compared with Plants and Living Creatures, and their Parts, Fruits and Excretions, have very little or no Smell at all. Fourthly, that ſuch Bodies as are of their own nature Odorous, become yet more Odorous when they are bruiſed. Fifthly, that when the breath is ſtopped (at leaſt in Men) nothing can be Smelt. Sixthly, that the ſenſe of Smelling is alſo taken away

away by the ſtopping of the Noſtrils, though the mouth be left open.

13 By the fourth and fifth *Phænomenon* it is manifeſt, that the firſt and immediate Organ of Smelling is the innermoſt cuticle of the Noſtrils, and that part of it which is below the paſſage common to the Noſtrils and the Palate. For when we draw breath by the Noſtrils, we draw it into the Lungs. That breath therefore which conveighs Smels, is in the way which paſſeth to the Lungs, that is to ſay, in that part of the Noſtrils which is below the paſſage through which the breath goeth. For nothing is Smelt, neither beyond the paſſage of the breath within, nor at all without the Noſtrils.

And ſeeing that from different Smels there muſt neceſſarily proceed ſome mutation in the Organ, and all mutation is motion; it is therefore alſo neceſſary that in Smelling, the parts of the Organ, that is to ſay, of that internal cuticle, and the nerves that are inſerted into it, muſt be diverſly moved by different Smels. And ſeeing alſo that it hath been demonſtrated, that nothing can be moved but by a Body that is already moved and contiguous; and that there is no other Body contiguous to the internal membrane of the noſtrils, but breath, that is to ſay, attracted aire, and ſuch little Solid inviſible Bodies (if there be any ſuch) as are intermingled with the aire; it follows neceſſarily, that the cauſe of Smelling is either the motion of that pure aire or æthereal Subſtance, or the motion of thoſe ſmall Bodies. But this motion is an effect proceding from the Object of Smell; and therefore either the whole Object it ſelf, or its ſeveral parts muſt neceſſarily be moved. Now we know, that Odorous Bodies make Odour though their whole bulk be not moved. Wherefore the cauſe of Odour is the motion of the inviſible parts of the Odorous Body. And theſe inviſible parts do either go out of the Object, or elſe retaining their former ſituation with the reſt of the parts, are moved together with them, that is to ſay, they have ſimple and inviſible motion. They that ſay there goes ſomething out of the Odorous Body, call it an *Effluvium*; which *Effluvium* is either of the æthereal ſubſtance, or of the ſmall Bodies, that are intermingled with it. But that all variety of Odours ſhould proceed from the

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Effluviums of thoſe ſmall Bodies that are intermingled with the æthereal ſubſtance, is altogether incredible, for theſe conſiderations; Firſt, that certain Unguents, though very little in quantity, do nevertheleſs ſend forth very ſtrong Odours, not onely to a great diſtance of place, but alſo to a great continuance of time, and are to be Smelt in every point both of that place and time; ſo that the parts iſſued out are ſufficient to fil ten thouſand times more ſpace, then the whole Odorous Body is able to fill, which is impoſſible. Secondly, that whether that iſſuing out be with ſtraight or with crooked motion, if the ſame quantity ſhould flow from any other Odorous Body with the ſame motion, it would follow, that all Odorous Bodies would yeild the ſame Smell. Thirdly, that ſeeing thoſe *Effluviums* have great Velocity of motion (as is manifeſt from this, that noyſome Odours proceeding from caverns are preſently Smelt at a great diſtance) it would follow, that by reaſon there is nothing to hinder the paſſage of thoſe *Effluviums* to the Organ, ſuch motion alone were ſufficient to cauſe Smelling. Which is not ſo; for we cannot Smell at all, unleſs we draw in our breath through our Noſtrils. Smelling therefore is not cauſed by the *Effluvium* of Atomes; nor, for the ſame reaſon is it cauſed by the *Effluvium* of æthereal ſubſtance; for ſo alſo we ſhould Smell without the drawing in of our breath. Beſides the æthereal ſubſtance being the ſame in all Odorous Bodies, they would always affect the Organ in the ſame manner, and conſequently the Odours of all things would be like.

It remains therefore, that the cauſe of Smelling muſt conſiſt in the Simple motion of the parts of Odorous Bodies, without any efflux or diminution of their whole ſubſtance. And by this motion, there is propagated to the Organ by the intermediate aire, the like motion, but not ſtrong enough to excite Senſe of it ſelf without the attraction of aire by reſpiration. And this is a poſſible cauſe of Smelling.

14 The cauſe why Smelling is hindred by Cold, and helped by Heat, may be this, that Heat (as hath been ſhewn in the 21 Chapter) generateth Simple motion; and therefore alſo whereſoever it is already, there it will encreaſe it; and the cauſe of Smelling being encreaſed the Smell it ſelf will alſo be encreaſed. As for the cauſe why

why the Wind blowing from the Object makes the Smell the ſtronger, it is all one with that for which the attraction of aire in reſpiration doth the ſame. For he that draws in the aire next to him, draws with it by ſucceſſion that aire in which is the Object. Now this motion of the aire is Wind, and when another Wind bloweth from the Object, will be encreaſed by it.

15 That Bodies which cõtain the leaſt quantity of air, as Stones and Metals, yeild leſs Smell then Plants and Living Creatures, the cauſe may be, that the motion which cauſeth Smelling, is a motion of the fluid parts onely; which parts, if they have any motion from the hard parts in which they are contained, they communicate the ſame to the open aire, by which it is propagated to the Organ. Where therefore there are no fluid parts, as in Metals; or where the fluid parts receive no motion from the hard parts, as in Stones, which are made hard by accretion, there can be no Smell. And therefore alſo the Water, whoſe parts have little or no motion, yeildeth no Smell. But if the ſame Water, by Seeds, and the heat of the Sunne, be together with particles of Earth raiſed into a Plant, and be afterwards preſſed out again, it will be Odorous, as Wine from the Vine. And as Water paſſing through plants is by the motion of the parts of thoſe plants made an Odorous liquour; ſo alſo of aire paſſing through the ſame plants whileſt they are growing, are made Odorous aires. And thus alſo it is with the Juices and Spirits which are bred in Living Creatures.

16 That Odorous Bodies may be made more Odorous by Contrition, proceeds from this, that being broken into many parts which are all Odorous, the aire which by reſpiration is drawn from the Object towards the Organ, doth in its paſſage touch upon all thoſe parts, and receives their motion. Now the aire toucheth the ſuperficies onely; and a Body having leſs ſuperficies whileſt it is whole, then all its parts together have after it is reduced to powder, it follows that the ſame Odorous Body yeildeth leſs Smell whileſt it is whole, then it will do after it is broken into ſmaller parts. And thus much of Smels.

17 The *Taſt* follows; whoſe generation hath this difference from that of the Sight, Hearing and Smelling, that by theſe we have Senſe of remote Objects; whereas we Taſt nothing but what is contiguous, and doth immediately touch either the Tongue or Palate,

Palate, or both. From whence it is evident, that the cuticles of the Tongue and Palate, and the Nerves inſerted into them are the firſt Organ of Taſt; and (becauſe from the concuſſion of the parts of theſe, there followeth neceſſarily a concuſſion of the *Pia Mater*) that the action communicated to theſe, is propagated to the Brain, and from thence to the fartheſt Organ, namely, the Hart; in whoſe reaction conſiſteth the nature of Senſe.

Now that Savours (as well as Odours) doe not onely move the Brain, but the Stomack alſo, as is manifeſt by the loathings that are cauſed by them both, they that conſider the Organ of both theſe Senſes will not wonder at all; ſeeing the Tongue, the Palate & the Noſtrils have one and the ſame continued cuticle, derived from the *Dura Mater*.

And that *Effluviums* have nothing to doe in the Senſe of Taſting, is manifeſt from this, that there is no Taſt where the Organ and the Object are not contiguous.

By what variety of motions the different kinds of Taſts (which are innumerable) may be diſtinguiſhed, I know not. I might (with others) derive them from the divers figures of thoſe Atomes, of which whatſoever may be Taſted conſiſteth; or from the diverſe motions which I might (by way of Suppoſition) attribute to thoſe Atomes; conjecturing (not without ſome likelyhood of truth) that ſuch things as taſt Sweet, have their particles moved with ſlow circular motion, and their figures Spherical, which makes them ſmooth and pleaſing to the Organ; that Bitter things have circular motion, but vehement, and their figures full of Angles; by which they trouble the Organ; and that Sowre things have ſtraight and reciprocal motion, and their figures long and ſmall, ſo that they cut and wound the Organ. And in like manner I might aſſigne for the cauſes of other Taſts ſuch ſeveral motions and figures of Atomes as might in probability ſeem to be the true cauſes. But this would be to revolt from Philoſophy to Divination.

18 By the *Touch*, we feel what Bodies are Cold or Hot, though they be diſtant from us. Others, as Hard, Soft, Rough and Smooth, we cannot feel, unleſs they be contiguous. The Organ of Touch, is every one of thoſe membranes, which being continued from the *Pia Mater*, are ſo diffuſed throughout the whole Body, as

that

that no part of it can be preſſed, but the *Pia Mater* is preſſed together with it. Whatſoever therefore preſſeth it, is felt as Hard or Soft, that is to ſay, as more or leſs Hard. And as for the Senſe of Rough, it is nothing elſe but innumerable perceptions of Hard and Hard ſucceeding one another by ſhort intervals both of time and place. For we take notice of Rough and Smooth, as alſo of Magnitude and Figure, not onely by the Touch, but alſo by Memory. For though ſome things are touched in one Point; yet Rough and Smooth, like Quantity and Figure, are not perceived but by the Flux of a Point, that is to ſay, we have no Senſe of them without Time; and we can have no Senſe of Time, without Memory.

CHAP. XXX.
Of Gravity.

1 *A Thick Body doth not contain more Matter (unleſs alſo more Place) then a Thinne.* 2 *That the Deſcent of Heavy Bodies proceeds not from their own Appetite, but from ſome Power of the Earth.* 3 *The difference of Gravities proceedeth from the difference of the* Impetus *with which the Elements whereof Heavy Bodies are made do fall upon the Earth.* 4 *The cauſe of the Deſcent of Heavy Bodies.* 5 *In what proportion the Deſcent of Heavy Bodies is accelerated.* 6 *Why thoſe that Dive do not when they are under Water, feel the waight of the Water above them.* 7 *The Waight of a Body that floateth, is equal to the Waight of ſo much Water as would fill the ſpace which the immerſed part of the Body takes up within the Water.* 8 *If a Body be Lighter then Water, then how big ſoever that Body be, it will float upon any quantity of Water, how little ſoever.* 9 *How Water may be lifted up and forced out of a Veſſel by Air.* 10 *Why a Bladder is Heavier when blown full of aire, then when it is empty.* 11 *The cauſe of the ejection upwards of Heavy Bodies from a Wind-Gun.* 12 *The cauſe of the aſcent of Water in a Weather-glaſs.* 13 *The cauſe of motion upwards in Living Creatures.* 14 *That there is in Nature a kind of Body Heavier then Aire, which nevertheleſs is not by Senſe diſtinguiſhable from it.* 15 *Of the cauſe of Magnetical vertue.*

1 IN the 21 Chapter I have defined Thick and Thinne (as that place required) ſo, as that by Thick was ſignified a more Reſiſting Body, and by Thinne a Body leſs Reſiſting; following the cuſtome of thoſe that have before me diſcourſed of Refraction. Now if we conſider the true and vulgar ſignification of thoſe words, we ſhall find them to be Names Collective, that is to ſay, Names of Multitude; as Thick to be that which takes up more parts of a ſpace given, & Thinne that which contains fewer parts of the ſame magnitude, in the ſame ſpace, or in a ſpace equal to it. Thick therefore is the ſame with *Frequent*, as a Thick Troop; And Thinne the ſame with *Unfrequent*, as a Thinne Rank, Thinne of Houſes; not that there is more matter in one place then in another e-

qual

qual place, but a greater quantity of ſome named Body. For there is not leſs matter or Body indefinitely taken, in a Deſert, then there is in a City; but fewer Houſes, or fewer Men. Nor is there in a Thick Rank a greater quantity of Body, but a greater number of Souldiers, then in a Thinne. Wherefore the multitude & paucity of the parts contained within the ſame ſpace, do conſtitute *Denſity* and *Rarity*, whether thoſe parts be ſeparated by *Vacuum*, or by Aire. But the conſideration of this is not of any great moment in Philoſophy; and therefore I let it alone, and paſs on to the ſearch of the cauſes of *Gravity*.

2 Now we call thoſe Bodies *Heavy*, which (unleſs they be hindred by ſome force) are carried towards the center of the Earth, and that by their own accord, for ought we can by Senſe perceive to the contrary. Some Philoſophers therefore have been of opinion, that the Deſcent of Heavy Bodies proceeded from ſome internal Appetite, by which when they were caſt upwards, they deſcended again, as moved by themſelves, to ſuch place as was agreeable to their nature. Others thought they were attracted by the Earth. To the former I cannot aſſent, becauſe I think I have already clearly enough demonſtrated, that there can be no beginning of motion, but from an external & moved Body; and conſequently, that whatſoever hath motion or endeavour towards any place, will alwayes move or endeavour towards that ſame place, unleſs it be hindered by the reaction of ſome external Body. Heavy Bodies therefore being once caſt upwards, cannot be caſt down again but by external motion. Beſides, ſeeing inanimate Bodies have no Appetite at all, it is ridiculous to think that by their own innate Appetite they ſhould to preſerve themſelves (not underſtanding what preſerves them) forſake the place they are in, and transferre themſelves to another place; whereas Man (who hath both Appetite and underſtanding) cannot for the preſervation of his own life, raiſe himſelfe by leaping above three or four feet from the ground. Laſtly, to attribute to created Bodies the power to move themſelves, what is it elſe then to ſay that there be creatures which have no dependance upon the Creator? To the later, who attribute the Deſcent of Heavy Bodies to the attraction of the Earth, I aſſent. But by what motion this is done, hath not as yet been explained by any man. I ſhall therefore in this place ſay ſomewhat

of the manner, and of the way by which the Earth by its action attracteth Heavy Bodies.

3 That by the ſuppoſition of ſimple motion in the Sunne, homogeneous Bodies are congregated, and heterogeneous diſſipated, has already been demonſtrated in the 5th Article of the 21 Chapter. I have alſo ſuppoſed, that there are intermingled with the pure Air, certain little Bodies, or (as others call them) Atomes, which by reaſon of their extreme ſmalneſs are inviſible, and differing from one another in *Conſiſtence, Figure, Motion & Magnitude*; from whence it comes to paſs, that ſome of them are congregated to the Earth, others to other Planets, and others are carried up and down in the ſpaces between. And ſeeing thoſe which are carried to the Earth, differ from one another in Figure, Motion and Magnitude, they will fall upon the Earth, ſome with greater, others with leſs *Impetus*. And ſeeing alſo that we compute the ſeveral degrees of Gravity no otherwiſe then by this their falling upon the Earth with greater or leſs *Impetus*; it follows, that we conclude thoſe to be the more Heavy that have the greater *Impetus*, and thoſe to be leſs Heavy that have the leſs *Impetus*. Our enquiry therefore muſt be, by what means it may come to paſs, that of Bodies which deſcend from above to the Earth, ſome are carried with greater, others with leſs *Impetus*; that is to ſay, ſome are more Heavy then others. We muſt alſo enquire, by what means ſuch Bodies as ſettle upon the Earth, may by the Earth it ſelf be forced to aſcend.

4 Let the Circle made upon the center C (in the 2d figure) be a great Circle in the Superficies of the Earth, paſſing through the points A and B. Alſo let any Heavy Body, as the ſtone A D be placed any where in the plain of the Æquator; and let it be conceived to be caſt up from A D perpendicularly, or to be carried in any other line to E, and ſuppoſed to reſt there. Therefore how much ſpace ſoever the ſtone took up in AD, ſo much ſpace it takes up now in E. And becauſe all place is ſuppoſed to be full, the ſpace A D will be filled by the aire which flows into it firſt from the neereſt places of the Earth, and afterwards ſucceſſively from more remote places. Upon the center C let a Circle be underſtood to be drawn through E; and let the plain ſpace which is between the Superficies of the Earth and that Circle, be divided into plain Orbs equal and concentrique; of which, let that be the firſt which is con-

contained by the two perimeters that pass through A & D. Whilst therefore the aire which is in the first Orbe, filleth the place AD, the Orbe it self is made so much less, and consequently its latitude is less then the straight line A D. Wherefore there will necessarily descend so much aire from the Orbe next above. In like manner, for the same cause, there will also be a descent of aire from the Orbe next above that; and so by Succession from the Orbe in which the Stone is at rest in E. Either thetefore the Stone it self, or so much aire will descend. And seeing aire is by the diurnal revolution of the Earth more easily thrust away, then the Stone, the aire which is in the Orbe that contains the Stone will be forced further upwards then the Stone. But this, without the admission of *Vacuum*, cannot be, unless so much aire descend to E from the place next above; which being done, the Stone will be thrust downwards. By this means therefore the Stone now receives the beginning of its Descent, that is to say, of its Gravity. Furthermore, whatsoever is once moved, will be moved continually (as hath been shewn in the 19th Article of the 8th Chapter) in the same way, and with the same celerity, except it be retarded or accelerated by some external Movent. Now the aire (which is the onely Body that is interposed between the Earth A and the stone above it E) will have the same action in every point of the straight line E A, which it hath in E. But it depressed the stone in E; and therefore also it will depres it equally in every point of the straight line E A. Wherefore the stone will descend from E to A with accelerated motion. The possible cause therefore of the Descent of Heavy Bodies under the Æquator, is the Diurnal motion of the Earth. And the same demonstration will serve, if the stone be placed in the plain of any other Circle parallel to the Æquator. But because this motion hath by reason of its greater slowness, less force to thrust off the aire in the parallel Circles then in the Æquator, and no force at all at the Poles, it may well be thought (for it is a certain consequent) that Heavy Bodies descend with less and less velocity, as they are more & more remote from the Æquator; & that at the Poles themselves they wil either not descend at all, or not descend by the Axis; which whether it be true or false, Experience must determine. But it is hard to make the experiment, both because the times of their descents

cents cannot be easily measured with sufficient exactness, and also because the places neer the Poles are inaccessible. Nevertheless, this we know, that by how much the neerer we come to the Poles, by so much the greater are the Flakes of the Snow that falls; and by how much the more swiftly such Bodies descend as are fluid and dissipable, by so much the smaller are the particles into which they are dissipated.

5. Supposing therefore this to be the cause of the Descent of Heavy Bodies; it will follow, that their motion will be accelerated in such manner, as that the spaces which are transmitted by them in the several times, will have to one another the same proportion which the odd numbers have in succession from Unity. For if the straight line E A be divided into any number of equal parts, the Heavy Body descending, will (by reason of the perpetual action of the Diurnal motion) receive from the aire in every one of those times, in every several point of the streight line E A, a several new and equal impulsion; and therefore also in every one of those times it will acquire a several and equal degree of celerity. And from hence it follows, by that which *Galilæus* hath in his Dialogues of Motion demonstrated, that Heavy Bodies descend in the several times with such differences of transmitted spaces as are equal to the differences of the square numbers that succeed one another from Unity; wnich square numbers being 1,4,9,16,&c. their differences are 3,5,7,&c; that is to say, the odd numbers which succeed another from Unity. Against this cause of Gravity which I have given, it will perhaps be objected, that if a Heavy Body be placed in the bottom of some hollow Cylinder of Iron or Adamant, and the bottom be turned upwards, the Body will descend, though the aire above cannot depress it, much less accelerate its motion. But it is to be considered, that there can be no Cylinder or Cavern, but such as is supported by the Earth, and being so supported, is together with the Earth carried about by its diurnal Motion. For by this means the bottom of the Cylinder will be as the Superficies of the Earth; and by thrusting off the next and loweſt aire, will make the uppermost aire depress the Heavy Body which is at the top of the Cylinder, in such manner as is above explicated.

6. The Gravity of Water being so great as by experience wee find it is, the reason is demanded by many, why those that Dive, how

how deep ſoever they go under water, do not at all feel the weight of the water which lyes upon them. And the cauſe ſeems to be this, that all Bodies by how much the Heavier they are, by ſo much the greater is the endeavour by which they tend downwards. But the Body of a Man is Heavier then ſo much water as is equal to it in magnitude, and therefore the endeavour downwards of a Mans Body is greater then that of water. And ſeeing all endeavour is motion, the Body alſo of a Man will be carried towards the bottom with greater Velocity then ſo much water. Wherefore there is greater Reaction from the bottom; and the Endeavour upwards is equal to the endeavour downwards, whether the water be preſſed by water, or by another Body which is Heavier then water. And therefore by theſe two oppoſite equal endeavours, the endeavour both ways in the water is taken away; and conſequently, thoſe that Dive are not at all preſſed by it.

Coroll. From hence alſo it is manifeſt, that water in water hath no Waight at all, becauſe all the parts of water (both the parts above, and the parts that are directly under) tend towards the bottom with equal endeavour and in the ſame ſtraight lines.

7 If a Body float upon the water, the waight of that Body is equal to the waight of ſo much water as would fill the place which the immerſed part of the Body takes up within the water.

Let E F (in the 3d figure) be a Body floating in the water A B C D; and let the part E be above, and the other part F under the water. I ſay the waight of the whole Body E F is equal to the waight of ſo much water as the Space F will receive. For ſeeing the waight of the Body E F forceth the water out of the ſpace F, and placeth it upon the Superficies A B, where it preſſeth downwards; it follows, that from the reſiſtance of the bottom there will alſo be an endeavour upwards. And ſeeing again that by this endeavour of the water upwards, the Body E F is lifted up; it follows, that if the endeavour of the Body downwards be not equal to the endeavour of the water upwards, either the whole Body E F will (by reaſon of that inequality of their endeavours or moments) be raiſed out of the water, or elſe it will deſcend to the bottom. But it is ſuppoſed to ſtand ſo, as neither to aſcend nor deſcend. Wherefore there is an *Æquilibrium* between the two endeavours; that is to ſay, the waight of the Body E F is equal to the

waight

waight of so much water as the Space F will receive; Which was to be proved.

8 From hence it follows, that any Body of how great magnitude soever, provided it consist of matter less Heavy then water, may nevertheless float upon any quantity of water how little soever.

Let A B C D (in the 4th figure) be a vessel; and in it let E F G H be a Body consisting of matter which is less Heavy then water; and let the space A G C F be filled with water. I say the Body E F G H will not sink to the bottom D C. For seeing the matter of the Body E F G H is less Heavy then Water, if the whole space without A B C D were full of Water, yet some part of the Body E F G H, as E F I K would be above the Water; and the waight of so much water as would fill the space I G H K would be equal to the waight of the whole Body E F G H; and consequently G H wonld not touch the bottom D C. As for the sides of the vessel, it is no matter whether they be hard, or fluid; for they serve onely to terminate the Water; which may be done as well by water as by any other matter how hard soever; and the water without the Vessel is terminated somewhere, so as that it can spread no further. The part therefore E F I G will be extant above the water A G C F which is contained in the vessel. Wherefore the Body E F G H will also float upon the water A G C F, how little soever that water be; which was to be demonstrated.

9 In the 4th Article of the 26th Chapter, there is brought for the proving of *Vacuum*, the experiment of water enclosed in a vessel; which water, the Orifice above being opened, is ejected upwards by the impulsion of the aire. It is therefore demanded (seeing water is Heavier then aire) how that can be done. Let the 2d. figure of the same 26th Chap. be considered, where the water is with great force injected by a Syringe into the space F G B. In that injection, the aire (but pure aire) goeth with the same force out of the vessel through the injected water. But as for those small Bodies which formerly I supposed to be intermingled with aire, & to be moved with simple motion, they can not together with the pure air penetrate the water; but remayning behind are necessarily thrust together into a narrower place, namely into the space which is above the water F G. The motions therefore of those small Bodies will be less and less free, by how much the quantity of the injected

jectedwater is greater and greater; so that by their motions falling upon one another the same small Bodies will mutually compress each other, and have a perpetual endeavour of regayning their liberty, and of depressing the water that hinders them. Wherefore, as soone as the orifice above is opened, the water which is next it will have an endeavour to ascend; and will therefore necessarily go out. But it cannot go out, unless at the same time there enter in as much aire; and therefore both the water will go out, and the aire enter in, till those small Bodies which were left within the vessel have recovered their former liberty of motion; that is to say, till the vessel be again filled with aire, and no water be left of sufficient height to stop the passage at B. Wherefore I have shewn a possible cause of this *Phænomenon*; namely, the same with that of Thunder. For as in the generation of Thunder, the small Bodies enclosed within the Clouds by being too closely pent together, do by their motion break the Clouds, and restore themselves to their natural liberty; so here also the small Bodies enclosed within the space which is above the straight line F G, do by their own motion expel the water as soon as the passage is opened above. And if the passage be kept stopped, and these small Bodies be more vehemently compressed by the perpetual forcing in of more water, they will at last break the vessel it self with great noise.

10 If Aire be blown into a hollow Cylinder, or into a Bladder, it will encrease the waight of either of them a little, as many have found by experience, who with great accurateness have tried the same. And it is no wonder, seeing (as I have supposed) there are intermingled with the common aire a great number of small hard Bodies which are Heavier then the pure aire. For the æthereal substance, being on all sides equally agitated by the motion of the Sunne, hath an equal endeavour towards all the parts of the Universe; and therefore it hath no Gravity at all.

11 We find also by experience, that by the force of air enclosed in a hollow Canon, a bullet of lead may with considerable violence be shot out of a Gunne of late invention, called the *Wind-Gun*. In the end of this Canon there are two holes with their Valves on the inside, to shut them close; one of them serving for the admission of aire, and the other for the letting of it out. Also to that

end which ſerves for the receiving in of aire, there is joyned another Canon of the ſame metal and bigneſs, in which there is fitted a Rammer, which is perforated, and hath alſo a Valve opening towards the former Canon. By the help of this Valve the Rammer is eaſily drawn back, and letteth in aire from without; and being often drawn back and returned again with violent ſtrokes, it forceth ſome part of that aire into the former Canon, ſo long, till at laſt the reſiſtance of the encloſed aire is greater then the force of the ſtroke. And by this means men think there is now a greater quantity of aire in the Canon then there was formerly, though it were full before. Alſo the aire thus forced in, how much ſoever it be, is hindered from getting out again by the foreſaid Valves, which the very endeavour of the aire to get out doth neceſſarily ſhut. Laſtly, that Valve being opened which was made for the letting out of the aire, it preſently breaketh out with violence, & driveth the bullet before it with great force and velocity.

As for the cauſe of this, I could eaſily attribute it (as moſt men do) to Condenſation, and think that the aire, which had at the firſt but its ordinary degree of Rarity, was afterwards by the forcing in of more aire condenſed, and laſt of all rarified again by being let out and reſtored to its natural liberty. But I cannot imagine how the ſame place can be alwayes full, and nevertheleſs contain ſometimes a greater, ſometimes a leſs quantity of matter; that is to ſay, that it can be fuller then full. Nor can I conceive, how Fulneſs can of it ſelf be an efficient cauſe of motion. For both theſe are impoſſible. Wherefore we muſt ſeek out ſome other poſſible cauſe of this *Phænomenon*. Whilſt therefore the Valve wch ſerves for the letting in of aire is opened by the firſt ſtroke of the Rammer, the aire within doth with equal force reſiſt the entering of the aire from without; ſo that the endeavours between the internal and external aire are oppoſite, that is, there are two oppoſite motions, whileſt the one goeth in and the other cometh out; but no augmentation at all of aire within the Canon. For there is driven out by the ſtroke as much pure aire which paſſeth between the Rammer and the ſides of the Canon, as there is forced in of aire impure by the ſame ſtroke. And thus by many forcible ſtrokes the quantity of ſmall hard Bodies will be encrea-

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sed within the Canon, and their motions also will grow stronger and stronger as long as the matter of the Canon is able to endure their force; by which if it be not broken, it will at least be urged every way by their endeavour to free themselves; and as soon as the Valve which serves to let them out is opened, they will fly out with violent motion, and carry with them the bullet which is in their way. Wherefore I have given a possible cause of this *Phænomenon*.

12 Water, contrary to the custome of Heavy Bodies, ascendeth in the Weather-glasse; but it doth it when the aire is cold; for when it is warme it descendeth againe. And this Organ is called a *Thermometer*, or *Thermoscope*, because the degrees of Heat and Cold are measured and marked by it. It is made in this manner. Let A B C D (in the 5th figure) be a vessel full of water, and E F G a hollow Cylinder of glasse, closed at E, and open at G. Let it be heated, and set upright within the water to F; and let the open end reach to G. This being done, as the aire by little and little grows colder, the water will ascend slowly within the Cylinder from F towards E; till at last the externall and internall aire coming to be both of the same temper, it will neither ascend higher, nor descend lower, till the temper of the aire be changed. Suppose it therefore to be setled any where, as at H. If now the heat of the aire be augmented, the water will descend below H; and if the heat be diminished, it will ascend above it. Which though it be certainely known to be true by experience, the cause nevertheless hath not as yet been discovered.

In the 6 and 7 articles of the 27th chapter (where I consider the cause of Cold) I have shewne, that fluid Bodies are made colder by the pressure of the aire, that is to say, by a constant Wind that presseth them. For the same cause it is, that the Superficies of the water is pressed at F; and having no place to which it may retire from this presure besides the cavity of the Cylinder between H and E, it is therefore necessarily forced thither by the Cold, and consequently it ascendeth more or lesse, according as the Cold is more or lesse encreased. And againe, as the Heat is more intense, or the Cold more remisse, the same water will be depressed more or lesse by its own Gravity, that is to say, by the cause of Gravity above explicated.

13 Also Living creatures, though they be Heavy, can by Leaping, Swimming & Flying raise themselvs to a certain degree of height. But they cannot do this except they be supported by some resisting Body, as the Earth, the Water and the Aire. For these motions have their beginning from the contraction (by the helpe of the Muscles) of the Body animate. For to this contraction there succeedeth a distension of their whole Bodies; by which distension, the Earth, the Water or the Aire which supporteth them, is pressed; and from hence, by the reaction of those pressed Bodies, Living Creatures acquire an endeavour upwards, but such, as by reason of the Gravity of their Bodies is presently lost againe. By this endeavour therefore it is, that Living creatures rayse themselues up a little way by Leaping, but to no great purpose; but by Swimming & Flying they raise themselves to a greater height; because before the effect of their endeavour is quite extinguished by the Gravity of their bodies, they can renew the same endeavour againe.

That by the power of the Soule, without any antecedent contraction of the Muscles, or the helpe of something to support him, any man can be able to raise his Body upwards, is a childish conceipt. For if it were true, a man might raise himselfe to what height he pleased.

14 The diaphanous Medium which surrounds the Eie on all sides, is invisible; Nor is Aire to be seen in Aire, nor Water in Water, nor any thing but that which is more opacous. But in the confines of two diaphanous Bodies, one of them may be distinguished from the other. It is not therefore a thing so very ridiculous for ordinary people to think all that Space empty, in which we say is Aire; it being the worke of Reason to make us conceive that the Aire is any thing. For by which of our Senses is it, that we take notice of the Aire, seeing we neither See, nor Hear, nor Tast, nor Smell, nor Feel it to be any thing? When we feel Heat, we do not impute it to the Air, but to the Fire; nor do we say the aire is Cold, but we our selves are Cold; and when we feel the Wind, we rather think something is comming, then that any thing is already come. Also we do not at al feel the waight of water in water, much lesse of air in air. That we come to know that to be a Body which we call Aire, it is by Reasoning; but it is from one

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Reaſon onely, namely, becauſe it is impoſſible for remote Bodies to work upon our Organs of Senſe but by the help of Bodies intermediate, without which we could have no ſenſe of them, till they came to be contiguous. Wherefore, from the Senſes alone, without reaſoning from effects, we cannot have ſufficient evidence of the nature of Bodies.

For there is under-ground in ſome Mines of Coles, a certain matter of a middle nature between Water and Aire, which nevertheleſs cannot by Senſe be diſtinguiſhed from aire; for it is as Diaphanous as the pureſt aire; and as farre as Senſe can judge, equally penetrable. But if we look upon the effect, it is like that of water. For when that matter breaks out of the Earth into one of thoſe Pits, it fils the ſame either totally, or to ſome degree; and if a Man, or Fire be then let down into it, it extinguiſhes them in almoſt as little time as water would do. But for the better underſtanding of this *Phænomenon*, I ſhall deſcribe the 6th figure. In which, let A B repreſent the pit of the Mine; and let part thereof, namely, C B be ſuppoſed to be filled with that matter. If now a lighted Cãdle be let down into it below C, it wil as ſuddenly be extinguiſhed, as if it were thruſt into water. Alſo if a grate filled with coles throughly kindled and burning never ſo brightly, be let down; as ſoon as ever it is below C, the fire will begin to grow pale, and ſhortly after (loſing its light) be extinguiſhed, no otherwiſe then if it were quenched in water. But if the grate be drawn up again preſently, whileſt the coles are ſtill very hot, the fire will by little and little be kindled again, and ſhine as before. There is indeed between this matter & water this conſiderable difference, that it neither wetteth, nor ſticketh to ſuch things as are put down into it, as water doth; which by the moiſture it leaveth, hindereth the kindling again of the matter once extinguiſhed. In like manner, if a Man be let down below C, he will preſently fall into a great difficulty of breathing, and immediatly after into a ſwoun, and die, unleſs he be ſuddenly drawn up again. They therefore that go down into theſe pits, have this cuſtome, that as ſoon as ever they feel themſelves ſick, they ſhake the rope by which they were let down, to ſignifie they are not well, and to the end that they may ſpeedily be pulled up again. For if a man be drawn out too late,

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void of ſenſe and motion, they digg up a Turff, and put his face and mouth into the freſh earth; by which means (unleſs he be quite dead) he comes to himſelf again by little and little, and recovers life by the breathing out (as it were) of that ſuffocating matter which he had ſucked in whileſt he was in the pit; almoſt in the ſame manner as they that are drowned come to themſelves again by vomiting up the water. But this doth not happen in all Mines, but in ſome onely; and in thoſe not alwayes, but often. In ſuch Pits as are ſubject to it, they uſe this remedy. They dig another pit, as D E, cloſe by it, of equal depth; and joyning them both together with one common channel E F, they make a Fire in the bottom E, which carries out at D the aire contained in the pit D E; and this draws with it the aire contained in the channel E F; which in like manner is followed by the noxious matter contained in C B; & by this means the pit is for that time made healthful. Out of this Hiſtory (which I write onely to ſuch as have had experience of the truth of it, without any deſigne to ſupport my Philoſophy with Stories of doubtful credit) may be collected the following poſſible cauſe of this *Phænomenon*; namely, that there is a certain matter, fluid, & moſt tranſparent, and not much lighter then water; which breaking out of the Earth fills the Pit to C; and that in this matter, as in water, both Fire and Living creatures are extinguiſhed.

15 About the nature of Heavy Bodies, the greateſt difficulty ariſeth from the contemplation of thoſe things which make other Heavy Bodies aſcend to them; ſuch are *Jet*, *Amber*, and the *Loadſtone*. But that which troubles men moſt is the Loadſtone, which is alſo called *Lapis Herculeus*; a ſtone, though otherwiſe deſpicable, yet of ſo great power, that it taketh up Iron from the Earth, and holds it ſuſpended in the aire, as *Hercules* did *Antæus*. Nevertheleſs, we wonder at it ſomewhat the leſs, becauſe we ſee Jet draw up Straws, which are Heavy Bodies, though not ſo Heavy as Iron. But as for Jet, it muſt firſt be excited by rubbing, that is to ſay, by motion to and fro; whereas the Loadſtone hath ſufficient excitation from its own nature, that is to ſay, from ſome internal principle of motion peculiar to it ſelf. Now whatſoever is moved, is moved by ſome contiguous and moved Body, as hath been formerly demonſtrated. And from hence it follows evidently, that the firſt endeavour

which

which Iron hath towards the Loadſtone, is cauſed by the motion of that aire which is contiguous to the Iron. Alſo that this motion is generated by the motion of the next aire, and ſo on ſucceſſively, till by this ſucceſſion we find that the motion of all the intermediate air taketh its beginning from ſome motion which is in the Loadſtone it ſelf; which motion (becauſe the Loadſtone ſeems to be at reſt) is inviſible. It is therefore certain, that the attractive power of the Loadſtone is nothing elſe but ſome motiõ of the ſmalleſt particles thereof. Suppoſing therefore that thoſe ſmall Bodies of which the Loadſtone is (in the bowels of the Earth) compoſed, have by nature ſuch motion or endeavour as was above attributed to Jet, namely a reciprocal motiõ in a line too ſhort to be ſeen, both thoſe ſtones wil have one & the ſame cauſe of attraction. Now in what manner, and in what order of working this cauſe produceth the effect of attraction, is the thing to be enquired. And firſt we know, that when the ſtring of a Lute or Viol is ſtricken, the Vibration, that is, the reciprocal motion of that ſtring in the ſame ſtraight Line, cauſeth like Vibration in another ſtring which has like tenſion. We know alſo, that the dregs or ſmall ſands which ſink to the bottom of a Veſſel, will be raiſed up from the bottom by any ſtrong and reciprocal agitation of the water ſtirred with the hand or with a ſtaff. Why therefore ſhould not reciprocal motion of the parts of the Loadſtone contribute as much towards the moving of Iron? For if in the Loadſtone there be ſuppoſed ſuch reciprocal motion, or motion of the parts forwards and backwards, it will follow, that the like motion will be propagated by the aire to the Iron, and conſequently that there will be in all the parts of the Iron the ſame reciprocations or motions forwards and backwards. And from hence alſo it will follow, that the intermediate aire between the Stone and the Iron will by little and little be thruſt away; and the aire being thruſt away, the Bodies of the Loadſtone and the Iron will neceſſarily come together. The poſſible cauſe therefore why the Loadſtone and Jet draw to them, the one Iron, the other Strawes, may be this, that thoſe attracting Bodies have reciprocal motion either in a ſtraight line, or in an Elliptical line, when there is nothing in the nature of the attracted Bodies which is repugnant to ſuch a motion.

But

But why the Loadstone (if with the help of Cork it float at liberty upon the top of the water) should from any position whatsoever so place it self in the plain of the Meridian, as that the same points which at one time of its being at rest respect the Poles of the Earth, should at all other times respect the same Poles, the cause may be this, That the reciprocal motion which I supposed to be in the parts of the Stone, is made in a line parallel to the Axis of the Earth, and has been in those parts ever since the Stone was generated. Seeing therefore the Stone whilest it remains in the Mine, and is carried about together with the Earth by its diurnal motion, doth by length of time get a habit of being moved in a line which is perpendicular to the line of its reciprocal motion, it will afterwards, though its axis be removed from the parallel situation it had with the axis of the Earth, retain its endeavour of returning to that situation again; and all endeavour being the beginning of motion, and nothing intervening that may hinder the same, the Loadstone will therefore return to its former situation. For any piece of Iron that has for a long time rested in the plain of the Meridian, whensoever it is forced from that situation, and afterwards left to its own liberty again, will of it self return to lie in the Meridian again; which return is caused by the endeavour it acquired from the diurnal motion of the Earth in the parallel circles which are perpendicular to the Meridians.

If Iron be rubbed by the Loadstone drawn from one Pole to the other, two things will happen; one, that the Iron will acquire the same direction with the Loadstone, that is to say, that it will lie in the Meridian, and have its Axis and Poles in the same position with those of the Stone; the other, that the like Poles of the Stone and of the Iron will avoid one another, and the unlike Poles approach one another. And the cause of the former may be this, that Iron being touched by motion which is not reciprocal, but drawn the same way from Pole to Pole, there will be imprinted in the Iron also an endeavour from the same Pole to the same Pole. For seeing the Loadstone differs from Iron no otherwise then as Ore from Metal, there will be no repugnance at all in the Iron to receive the same motion which is in the Stone. From whence it follows, that seeing they are both affected alike by the diurnal motion

on of the Earth, they will both equally return to their ſituation in the Meridian whenſoever they are put frō the ſame. Alſo of the later this may be the cauſe, that as the Loadſtone in touching the Irō doth by its action imprint in the Iron an endeavour towards one of the Poles, ſuppoſe towards the North Pole; ſo reciprocally, the Iron by its action upon the Loadſtone doth imprint in it an endeavour towards the other Pole, namely towards the South Pole. It happens therefore in theſe reciprocations or motions forwards and backwards of the particles of the Stone and of the Iron betwixt the North & the South, that whileſt in one of them the motion is from North to South, and the return from South to North, in the other the motion wil be from South to North, & the return frō North to South; which motions being oppoſite to one another, and communicated to the Air, the North Pole of the Iron (whileſt the attraction is working) will be depreſſed towards the South Pole of the Loadſtone; or contrarily the North Pole of the Loadſtone will be depreſſed towards the South Pole of the Iron; and the Axes both of the Loadſtone and of the Iron will be ſituate in the ſame ſtraight line. The truth whereof is taught us by experience.

As for the propagation of this Magnetical vertue, not onely through the Aire, but through any other Bodies how hard ſo ever, it is not to be wondred at, ſeeing no motion can be ſo weak, but that it may be propagated infinitely through a ſpace filled with Body of any hardneſs whatſoever. For in a full Medium, there can be no motion which doth not make the next part yeild, and that the next, and ſo ſucceſſively without end; ſo that there is no effect whatſoever but to the production thereof ſomething is neceſſarily contributed by the ſeveral motions of all the ſeveral things that are in the World.

And thus much concerning the nature of Body in general; with which I conclude this my firſt Section of the Elements of Philoſophy. In the firſt, ſecond and third Parts, where the Principles of Ratiocination conſiſt in our own Underſtanding, that is to ſay, in the legitimate uſe of ſuch Words as we our ſelves conſtitute, all the Theoremes (if I be not deceived) are rightly demonſtrated. The fourth Part depends upon *Hypotheſes*; which unleſs we know them to be true, it is impoſſible for us to demonſtrate that thoſe

Causes which I have there explicated, are the true Causes of the things whose productions I have derived from them.

Nevertheless, seeing I have assumed no *Hypothesis*, which is not both possible and easie to be comprehended; and seeing also that I have reasoned aright from those Assumptions, I have withall sufficiently demonstrated that they may be the true Causes; wch is the end of Physical Contemplation. If any other man from other *Hypotheses* shall demonstrate the same, or greater things, there wil be greater praise and thanks due to him then I demand for my self, provided his *Hypotheses* be such as are conceivable. For as for those that say any thing may be moved or produced by *it Self*, by *Species*, by *its own Power*, by *Substantial Forms*, by *Incorporeal Substances*, by *Instinct*, by *Anteperistasis*, by *Antipathy*, *Sympathy*, *Occult Quality*, and other empty words of Schoolmen, their saying so is to no purpose.

And now I proceed to the *Phænomena* of Mans Body; Where I shall speak of the *Opticks*, and of the *Dispositions*, *Affections*, *and Manners* of Men (if it shall please God to give me life), and shew their Causes.

AD Cap XXVIII et XXX



fig: 3

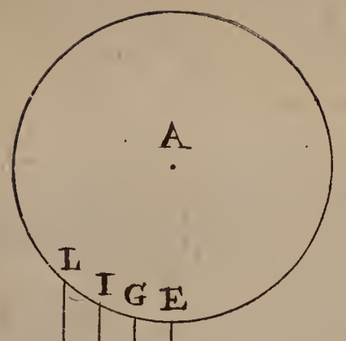


fig: 1.

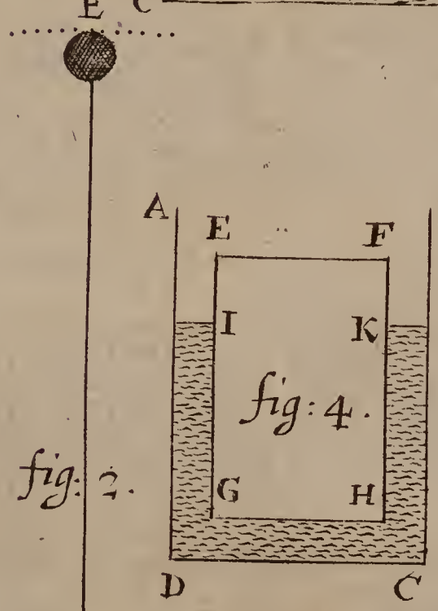


fig: 4.



fig: 5.

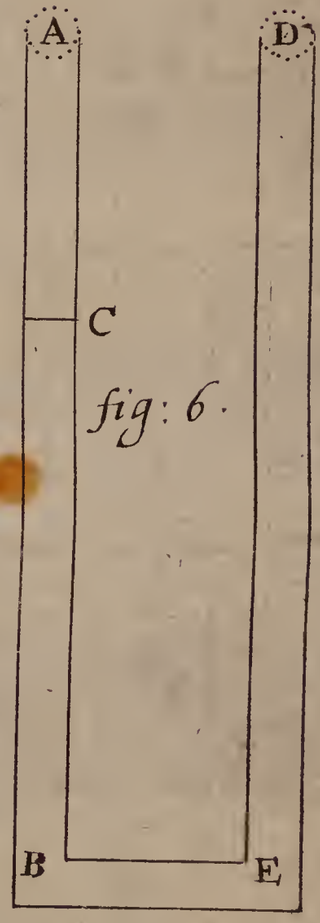


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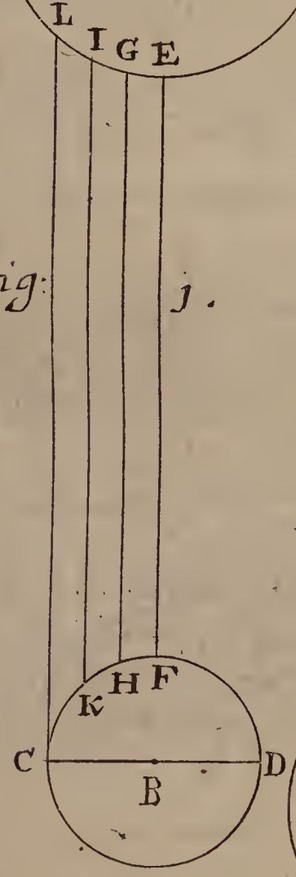


fig: 2.

SIX LESSONS

To the PROFESSORS of the

MATHEMATIQUES,

ONE OF

GEOMETRY,

THE OTHER OF

ASTRONOMY:

In the Chaires set up by the Noble and Learned Sir HENRY SAVILE, in the University of *Oxford*.

LONDON,
Printed by *J. M.* for *Andrew Crook*, at the *Green-Dragon*, in *Pauls* Church-yard.

SIX LESSONS

To be Perused at the

MATHEMATICAL

ONE OF

GEOMETRY,

THE OTHER OF

ASTRONOMY:

In the Chaires for up-by the Nobs and Learned Sir Henry Savile, in the University of Oxford.



Printed by J. M. for Edward Croft, at the Green-Dragon, in Pauls Church-yard.

To the Right Honourable,

Henry Lord Pierrepont,

Viſcount *Newarke*, Earle of *Kingſtone*, and Marquis of *Dorcheſter*.

My moſt Noble Lord,

NOt knowing on my own part any cauſe of the favour your Lordſhip has been pleaſed to expreſs towards me, unleſs it be the Principles, Method, and Manners you have obſerved and approved in my Writings; and ſeeing theſe have all been very much reprehended by men to whom the name of Publique Profeſſors hath procured reputation in the Univerſity of *Oxford*; I thought it would be a forfeiture of your Lordſhips good opinion, not to juſtifie my ſelf in publique alſo againſt them. Which, whether I have ſufficiently performed or not in the ſix following Leſſons addreſſed to the ſame Profeſſors, I humbly pray your Lordſhip to conſider. The volume it ſelf is too ſmall to be offered to you as a Preſent; but to be brought before you as a Controverſie it is perhaps the better for being ſhort. Of Arts, ſome are demonſtrable, others indemonſtrable; and demonſtrable are thoſe the conſtruction of the Subject whereof is in the power of the Artiſt himſelf; who in his demonſtration does no more but deduce the Conſequences of his own operation. The reaſon whereof is this, that the Science of every Subject is derived from a præcognition of the Cauſes, Generation, and Conſtruction of the ſame; and conſequently where the Cauſes are known, there is place for Demonſtration; but not where the Cauſes are to ſeek for. Geometry therefore is demonſtrable; for the Lines and Figures from which we reaſon are drawn and deſcribed by our ſelves; and Civill

vill Philoſophy is demonſtrable becauſe, we make the Common-wealth our ſelves. But becauſe of Naturall Bodies we know not the Conſtruction, but ſeek it from the Effects, there lyes no demonſtration of what the Cauſes be we ſeek for, but onely of what they may be.

And where there is place for Demonſtration, if the firſt Principles, that is to ſay, the Definitions contain not the Generation of the Subject; there can be nothing demonſtrated as it ought to be. And this in the three firſt Definitions of *Euclide* ſufficiently appeareth. For ſeeing he maketh not, nor could make any uſe of them in his Demonſtrations, they ought not to be numbered among the Principles of Geometry. And *Sextus Empiricus* maketh uſe of them (miſunderſtood, yet ſo underſtood as the ſaid Profeſſors underſtand them) to the overthrow of that ſo much renouned Evidence of Geometry. In that part therefore of my Book where I treat of Geometry, I thought it neceſſary in my Definitions to expreſs thoſe Motions by which Lines, Superficies, Solids and Figures were drawn and deſcribed; little expecting that any Profeſſor of Geometry ſhould finde fault therewith; but on the contrary ſuppoſing I might thereby not only avoid the Cavils of the Scepticks, but alſo demonſtrate divers Propoſitions which on other Principles are indemonſtrable. And truly, if you ſhall finde thoſe my Principles of Motion made good, you ſhall find alſo that I have added ſomething to that which was formerly extant in Geometry.

For firſt from the ſeventh Chapter of my Book *de Corpere* to the thirteenth, I have rectified and explained the Principles of the Science, *id eſt*, I have done that buſineſs for which Doctor *Wallis* receives the wages. In the ſeventh, I have exhibited and demonſtrated the proportion of the Parabola and Parabolaſters to the Parallelograms of the ſame height and baſe; which (though ſome of the propoſitions were extant without their demonſtration) were never before demonſtrated, nor are by any other then this method demonſtrable.

In the eighteenth, (as it is now in Engliſh) I have demonſtrated the (for any thing I yet perceive) Equation between the crooked line of a Parabola or any Parabolaſter and a ſtraight line.

In

In the twenty-third, I have exhibited the Center of Gravity of any Sector of a Sphere.

Laſtly, the twenty-fourth, (which is of the nature of Refracti- and Reflexion) is almoſt all new.

But your Lordſhip will ask me what I have done in the twentieth, about the Quadrature of the Circle. Truely, my Lord, not much more then before. I have let ſtand there that which I did before condemn, not that I think it exact, but partly becauſe the Diviſion of Angles may be more exactly performed by it then by any organicall way whatſoever; and I have attempted the ſame by another Method, which ſeemeth to me very naturall, but of calculation difficult and ſlippery. I call them only Aggreſſions, retaining nevertheleſs the formall manner of Aſſertion uſed in Demonſtration. For I dare not uſe ſuch a doubtfull word as *Videtur*, becauſe the Profeſſors are preſently ready to oppoſe me with a *Videtur quod non*. Nor am I willing to leave thoſe Aggreſſions out, but rather to try if it may be made paſs for lawfull (in ſpight of them that ſeek honour not from their own performances but from other mens failings) amongſt many difficult undertakings carryed through at once) to leave one and the greateſt for a time behind; and partly becauſe the method is ſuch as may hereafter give further light to the finding out of the exact truth.

But the Principles of the Profeſſors that reprehend theſe of mine, are ſome of them ſo void of ſenſe, that a man at the firſt hearing, whether Geometrician or not Geometrician muſt abhor them. As for example,

1. That two equall Proportions are not double to one of the ſame Proportions.

2. That a Proportion is double, triple, &c. of a Number, but not of a Proportion.

3. That the ſame Body, without adding to it, or taking from it, is ſometimes Greater, and ſometimes leſs.

4. That a Quantity may grow leſs and leſs Eternally, ſo as at laſt to be equall to another Quantity; or (which is all one) that there is a Laſt in Eternity.

5. That the nature of an Angle conſiſteth in that which lyes between

between the lines that comprehend the Angle in the very point of their concourſe; that is to ſay, An Angle is the Superficies which lyes between the two Points which touch, or (as they underſtand a Point) the Superficies that lyes between the two Nothings which touch.

6. That the Quotient is the Proportion of the Diviſion to the Dividend.

Upon theſe and ſome ſuch other Principles is grounded all that Doctor *Wallis* has ſaid not onely in his *Elenchus* of my Geometry, but alſo in his Treatiſes of the Angle of Contact, and in his *Arithmetica Infiniterum*; which two laſt I have here in two or three leaves wholly and cleerly confuted. And I verily believe that ſince the beginning of the world there has not been, nor ever ſhall be ſo much abſurdity written in Geometry, as is to be found in thoſe books of his; with which there is ſo much preſumption joyned, that an ἀποκατάστασις of the like conjunction cannot be expected in leſs then a Platonick year. The cauſe whereof I imagine to be this, that he miſtook the ſtudy of *Symboles* for the ſtudy of *Geometry*, and thought *Symbolicall* writing to be a new kinde of Method, and other mens Demonſtrations ſet down in *Symboles* new Demonſtrations. The way of *Analyſis* by Squares, Cubes, &c. is very antient, and uſefull for the finding out whatſoever is contained in the nature and generation of rectangled Plains (which alſo may be found without it) and was at the higheſt in *Vieta*; but I never ſaw any thing added thereby to the Science of Geometry, as being a way wherein men go round from the Equality of rectangled Plains to the Equality of Proportion, and thence again to the Equality of rectangled Plains; wherein the *Symboles* ſerve only to make men go faſter about, as greater Winde to a Winde-mill.

It is in Sciences as in Plants; Growth and Branching is but the Generation of the Root continued; nor is the Invention of Theoremes any thing elſe but the knowledge of the Conſtruction of the Subject proſecuted. The unſoundneſs of the Branches are no prejudice to the Roots; nor the Faults of Theoremes to the Principles. And active Principles will correct falſe Theo-

remes

remes if the Reaſoning be good; but no Logique in the world is good enough to draw evidence out of falſe or unactive Principles. But I detain your Lordſhip too long. For all this will be much more manifeſt in the following Diſcourſes: wherein I have not onely explained and rectified many of the moſt important Principles of Geometry: but alſo by the examples of thoſe errors which have been committed by my Reprehenders, made manifeſt the evil Conſequence of the Principles they now proceed on. So that it is not only my own Defence that I here bring before you, but alſo a poſitive doctrine concerning the true Grounds, or rather Atomes of Geometry: which I dare only ſay are very ſingular: but whether they be very good or not, I ſubmit to your Lordſhips judgement. And ſeeing you have been pleaſed to beſtow ſo much time (with great ſucceſs) in the reading of what has been written by other men in all kindes of Learning. I humbly pray your Lordſhip to beſtow alſo a little time upon the reading of theſe few and ſhort Leſſons; and if your Lordſhip finde them agreeable to your Reaſon and Judgement, let me (notwithſtanding the clamour of my Adverſaries) be continued in your good opinion, and ſtill retain the honour of being

My moſt Noble Lord,

Your Lordſhips moſt
Humble and Obliged
Servant.
THOMAS HOBBES.

London, *June* 10.
1656.

ERRATA.

PAge 2. l. 11. for *Art*, r. *act*, l. 12. for ἐφάμοσις, r. εφαρμοσις, p. 6. l. 23. for *Mr.* r. *Sir*, p. 21. l. 22. for *Propoſition*, r. *Proportion*, p. 55. l. 20. for *Senctoribus*, r. *Senatoribus*, p. 56. l. 8. for *Serberius*, r. *Serberius*.

LESSONS
OF THE
PRINCIPLES
OF
Geometry, &c.

To the egregious Professors of the Mathematicks, one of Geometry, the other of Astronomy, in the Chairs set up by the Noble and Learned Sir *Henry Savile*, in the University of *Oxford*.

Lesson I.

Suppose (most egregious Professors) you know already that by Geometry (though the word import no more but the measuring of Land) is understood no less the measuring of all other Quantity, then that of Bodies. And though the Definition of Geometry serve not for proof, nor enter into any Geometricall Demonstration, yet for understanding of the Principles of the Science, and for a Rule to judge by, who is a Geometrician, and who is not, I hold it necessary to begin therewith.

Geometry is the Science of determining the quantity of any thing, not measured, by comparing it with some other Quantity, or Quantities measured. Which Science therefore whosoever shall go about to teach, must first be able to tell his Disciple what Measuring or Dimension is; by what each several kind of Quantity is Measured; what Quantity is; & what are the several kinds thereof. Therefore as they who handle any one part of Geometry, determine by Definition the signifie

cation of every word which they make the Subject, or Prædicate of any Theoreme they undertake to demonſtrate; ſo muſt he which intendeth to write a whole body of Geometry, Define and Determine the meaning of whatſoever word belongeth to the whole Science; The deſign of *Euclid* was to demonſtrate the Properties of the five regular bodies mentioned by *Plato*; in which Demonſtrations there was no need to alledge for Argument the Definition of Quantity, which it may be was the cauſe he hath not any where Defined it, but done what he undertook without it. And though having perpetually occaſion to ſpeak of meaſure, he hath not Defined Meaſure; yet inſtead thereof he hath in the beginning of his firſt Element, aſſumed an Axiome which ſerveth his turn ſufficiently, as to the meaſure of lines, which is the eighth Axiome; That thoſe things which lye upon one another all the way (called by him ἐφαρμόζοντα) are equall. Which Axiome is nothing elſe but a deſcription of the Art of Meaſuring Length, and Superficies. For this ἐφάρμοσις can have no place in ſolid bodies, unleſs two bodies could at the ſame time be in one place. But amongſt the Principles of Geometry univerſall, the Definitions are neceſſary, both of Quantity and Dimenſion.

Quantity is that which is ſignified by what we anſwer to him that asketh, *How much* any thing is, and thereby determine the magnitude thereof. For magnitude being a word indefinite, if a man ask of a thing *Quantum eſt*, that is, *How much it is*; we do not ſatisfie him by ſaying it is magnitude or quantity, but by ſaying it is *Tantum*, *ſo much*. And they that firſt called it in Greek ποσότης, and in Latine *Quantity*, might more properly have called it, in Latine *Tantity*, and in Greek τοσότης; and we, if we allowed our ſelves the Eloquence of the Greeks and Latines, ſhould call it the *So-muchneſs*.

There is therefore to every thing concerning which a man may ask without abſurdity how *much it is*, a certain Quantity belonging; determining the magnitude to be *ſo much*. Alſo whereſoever there is *more* and *leſs*, there is one kinde of Quantity or other. And firſt, there is the Quantity of Bodies, and that of three kindes; Length, which is by one way of Meaſurings; Superficies made of the complication of two Lengths, or the Meaſure taken, two wayes; and Solid which is the complication of three lengths, or of the meaſure taken three wayes; for breadth or thickneſs are but other Lengths. And the Science of Geometry ſo far forth as it contemplateth Bodies onely, is no more but by Meaſuring the length of one or more lines, and by the poſition of others known in one and the ſame Figure, to Determine by ratiocination, how much is the Superficies; and by Meaſuring Length, Breadth, and Thickneſs, to determine the Quantity of the whole Body. Of this kinde of Magnitudes and Quantities the Subject is Body.

And becauſe for the computing of the Magnitudes of Bodies, it is not neceſſary that the Bodies themſelves ſhould be preſent (the Ideas and memory of them ſupplying their preſence) we reckon upon thoſe Imaginary Bodies, which are the Quantities themſelves, and ſay the length is ſo great, the breadth ſo great, &c. which in truth is no better then to ſay the length is ſo long, or the breadth ſo broad, &c. But in the mind of an inteligent man it breedeth no miſtake.

Beſides the Quantity of Bodies, there is a Quantity of Time. For ſeeing men, without abſurdity do ask how much it is; by anſwering *Tantum*, *ſo much*, they make manifeſt there is a a quantity that belongeth unto Time, namely, a Length. And becauſe Length cannot be an accident of Time, which is it ſelf an accident, it is the accident of a Body; and conſequently the length of the Time, is the Length of the Body; by which Length or Line, we determine how much the Time is, ſuppoſing ſome Body to be moved over it.

Alſo we not improperly ask with *how much* Swiftneſs a Body is moved; and conſequently there is a Quantity of Motion or Swiftneſs, and the meaſure of that Quantity is alſo a line. But then again, we muſt ſuppoſe another motion, which determineth the time of the former. Alſo of Force there is a Queſtion of *How much*, which is to be anſwered by *So much*; that is, by Quantity. If the Force conſiſt in Swiftneſs, the Determination is the ſame with that of Swiftneſs, namely, by a Line; if in Swiftneſs and Quantity of Body joyntly, then by a Line and a Solid; or if in quantity of Body onely (as Weight) by a Solid onely.

So

So also is Number Quantity; but in no other sense then as a line is Quantity divided into equall parts.

Of an Angle, which is of two Lines whatsoever they be, meeting in one point the digression or openess in other points, it may be asked how great is that digression; This Question is answered also by Quantity. An Angle therefore hath Quantity, though it be not the subject of Quantity; for the body onely can be the subiect, in which Body those stradling line are marked.

And because two lines may be made to divaricate by two causes; one, when having one end common, and immoveable, they depart one from another at the other ends circularly, and this is called simply an Angle; and the Quantity thereof is the Quantity of the Arch, which the two lines intercept.

The other cause is the bending of a straight line into a circular or other crooked line, till it touch the place of the same line, whilst it was straight, in one onely point. And this is called an Angle of contingence. And because the more it is bent, the more it digresseth from the Tangent, it may be asked *how much* more? and therefore the answer must be made by Quantity; and consequently an Angle of Contingence hath its Quantity as well as that which is called simply an Angle. And in case the digression of two such crooked lines from the Tangent be uniform as in Circles, the Quantity of their digression may be determined. For if a straight line be drawn from the point of Contact, the digression of the lesser Circle will be to the digression of the greater Circle, as the part of the line drawn from the point of Contact, and intercepted by the Circumference of the greater Circle is to the part of the same line intercepted by the Circumference of the Lesser Circle, or, which is all one, as the greater Radius is to the lesser Radius. You may guess by this what will become of that part of your last Book, where you handle the Question of the Quantity of an Angle of Contingence.

Also there lyeth a Question of *how much Comparatively* one magnitude is to another magnitude, as how much water is in a Tun in respect of the Ocean, how much in respect of a Pint; *little* in the first respect, *much* in the Latter. Therefore the Answer must be made by some respective Quantity. This respective Quantity is called *Ratio* and Proportion, and is determined by the Quantity of their differences; and if their differences be compared also with the Quantities themselves that differ, it is called simply Proportion, or Proportion Geometricall. But if the differences be not so compared, then it is called Proportion Arithmeticall. And where the difference is none, there is no Quantity of the Proportion, which in this case is but a bare comparison.

Also concerning Heat, Light, and divers other Qualities, which have degrees, there lyeth a question of *how much*, to be answered by a *so much*, and consequently they have their Quantities, though the same with the Quantity of Swiftness: because the intensions and remissions of such Qualities are but the intensions and remissions of the Swiftness of that motion by which the Agent produceth such a quality. And as Quantity may be considered in all the operations of Nature, so also doth Geometry run quite thorow the whole body of Naturall Philosophy.

To the Principles of Geometry the definition appertaineth also of a *Measure*, which is this, *One Quantity is the Measure of another Quantity, when it, or the Multiple of it, is Coincident in all points, with the other Quantity.* In which Definition in stead of that ἐφαρμογὴ of *Euclid*, I put Coincidence. For that superposition of Quantities by which they render the word ἐφαρμογὴ cannot be understood of Bodies, but only of Lines and Superficies. Nevertheless many Bodies may be Coincident successively with one and the same place, and that place will be their Measure, as we see practised continually in the measuring of Liquid Bodies, which Art of Measuring may properly be called ἐφαρμοσις, but not Superposition.

Also the definitions of *Greater*, *Less*, and *Equall*, are necessary Principles of Geometry. For it cannot be imagined that any Geometrician should demonstrate to us the Equality, and Inequali-

quality of magnitudes, except he tell us firſt what thoſe words do ſignifie. And it is a wonder to me, that *Euclide* hath not any where defined what are Equals, or at leaſt, what are Equall Bodies, but ſerveth his turn throughout with that forementioned ἐφάρμοσις, which hath no place in Solids, nor in Time, nor in Swiftneſs, nor in Circular, or other crooked lines; and therefore no wonder to me, why Geometry hath not proceeded to the calculation neither of crooked lines, nor ſufficiently of Motion, nor of many other things, that have proportion to one another.

Equall Bodies, Superficies, and Lines are thoſe of which every one is capable of being co-incident with the place of every one of the reſt. And Equall Times wherein with one and the ſame Motion Equall lines are deſcribed. And Equally Swift are thoſe Motions by which we run over equall ſpaces in any time determined by any other motion. And univerſally all Quantities are Equall, that are meaſured by the ſame number of the ſame Meaſures.

It is neceſſary alſo to the Science of Geometry, to define what Quantities are of one and the ſame kind, which they call *Homogeneous*; the want of which definitions hath produced thoſe wranglings (which your Book *De Angulo Contactus* will not make to ceaſe) about the Angle of Contingence.

Homogeneous quantities are thoſe which may be compared by (ἐφάρμοσις) application of their Meaſures to one another; So that Solids and Superficies are Heterogeneous quantities, becauſe there is no coincidence or application of thoſe two dimenſions.

No more is there of Line and Superficies, nor of Line and Solid, which are therefore Heterogeneous. But Lines and Lines, Superficies and Superficies, Solids and Solids, are Homogeneous.

Homogeneous alſo are Line, and the Quantity of Time; becauſe the Quantity of Time is meaſured by application of a Line to a Line; for though Time be no Line, yet the Quantity of Time is a Line, and the length of two Times is compared by the length of two Lines.

Weight, and Solid have their Quantity Homogeneous, becauſe they meaſure one another by application, to the beam of a Ballance. Line and Angle ſimply ſo called have their Quantity Homogeneous, becauſe their meaſure is an Arch or Arches of a Circle applicable in every point to one another.

The Quantity of an Angle ſimply ſo called, and the Quantity of an Angle of Contingence are Heterogeneous. For the meaſures by which two Angles ſimply ſo called are compared, are in two coincident Arches of the ſame Circle; but the meaſure by which an Angle of Contingence is meaſured, is a ſtraight line intercepted between the point of Contact, and the Circumference of the Circle; and therefore one of them is not applicable to the other; and conſequently of theſe two ſorts of Angles, the Quantities are Heterogeneous. The Quantities of two Angles of Contingence are Homogeneous; for they may be meaſured by the ἐφάρμοσις of two Lines, whereof one extream is common, namely, the point of Contact, the other Extreams, are in the Arches of the two Circles.

Beſides this knowledge of what is Quantity and Meaſure, and their ſeverall ſorts, it behoveth a Geometrician to know why, and of what they are called Principles. For not every Propoſition that is evident, is therefore a Principle. A Principle is the beginning of ſomething. And becauſe Definitions are the beginnings or firſt Propoſitions of Demonſtration, they are therefore called Principles, Principles, I ſay, of Demonſtration. But there be alſo neceſſary to the teaching of Geometry other Principles, which are not the beginnings of Demonſtration, but of Conſtruction, commonly called Petitions; as that it may be granted *that a man can draw a ſtraight Line, and produce it; and with any* Radius, *on any Center deſcribe a Circle,* and the like. For that a man may be able to deſcribe a ſquare, he muſt firſt be able to draw a ſtraight line; and before he can deſcribe an Æquilaterall Triangle, he muſt be able firſt to deſcribe a Circle. And theſe Petitions are therefore properly called Principles (not of Demonſtration, but) of Operation. As for the commonly received third ſort of Principles, called *Common Notions,* they are Principles, onely by permiſſion of him that is the Diſciple; who being ingenuous, and comming not to cavill but to learn, is content to receive them (though Demonſtrable.)

Demonstrable) without their Demonstrations. And though Definitions be the onely Principles of Demonstration, yet it is not true that every Definition is a Principle. For a man may so precisely determine the signification of a word, as not to be mistaken, yet may his Definition be such, as shall never serve for proof of any Theoreme, nor ever enter into any demonstration (such as are some of the Definitions of *Euclide*) and consequently can be no beginnings of Demonstration, that is to say, no Principles.

All that hitherto hath been said, is so plain and easie to be understood, that you cannot (most Egregious Professors) without discovering your ignorance to all men of reason, though no Geometricians, deny it. And the same (saying that the words are all to be found in Dictionaries) new; also to him that means to learn, not onely the Practice, but also the Science of Geometry necessary, and (though it grieve you) mine. And now I come to the Definitions of *Euclide*.

The first is of a Point. Σημεῖον &c. *Signum est, cujus est pars nulla*, that is to say, *a Marke is that of which there is no part*. Which definition, not onely to a candid, but also to a rigid construer is sound and usefull. But to one that neither will interpret candidly, nor can interpret accurately, is neither usefull nor true. Theologers say the Soul hath no part, and that an Angel hath no part, yet do not think that Soul or Angel is a point. A mark, or as some put in stead of it σίγμη, which is a mark with a hot Iron, is visible; if visible then it hath Quantity, and consequently may be divided into parts innumerable. That which is indivisible is no Quantity; and if a point be not Quantity, seeing it is neither substance nor Quality, it is nothing. And if *Euclide* had meant it so in his definition, (as you pretend he did) he might have defined it more briefly, (but ridiculously) thus, *a Point is nothing*. Sir *Henry Savile* was better pleased with the Candid interpretation of *Proclus*, that would have it understood respectively to the matter of Geometry. But what meaneth this *respectively to the matter of Geometry*? It meaneth this, that no Argument in any Geometricall demonstration should be taken from the Division, Quantity, or any part of a Point; which is as much as to say, a Point is that whose Quantity is not drawn into the demonstration of any Geometricall conclusion; or (which is all one) whose Quantity is not considered.

An accurate interpreter might make good the definition thus; *a Point is that which is undivided*; and this is properly the same with *cujus non est pars*: for there is a great difference between *undivided*, and *indivisible*, that is, between *cujus non est pars*, and *cujus non potest esse pars*. Division is an Act of the understanding; the understanding therefore is that which maketh parts, and there is no part where there is no consideration but of one. And consequently *Euclides* definition of a Point, is accurately true, and the same with mine, which is, that *a Point is that Body whose Quantity is not considered*. And *considered*, is that (as I have defined it, *Chap.* 1. at the end of the third Article) which is not put to account in demonstration.

Euclide therefore seemeth not to be of your opinion, that say a Point is nothing. But why then doth he never use this definition in the Demonstration of any Proposition? Whether he useth it expressly or no, I remember not; but the 16th. Proposition of the third Book without the force of this definition is undemonstrated.

The second Definition is of a Line. γραμμὴ δὲ μῆκος ἀπλατες: *Linea est longitudo latitudinis expers, a Line is length which hath no breadth*; and if candidly interpreted, sound enough, though rigorously, not so. For to what purpose is it to say *Length not Broad*, when there is no such thing as a *broad length*. One Path may be broader then another Path, but not one Mile then another Mile; and it is not the Path but the Mile which is the ways Length. If therefore a man have any ingenuity he will understand it thus, *That a Line is a Body whose length is considered without its breadth*, else we must say absurdly a *broad length*; or untruly, that there be bodies which have length and yet no breadth; and this is the very sense which *Apollonius* (saith *Proclus*) makes of this Definition; *when we measure*, sayes he, *the length of a way, we take not in the breadth or depth, but consider onely one Dimension*. See this of

Proclus

Proclus cited by Sir *Henry Savile*, where you ſhall finde the very word *conſider*.

The fourth Definition is of a ſtraight line, thus. Εὐθεῖα γραμμή ἐστιν, &c. *Recta linea eſt quæ ex æquo ſua ipſius puncta inter jacet. A ſtraight line is that which lieth equally* (or perhaps evenly) *between its own Points.* This Definition is inexcuſable. Between what Points of its own can a ſtraight line lye but between its extreams? And how lies it evenly between them, unleſs it ſwarve no more from ſome other line which hath the ſame Extreams, one way then another? And then why are not between the ſame Points both the lines ſtraight? How bitterly, and with what inſipide jeſts, would you have reviled *Euclide* for this, if living now he had written a Leviathan. And yet there is ſomewhat in this Definition to help a man, not onely to conceive the nature of a ſtraight line (for who doth not conceive it?), but alſo to expreſs it. For he meant perhaps to call a ſtraight line that which is all the way from one Extream to another, equally diſtant from any two or more ſuch Lines as being like and equall have the ſame Extreams. So the Axis of the Earth is all the way equally diſtant from the circumference of any two or more Meridians. But then before he had defined a ſtraight line, he ſhould have defined what lines are *like*, and what are *equall*. But it had been beſt of all, firſt to have defined crooked lines, by the poſſibility of a diduction or ſetting further aſunder of their extreams; and then ſtraight Lines, by the impoſſibility of the ſame.

The ſeventh Definition, which is that of a plain Superficies, hath the ſame fault.

The Eighth is of a plain Angle. Ἐπίπεδος γωνία ἐστὶν ἡ ἐν ἐπιπέδῳ &c. *Angulus Planus eſt duarum Linearum in plano ſe mutuo tangentium, & non indirectum jacentium, alterius ad alteram inclinatio. A Plain Angle is the inclination one towards another of two lines that touch one another in the ſame Plain, and lye not in the ſame ſtraight line*: Beſides the faults here obſerved by Mr. *Henry Savile*, as the clauſe of not lying in the ſame ſtraight line, and the obſcurity or æquivocation of the word *inclination*, there is yet another, which is, that by this Definition two Right Angles together taken, are no Angle; which is a fault which you ſomewhere (asking leave to uſe the word *Angle* καταχρησικῶς, acknowledge, but avoid not. For in Geometry, where you confeſſe there is required all poſſible accurateneſs, every καταχρῆσις is a fault. Beſides you ſee by this Definition, that *Euclide* is not of your, but of *Clavius* his opinion. For it is manifeſt that the two lines which contain an Angle of Contact, incline one towards another, and come together in a Point, and lye not in the ſame ſtreight line, and conſequently make an Angle.

The thirteenth Definition is exact, but makes againſt your Doctrine, that a Point is nothing. Examine it. Ὅρος ἐστὶν ὅ τινός ἐστι πέρας. *Terminus eſt quod alicujus extremum eſt. A Term or Bound is that which is the Extream of any thing* We had before, *The extreams of a Line are Points.* But what is in a Line the extream, but the firſt or laſt *part*, though you may make that part as ſmall as you will? A point is therefore a part, and nothing is no extream.

The fourteenth, Σχῆμα ἐστὶ τὸ ὑπό τινος ἤ τινων ὅρων περιεχόμενον. *Figura eſt* (*ſubaudi Quantitas*) *quæ ab aliquo, vel aliquibus terminis undiq; continetur ſive clauditur. A Figure is Quantity contained within ſome bound or bounds.* Or ſhortly thus, *A Figure is Quantity every way determined*, is in my opinion as exact a Definition of a Figure as can poſſibly be given, though it muſt not be ſo in yours. For this *determination* is the ſame thing with *circumſcription*; and whatſoever is any where (*ubicunq;*) *definitivè* is there alſo *circumſcriptivè*; and by this means, the diſtinction is loſt, by which Theologers, when they deny God to be in any place, ſave themſelves from being accuſed of ſaying he is no where; for that which is no where is nothing. This definition of *Euclide* cannot therefore poſſibly be embraced by you that carry double, namely, *Mathematicks* and *Theology*. For if you reject it, you will be caſt out of all Mathematick Schooles; and if you maintain it, from the Society of all School-Divines, and loſe the thanks of the Favour you have ſhewn (you the Aſtronomer) to Biſhop *Bramhall*.

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The fifteenth is of a Circle, Κύκλος ἐστὶ σχῆμα ἐπίπεδον, &c. *A Circle is a plain Figure comprehended by one line which is called the Circumference, to which Circumference all the ſtraight lines drawn from one of the points within the Figure, are equall to one another.* This is true. But if a man had never ſeen the generation of a Circle by the motion of a Compaſs or other æquivalent means, it would have been hard to perſwade him, that there was any ſuch Figure poſſible. It had been therefore not amiſs firſt to have let him ſee that ſuch a Figure might be deſcribed. Therefore ſo much of Geometry is no part of Philoſophy, which ſeeketh the proper paſſions of all things in the generation of the things themſelves.

After the fifteenth till the laſt or thirty fifth Definition all are moſt accurate, but the laſt, which is this, *Parallel ſtraight lines are thoſe which being in the ſame Plane, though infinitely produced both wayes ſhall never meet.* Which is leſſe accurate. For how ſhall a man know that there be ſtraight lines, which ſhall never meet, though both wayes infinitely produced? Or how is the Definition of Parallels, that is, of lines perpetually æquidiſtant good, wherein the nature of æquidiſtance is not ſignified? Or if it were ſignified, why ſhould it not comprehend, as well the Paralleliſm of Circular and other crooked lines, as of ſtreight, and as well of Superficies, as of lines? By Parallels is meant æquidiſtant both Lines and Superficies, and the word is therefore not well defined without defining firſt equality of diſtance. And becauſe the diſtance between two lines or Superficies, is the ſhorteſt line that can joyn them, there either ought to be in the definition, the *ſhorteſt diſtance*, which is that of the Perpendicular, and without inclination, or the diſtance in equall inclination, that is, in equall Angles. Therefore if Parallels be defined to be thoſe Lines or Superficies, where the Lines drawn from one to an other in equal Angles be equal, the Definition, as to like Lines, or like Superficies, will be Univerſall and Convertible. And if we add to this Definition, that the equall Angles be drawn not oppoſite wayes, it will be abſolute, and Convertible in all Lines and Superficies; and the definition will be this, *Parallels are thoſe Lines and Superficies between which every line drawn, in any Angle, is equall to any other line drawn in the ſame Angle the ſame way.* For by this Definition the diſtance between them will perpetually be equall, and conſequently they will never come nearer together, how much, or which way ſoever they be produced. And the converſe of it will be alſo true, *If two Lines, or two Superficies be Parallel, and a ſtraight line be drawn from one to the other, any other ſtraight line, drawn from one to the other in the ſame Angle, and the ſame way, will be equall to it.* This is manifeſtly true, and (moſt egregious Profeſſors) new, at leaſt to you.

And thus much for the Definitions placed before the firſt of *Euclides* Elements.

Before the third of his Elements is this Definition. *In circulo æqualiter diſtare à centro rectæ lineæ dicuntur, cum prependiculares quæ à centro in ipſas ducuntur ſunt æquales. In a circle two ſtraight Lines are ſaid to be equally diſtant from the Center, upon which the perpendiculars drawn from the Center are equall.* This is true; but it is rather an Axiome then a Definition, as being demonſtrable that the Perpendicular is the meaſure of the diſtance between a Point and a ſtraight or crooked Line.

Before the fifth Element the firſt Definition is of a Part. *Pars eſt magnitudo magnitudinis, minor majoris, cum minor metitur majorem. A Part is one magnitude of another, the leſſe of the greater, when the leſſe meaſureth the greater.* From which Definition it followeth, that more then the half is not a part of the whole. But becauſe *Euclide* meaneth here an Aliquot part, as a half, a third, or a fourth, &c. It may paſs for the Definition of a meaſure under the name of part; as thus, *a Meaſure is a Part of the whole, when multiplyed, it may be equall to the whole*, though properly a Meaſure is externall to the thing meaſured, and not the Aliquot part it ſelf; but equall to an Aliquot part.

But the third Definition is intollerable, It is the definition of λόγος, in Latin *Ratio*, in Engliſh *Proportion*, in this manner; λόγος ἐστὶ δύο μεγεθῶν ὁμογενῶν ἡ κατὰ πηλικότητα πρὸς ἄλληλα ποιὰ σχέσις. *Ratio eſt duarum magnitudinum ejuſdem generis mutua quædam ſecundum quantitatem habitudo. Proportion is a certain mutuall habitude in Quantity, of two magnitudes*

nitudes of the same kinde, one to another. First, we have here *ignotum per ignotius*; for every man understandeth better what is meant by *Proportion* then by Habitude. But it was the phrase of the Greeks when they named like Proportions, to say, the first to the second οὕτως ἔχει, *id est, Ita se habet,* and in English, *is as,* the third to the fourth. As for Example (in the Proportions of two to four, and three to six) to say two to four οὕτως ἔχει, *id est, ita se habet, id est, is as*; three to six. From which phrase *Euclide* made this his Definition of Proportion by ποιὰ σχέσις, which the Latines translate *quædam habitudo. Quædam* in a Definition is a most certain note of not understanding the word *defined*; And in Greek ποιὰ σχέσις is much worse; for to render rightly the Greek definition, we are to say in English, that Proportion is a what-shall-I-call-it *ifneſſe*, or *foneſs* of two magnitudes &c. Then which nothing can be more unworthy of *Euclide*. It is as bad as any thing was ever said in Geometry by *Orontius*, or by Dr. *Wallis*, That Proportion is Quantity compared, that is to say, little or great in respect of some other Quantity (as I have above defined it) is I think intelligible.

The fourth is, Ἀναλογία δέ ἐστιν ἡ τῶν λόγων ὁμοιότης. *Proportio verò est rationum similitudo.* Here we have no one word by which to render Ἀναλογία; for our word *Proportion*, is already bestowed upon the rendring of λόγος. Nevertheless the Greek may be translated into English thus, *Iterated Proportion is the similitude of Proportions.* But Iterated Proportion is the same with *eadem Ratio.* To what purpose then serveth the sixth Definition, which is of *eadem Ratio*? For Ἀναλογία, and *eadem Ratio*, and *Similitudo Rationum*, are the same thing, as appeareth by *Euclide* himself; where he defines those Quantities, that are in the same Proportion by ἀνάλογον. Therefore the sixth Definition is but a *Lemma*, and assumed without demonstration.

The fourteenth, *Compositio Rationis est sunptio Antecedentis cum Consequente, ceu unius, ad ipsum consequentem. To Compound Proportion, is to take both Antecedent, and Consequent together, as one magnitude, and compare it to the consequent,* Is good; though he might have compared it as well with the Antecedent; For both wayes it had been a Composition of Proportion. We are to note here, that the Composition defined in this place by *Euclide*, is not adding together of Proportions, but of two Quantities that have Proportion. And therefore it is not the same Composition which he defineth in the fourth place before the sixth Element; for there

He defineth the addition of one Proportion to another Proportion in this manner λόγος ἐκ λόγων συγκεῖσθαι λέγεται, &c. *A Proportion is said to be compounded of Proportions, when their Quantities multiplyed into one another make a Proportion*; as when we would compound or add together, the Proportions, of three to two, and of four to five, we must multiply three and four, which maketh twelve, and two and five which maketh ten. And then the Proportion of twelve to ten, is the sum of the Proportions, of three to two, and of four to five; which is true; but not a definition; for it may and ought to be demonstrated. For to define what is addition of two Proportions (which are alwayes in four Quantities, though sometimes one of them be twice named) we are to say, that they are then added together when we make the second to another in the same Proportion, which the third hath to the fourth.

And thus much of the Definitions; of which, some, very few, you see are faulty; the rest either accurate, or good enough if well interpreted. For before the rest of the Elements all are accurate, notwithstanding, that you allow not for good any definition in Geometry that hath in it the word *motion*: of which there be divers before the Eleventh Element. But I must here put you in minde, that Geometry being a Science, and all Science proceeding from a precognition of causes, the definition of a Sphere, and also of a Circle, by the generation of it, that is to say, by motion, is better then by the equality of distance from a Point within.

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The second sort of Principles, are those of Construction, usually called *Postulata*, or Petitions. For as for those *notiones communes*, called *Axiomes*, they are from the definitions of their terms demonstrable, though they be so evident as they need not demonstration. These Petitions are by *Euclide* called Αἰτήματα, such as are granted by favour, that is, simply Petitions whereas by Axiome is understood that which is claimed as due. So that between Ἀξίωμα and Αἴτημα there is this other difference, that this later is simply a Petition, the former a Petition of Right.

Of Petitions simply, the first is, *That from any Point to any Point may be drawn a straight Line.* The second, *That a finite straight Line may be produced.* The third, *That upon any Center, at any distance may be described a Circle.* All which are both evident, and necessary to be granted.

And by all these a man may easily perceive that *Euclide* in the definitions of a Point, a Line, and a Superficies, did not intend that a Point should be Nothing, or a Line be without Latitude, or a Superficies without Thickness; for if he did, his Petitions are not onely unreasonable to be granted, but also impossible to be performed. For Lines are not drawn but by Motion; and Motion is of Body only. And therefore his meaning was, that the Quantity of a Point, the Breadth of a Line, and the Thickness of a Superficies were not to be *considered*, that is to say, not to be reckoned in the demonstration of any Theoreme concerning the Quantity of Bodies, either in Length, Superficies, or Solid.

Of the Faults that Occurre in Demonstration.

To the same egregious Professors of the Mathematicks in the University of *Oxford*.

Lesson II.

THere be but two causes from which can spring an error in the demonstration of any conclusion in any Science whatsoever. And those are Ignorance or want of understanding, & Negligence. For as in the adding together of many and great Numbers, he cannot fail, that knoweth the Rules of Addition, and is also all the way so carefull, as not to mistake one number, or one place for another; so in any other Science, he that is perfect in the Rules of Logick, and is so watchfull over his Pen, as not to put one word for another, can never fail of making a true, though not perhaps the shortest and easiest demonstration.

The Rules of Demonstration are but of two kindes; One, that the Principles be true and evident Definitions; the other, that the Inferences be necessary. And of true and evident definitions, the best are those which declare the cause or generation of that Subject, whereof the proper passions are to be demonstrated. For Science is that knowledge which is derived from the comprehension of the cause. But when the cause appeareth not, then may, or rather must we define some known property of the Subject, and from thence derive some possible way, or wayes of the

generation. And the more waies of generation are explicated, the more eaſie will be the derivation of the Properties, whereof ſome are more immediate to one, ſome to another generation. He therefore that proceedeth from untrue, or not underſtood definitions, is ignorant of that he goes about; which is an il-favoured fault, be the matter he undertaketh eaſie or difficult; becauſe he was not forced to undergo a greater charge then he could carry through. But he that from right definitions maketh a falſe concluſion, erreth through humane frailty, as being leſs awake, more troubled with other thoughts, or more in haſte when he was in writing. Such faults, unleſs they be very frequent, are not attended with ſhame, as being common to all men, or are at leſt leſs ugly then the former; except then, when he that committeth them reprehendeth the ſame in other men. For that is in every man intolerable, which he cannot tolerate in another. But to the end that the faults of both kinds may by every man be well underſtood, it will not be amiſs to examine them by ſome ſuch Demonſtrations, as are publikely extant. And for this purpoſe I will take ſuch as are in mine and in your Books, and begin with your *Elenchus* of the Geometry contained in my Book *de Corpore*; to which I will alſo joyn your Book lately ſet forth concerning the *Angle of Contact*, *Conique ſections*, and your *Arithmetica Infinitorum*; and then examine the reſt of my Philoſophy, and yours that oppugne it. For I will take leave to conſider you both every where, as one Author, becauſe you publikely declare your approbation of one anothers doctrine.

My firſt Definition is of a Line, of Length, and of a Point; *The way (ſay I) of a Body moved, in which magnitude (though it alwayes have ſome magnitude) is not conſidered, is called a Line; and the ſpace gone over by that motion, Length, or one and a ſimple Dimenſion.* To this Definition you ſay, Firſt, *what Mathematician did ever thus define a Line or Length?* Whether you call in others for help or teſtimony, it is not done like a Geometrician; for they uſe not to prove their concluſions by witneſſes, but relye upon the ſtrength of their own reaſon; and when your witneſſes appear, they will not take your part. Secondly, you grant that what I ſay is true, but not a Definition. But to tell you truly what it is which we call a Line, is to define a Line. Why then is not this a Definition? *Becauſe* (ſay you in the firſt place) *it is not a reciprocall propoſition.* But by your favour it is reciprocall. For not only the way of a Body whoſe Quantity is not conſidered, is a Line; but alſo every Line is (or may be conceived to be) the way of a Body ſo moved. And if you object that there is a difference between *is* and *may be conceived to be*, *Euclide* whom you call to your aid, will be againſt you in the fourteenth Definition before his eleventh Element; Where he defines a Sphere juſt as convertibly as I define a Line; except you think the Globes of the Sun and Stars cannot be Globes, unleſs they were made by the circumduction of a Semicircle; And again in the eighteenth definition, which is of a Cone, unleſs you admit no Figure for a Cone, which is not generated by the Revolution of a Triangle; And again, in the twentieth Definition, which is of a Cylinder, except it be generated by the circumvolution of a Parallelogram. *Euclide* ſaw that what proper paſſion ſoever ſhould be derived from theſe his Definitions, would be true of any other Cylinder, Sphere, or Cone, though it were otherwiſe generated; And the deſcription of the generation of any one being by the imagination applicable to all (which is equivalent to convertible) he did not believe that any rationall man could be miſled by learning Logick, to be offended with it. Therefore this exception proceedeth from want of underſtanding, that is, from ignorance of the nature, and uſe of a Definition.

Again, you object and ask *What need is there of motion, or of Body moved, to make a man underſtand what is a Line? Are not Lines in a Body at reſt, as well as in a Body moved? And is not the diſtance of two reſting points Length, as well as the meaſure of the paſſage? Is not Length one and a ſimple dimenſion, and one and a ſimple dimenſion Line? Why then is not Line and Length all one?* See how impertinent theſe queſtions are. *Euclide* defines a Sphere to be a Solid Figure deſcribed by the revolution of a Semicircle, about the unmoved Diameter. *Why do you not ask what need there is to the underſtanding of what a Sphere is, to bring in the motion of a Semicircle? Is not a Sphere to be underſtood without ſuch motion? Is not*

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the Figure so made, a Sphere without this motion? And where he defines the Axis of a Sphere to be that unmoved Diameter, may not you ask, whether there be no Axis of a Sphere, when the whole Sphere, Diameter and all is in motion? But it is not my purpose to defend my definition by the example that of *Euclide*. Therefore first, I say, to me howsoever it may be to others, it was fit to define a Line by Motion. For the generation of a Line is the Motion that describes it. And having defined Philosophy in the beginning, to be the knowledge of the properties from the generation, it was fit to define it by its generation. And to your question, *Is not distance Length*? I answer, that though sometimes distance be æquivalent to Length, yet certainly the distance between the two ends of a thread wound up into a Clew is not the length of the thread; for the length of the thread is equall to all the windings whereof the Clew is made. But if you will needs have distance and length to be all one, tell me of what, the distance between any two Points is the Length. Is it not the length of the way? And how is that called Way, which is not defined by some motion? And have not severall wayes between the same places, as by Land and by Water, severall lengths? But they have but one distance, because the distance is the shortest way. Therefore between the length of the Path, and the distance of the Places, there is a reall difference in this case, and in all cases a difference of the consideration. Your objection, that Line is Longitude, proceeds from want of understanding English. Do men ever ask what is the *Line* of a thread, or the Line of a Table, or of any other Body? Do they not always ask what is the length of it? And why, but because they use their own judgements, nor yet corrupted by the subtilty of mistaken Professors. *Euclide* defines a Line, *to be length without breadth*. If those terms be all one, why said he not that a *Line, is a Line without breadth*? But what Definition of a Line give you? None. Be contented then with such as you receive, and with this of mine, which you shall presently see is not amiss.

Your next objections are to my Definition of a Point. Which Definition adhereth to the former in these words, *and the Body it self is called a Point*. Here again you call for help; *Quis unquum mortalium, &c. What mortall man, what sober man did ever so Define a Point*? 'Tis well, and I take it to be an honour to be the first that do so. But what objection do you bring against it? This, *that a point added to a Point (if it have magnitude) makes it greater*. I say it doth so, but then presently it loseth the name of a Point, which name was given to signifie that it was not the meaning of him that us'd it in demonstration, to add, substract, multiply, divide, or any way compute it. Then you come in with *perhaps you will say though it have magnitude, that magditude is not considered*. You need not say *perhaps*. You know I affirm it; and therefore your Argument might have been left out, but that it gave you occasion of a digression into scurvie language.

And whereas you ask why I defined not a Point thus, *Punctum est corpus quod non consideratur esse corpus, & magnum quod non consideratur esse magnum*. I will tell your why. First, because it is not Latine. Secondly, because when I had defined it by *corpus*, there was no need to Define it again by *magnum*. I understand very well this language, *Punctum est corpus, quod non consideratur ut corpus*. A Point is a Body not considered as Body. But *Punctum est corpus, quod non consideratur esse corpus, vel esse magnum*, is not Latine, nor the version of it, *A Point is a Body which is not considered to be a Body*, English. My Definition was, that a Point is that Body whose magnitude is not considered; not reckoned, not put to account in Demonstration. And I exemplified the same by the Body of the Earth describing the Ecliptique Line; because the magnitude is not there reckoned nor chargeth the Ecliptique Line with any breadth. But I perceive you understand not what the word *consideration* signifieth, but take it for comparison or relation; and say I ought to define a Point simply, and not by relation to a greater Body; as if to reckon, and to compare were the same thing. *Omnia mihi* (saith *Cicero*) *provisa & considerata sunt*. I have provided and reckoned every thing. There is a great difference between Reckoning and Relation.

Again, you ask why *Corpus motum*, a Body moved. Ile tell you; because the motion was

neceſſary for the generation of a Line. And though after the generation of the Line, the Point ſhould reſt, yet it is not neceſſary from this Definition that it ſhould be no more a Point; nor when *Euclide* defines a Sphere by the circumduction of a Semicircle upon an Axis that reſteth, doth it follow from thence when the Sphere, Axis, Center and all (as that of the Earth) is moved from place to place, that it is no more an Axis.

Laſtly, you object *that motion is accidentary to a Point, and conſequently not eſſentiall, not to be put into the Definition.* And is not the circumduction of a Semicircle accidentary to a Sphere? Or do you think the Sphere of the Sun was generated by the revolution of a Semicircle? And yet it was thought no fault in *Euclide* to put that motion into the Definition of a Sphere.

The conceit you have concerning Definitions, that they muſt explicate the eſſence of the thing defined, and muſt conſiſt of a *genus*, and a *difference*, is not ſo univerſally true as you are made believe, or elſe there be very many inſufficient Definitions that paſs for good with you in *Euclide*. You are much deceived if you think theſe wofull notions of yours, and the Language that doth every where accompany them, ſhew handſomely together. Or that ſuch grounds as theſe be able to ſuſtain ſo many, and ſo haughty reproaches as you advance upon them, ſo as they fall not (as you ſhall ſee immediately) upon your own head. I ſay a point hath Quantity, but not to be reckoned in Demonſtrating the properties of Lines, Solids, or Superficies; You ſay it hath no Quantity at all, but is plainly Nothing.

The firſt of the Petitions of *Euclide* is, *that a Line may be drawn from Point to Point at any diſtance.* The ſecond, *that a ſtraight Line may be produced.* The third, *that on any Center a circle may be deſcribed at any diſtance.* And the eighth Axiome (which Sir *H. Savile* obſerves to be the foundation of all Geometry) is this, *Quæ ſibi mutuo congruunt, &c. Thoſe things that are applyed one to another in all Points, are equall.* All or any of theſe Principles being taken away, there is not in *Euclide* one Propoſition Demonſtrated, or Demonſtrable. If a Point have not Quantity, a Line can have no Latitude; and becauſe a Line is not drawn but by motion, by motion of a Body, and Body imprinteth Latitude all the way, it is impoſſible to draw or produce a ſtraight Line, or to deſcribe a circular Line without Latitude. Alſo if a Line have no Latitude, one ſtraight Line cannot be applyed to another. To them therefore that deny a Point to have Quantity, that is, a Line to have Latitude, the forenamed Principles are not poſſible, and conſequently no propoſition in Geometry is demonſtrated, or demonſtrable. You therefore that deny a Point to have Quantity, and a Line to have Breadth, have nothing at all of the Science of Geometry. The practiſe you may have; but ſo hath any man that hath learned the bare Propoſitions by heart; but they are not fit to be Profeſſors either of Geometry, or of any other Science that dependeth on it. Some man perhaps may ſay that this controverſie is not much worth, and that we both mean the ſame thing. But that man (though in other things prudent enough) knoweth little of Science and Demonſtration. For Definitions are not onely uſed to give us the Notions of thoſe things whoſe appellations are defined; for many times they that have no Science have the Ideas of things more perfect then ſuch as are raiſed by Definitions. As who is there that underſtandeth not better what a ſtraight Line is, or what Proportion is, and what many other things are without Definition, then ſome that ſet down the Definitions of them. But their uſe is, when they are truly and clearly made, to draw Arguments from them for the Concluſions to be proved. And therefore you that in your following cenſures of my Geometry, take your Argument ſo often from this, That a Point is nothing, and ſo often revile me for the contrary, are not to be allowed ſuch an excuſe. He that is here miſtaken, is not to be called Negligent in his Expreſſion, but Ignorant of the Science.

In the next place, you take exceptions to my Definition of *Equall Bodies*, which is this, *Corpora æqualia ſunt quæ eundem locum poſſidere poſſunt. Equall bodies are thoſe which may have the ſame place.* To which you object impertinently, that I may as well define a man to be *He that may be Prince of Tranſilvania*, Wittily, as you count wit. Formerly in every Definition, you exacted an Explication of the Eſſence. You are therefore of opinion that the Poſſibility of be-

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;ng Prince of *Tranſilvania*, is no leſs Eſſentiall to *a man*, then the Poſſibility of the being of two Bodies ſucceſſively in the ſame place, is Eſſentiall to *Bodies equall.*

You take no notice of the twenty third Article of this ſame Chapter, where I define what it is we call Eſſence; namely, that Accident, for which we give the thing its name. As the Eſſence of a man is his Capacity of reaſoning, the Eſſence of a white-body, whiteneſs, &c. becauſe we give the name of *man* to ſuch Bodies as are capable of Reaſoning, for that their capacity; and the name of *White* to ſuch Bodies as have that colour, for that colour. Let us now examine why it is that men ſay Bodies are one to another Equall; and thereby we ſhall be able to determine whether the *poſsibility of having the ſame place*, be Eſſentiall or not to *Bodies equall*, and conſequently whether this Definition be ſo like to the Defining of a man by the *Poſsibility of being Prince of Tranſilvania*, as you ſay it is. There is no man (beſides ſuch Egregious Geometricians as your ſelves) that inquireth the equality of two bodies, but by meaſure. And for Liquid Bodies, or the Aggregates of innumerable ſmall Bodies, men (men I ſay) meaſure them by putting them one after another into the ſame veſſell, that is to ſay, into the ſame place (as *Ariſtotle* defines place) or into the ſpace determined by the veſſell, as I define place. And the Bodies that ſo fill the veſſell, they acknowledge, and receive for equall. But though, when hard Bodies cannot be ſo meaſured, without the incommodity, or trouble of altering their Figure, they then enquire (if the Bodies are both of the ſame kind) their equality by weight, knowing (without your teaching) that equall bodies of the ſame nature, weigh Proportionably to their magnitudes; yet they do it not for fear of miſſing of the equality, but to avoid inconvenience, or trouble But you, (you I ſay) that have no Definition of Equalls, neither received from others, nor framed by your ſelves, out of your ſhallow meditation, and deep conceit of your own Wits, contend againſt the common light of Nature. So much is unheedy learning a hinderance to the knowledge of the truth, and changeth into Elves thoſe that were beginning to be men.

Again, when men inquire the equality of two Bodies in length, they meaſure them by a common meaſure; in which meaſure they conſider neither breadth nor thickneſs, but how the length of it agreeth; firſt, with the length of one of the Bodies, then with the length of the other. And both the Bodies whoſe lengths are meaſured, are ſucceſſively in the ſame place under their common meaſure. *Place* therefore in Lines alſo, is the proper *Index* and diſcoverer of Equality, and Inequality. And as in length, ſo it is in breadth and thickneſs, which are but Lengths otherwiſe taken in the ſame Solid Body. But now when we come from this Equality, and Inequality of Lengths known by meaſure, to determine the Proportions of Superficies, and of Solids, by ratiocination, then it is that we enter into Geometry; for the making of Definitions, in whatſoever Science they are to be uſed, is that which we call *Philoſophia prima*. It is not the work of a Geometrician, as a Geometrician, to Define what is Equality, or Proportion, or any other word he uſeth, though it be the work of the ſame man, as a man. His Geometricall part is, To draw from them, as many true and uſefull Theoremes as he can.

You object ſecondly, That a Pyramis may be equall to a Cube, whilſt it is a Pyramis. True, And ſo alſo whilſt it is a Pyramis it hath a poſſibility by flexion and tranſpoſition of parts to become a Cube, and to be put into the place where another Cube equall to it was before. This is to argue like a child that hath not yet the perfect underſtanding of any Language.

In the third and fourth objection, you teach me to define Equall Bodies (if I will needs define them by place) by the *Equality of place*, and to ſay *that Bodies are Equall that have Equall places.* Teach others, if you can, to meaſure their grain, not by the ſame, but Equall Buſhels.

In the fifth objection, you except againſt the word *can*, in that I ſay that Bodies are Equall, which *can* fill the ſame place. For the greater Body *can* (you ſay) fill the place of the leſs, though not reciprocally the leſs of the greater; It is true, that though the place of the leſs, can never be the place of the greate r yet it may be filled by a part of the greater. But 'tis not then the greater Body that filleth the place of the leſs, but a part of it, that is to ſay, a leſs Body. Howſoever-

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to take away from ſimple men this ſtraw they ſtumble at, I have now put the Definition of Equal Bodies into theſe words, *Equall Bodies are thoſe whereof every one can fill the place of every other.* And if my Definition diſpleaſe you, propound your own, either of *Equall Bodies*, or of *Equals* ſimply. But you have none. Take therefore this of mine.

The ſixth is a very admirable exception. *What* (ſay you) *if the ſame Body can ſometimes take up a greater, ſometimes a leſſer place, as by Rarefaction and Condenſation?* I underſtand very well that Bodies may be ſometimes thin and ſometimes thick, as they chance to ſtand cloſer together, or further from one another. So in the Mathematick-Schools, when you read your Learned Lectures, you have a thick or thronging Audience of Diſciples, which in a great Church, would be but a very thin company. I underſtand how thick and thin may be attributed to bodies in the Plurall, as to a company, but I underſtand not how any one of them is thicker in the School, then in the Church; or how any one of them taketh up a greater room in the School (when he can get in) then in the ſtreet. For I conceive the Dimenſions of the Body, and of the Place, whether the place be filled with Gold or with Air, to be coincident and the ſame; and conſequently both the Quantity of the Air, and the Quantity of the Gold to be ſeverally equall to the Quantity of the place; and therefore alſo (by the firſt Axiome of *Euclide*) equall to one another; inſomuch, as if the ſame Air ſhould be by Condenſation contained in a part of the place it had, the dimenſions of it would be the ſame with the dimenſions of part of the place, that is, ſhould be leſs then they were, and by conſequence the Quantity leſs. And then either the ſame body muſt be leſs alſo, or we muſt make a difference between greater Bodies, and Bodies of greater Quantity; which no man doth that hath not loſt his wits by truſting them with abſurd teachers. When you receive Salary, if the Steward give you for every ſhilling a piece of ſix pence, and then ſay, every ſhilling is condenſed into the room of a ſix pence, I believe you would like this Doctrine of yours much the worſe. You ſee how by your ignorance you confound the affairs of mankind, as far forth as they give credit to your opinions, though it be but little. For nature abhorres even empty words, ſuch as are (in the meaning you aſſign them) *Rarefying* and *Condenſing*. And you would be as well underſtood if you ſhould ſay (coining words by your own power) that the ſame Body might take up ſometimes a greater, ſometimes a leſſer place, by Wallifaction and Wardenſation, as by Rarefaction and Condenſation. You ſee how admirable this your objection is.

In the ſeventh objection you bewray another kind of Ignorance, which is the Ignorance of what are the proper works of the ſeverall parts of Philoſophy. *Though it were out of doubt* (ſay you) *that the ſame Body cannot have ſeveral Magnitudes yet ſeeing it is matter of Natural Philoſophy, nor hath any thing to do with the preſent buſineſs, to what purpoſe is it to mention it in a Mathematicall Definition?* It ſeems by this, that all this while you think it is a piece of the Geometry of *Euclide*, no leſs to make the Definitions he uſeth, then to infer from them the Theorems he demonſtrateth. Which is not true. For he that telleth you in what ſenſe you are to take the Appellations of thoſe things which he nameth in his diſcourſe, teacheth you but his Language, that afterwards he may teach you his Art. But teaching of Language is not Mathematick, nor Logick, nor Phyſick, nor any other Science; and therefore to call a Definition (as you do) Mathematicall, or Phyſicall, is a mark of Ignorance (in a Profeſſor) unexcuſable. All Doctrine begins at the underſtanding of words, and proceeds by Reaſoning till it conclude in Science. He that will learn Geometry muſt underſtand the Termes before he begin, which that he may do, the Maſter demonſtrateth nothing, but uſeth his Naturall prudence onely, as all men do, when they endeavour to make their meaning clearly known. For words underſtood are but the ſeed, and no part of the harveſt of Philoſophy. And this ſeed was it, which *Ariſtotle* went about to ſow in his twelve Books of Metaphyſicks, and in his eight Books concerning the Hearing of *Naturall Philoſophy*. And in theſe Books he defineth Time, Place, Subſtance or Eſſence, Quantity, Relation, &c. that from thence might be taken the Definitions of the moſt generall words for Principles in the ſeverall parts of Science. So that all Definitions proceed from common underſtanding; of which, if any man rightly write, he may properly call his

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writing *Philoſophia prima*, that is, the Seeds, or the Grounds of Philoſophy. And this is the Method I have uſed, defining Place, Magnitude, and the other the moſt generall Appellations in that part which I intitle *Philoſophia prima*. But you now not underſtanding this, talk of Mathematicall Definitions. You will ſay perhaps that others do the ſame as well as you. It may be ſo. But the appeaching of others does not make your ignorance the leſs.

In the eighth place you object not, but ask me *why I define equall Bodies apart.* I will tell you. Becauſe all other things which are ſaid to be equall, are ſaid to be ſo, from the equality of Bodies; as two lines are ſaid to be equall, when they be coincident with the Length of one and the ſame Body; and equall Times, which are meaſured by equall Lengths of Body, by the ſame Motion. And the reaſon is, becauſe there is no Subject of Quantity, or of Equality, or of any other accident but Body; all which I thought certainly was evident enough to any uncorrupted Judgement; and therefore, that I needed firſt to define Equality in the Subject thereof, which is Body, and then to declare in what ſenſe it was attributed to Time, Motion, and other things that are not Body.

The ninth objection is an egregious cavill. Having ſet down the Definition of *Equall Bodies*, I conſidered that ſome men might not allow the attribute of Equality to any things, but thoſe which are the Subjects of Quantity, becauſe there is no Equality, but in reſpect of Quantity. And to ſpeak rigidly, *Magnum & Magnitudo* are not the ſame thing; for that which is great, is properly a Body, whereof greatneſs is an Accident. In what ſenſe therefore (might you object) can an Accident have Quantity? For their ſakes therefore that have not Judgement enough to perceive in what ſenſe men ſay the Length is ſo Long, or the Superficies ſo broad, &c. I added theſe words, *Eâdem ratione (quâ ſcilicet corpora dicuntur æqualia) Magnitudo Magnitudini æqualis dicitur*, that is, *in the ſame manner, as Bodies are ſaid to be equal, their magnitudes alſo are ſaid to be equall.* Which is no more then to ſay, *when Bodies are Equall, their Magnitudes alſo are called Equall. When Bodies are Equall in Length, their Lengths are alſo called Equall. And when Bodies are Equall in Superficies, their Superficies are alſo called Equall.* All which is common ſpeech, as well amongſt Mathematicians, as amongſt common people; and (though improper) cannot be altered, nor needeth to be altered to intelligent men. Nevertheleſs I did think fit to put in that clauſe, that men might know what it is we call Equality, as well in Magnitudes as in *Magnis*, that is, in *Bodies*. Which you ſo interpret, as if it bore this ſenſe, *that when Bodies are Equall, their Superficies alſo muſt be Equall*, contrary to your own knowledge, onely to take hold of a new occaſion of reviling. How unhandſome, and unmanly this is, I leave to be judged by any Reader that hath had the fortune to ſee the world, and converſe with honeſt men.

Againſt the fourteenth Article, where I prove that the ſame Body hath alwayes the ſame magnitude, you object nothing but this, *that though it be granted, that the ſame Body hath the ſame magnitude, while it reſteth, yet I bring nothing to prove that when it changeth place, it may not alſo change its Magnitude by being enlarged or contracted.* There is no doubt, but to a Body (whether at reſt or in motion) more Body may be added, or part of it taken away. But then it is not the ſame Body, unleſs the Whole, and the Part be all one. If the Schools had not ſet your wit awry, you could never have been ſo ſtupid as not to ſee the weakneſs of ſuch objections. That which you add in the end of your objections to this eighth Chapter, *that I allow not* Euclide *this Axiom gratis, that the Whole is greater then a Part*, you know to be untrue.

At my eleventh Chapter, you enter into diſpute with me, about the nature of Proportion. Upon the truth of your Doctrine therein, and partly upon the truth of your opinions concerning the Definitions of a Point, and of a Line, dependeth the Queſtion whether you have any Geometry, or none; and the truth of all the Demonſtrations you have in your other Books, namely, of the *Angle of Contact*, and *Arithmetica Infinitorum*. Here I ſay you enter, how you will get out (your reputation ſaved) we ſhall ſee hereafter.

When a man asketh what Proportion one Quantity hath to another, he asketh how great or how-

how little the one is comparatively to, or inreſpect of the other. When a Geometrician prefixeth before his Demonſtrations a Definition, he doth it not as a part of his Geometry, but of naturall evidence, not to be demonſtrated by Arguments, but to be underſtood in underſtanding the Language wherein it is ſet down; though the matter may nevertheleſs (if beſides Geometry he have wit) be of ſome help to his Diſciple to make him underſtand it the ſooner. But when there is no ſignificant Definition prefixed (as in this caſe, where *Euclides* Definition of Proportion, That it is *a whatſhicalt habitude of two Quantities*, &c. is inſignificant, and you alledge no other) every one that will learn Geometry, muſt gather the Definition from obſerving how the word to be defined is moſt conſtantly uſed in common ſpeech. But in common ſpeech if a man ſhall ask how much (for example) is ſix in reſpect of four, and one man anſwer that it is greater by two, and another that it is greater by half of four, or by a third of ſix, he that asked the queſtion, will be ſatisfied by one of them, though perhaps by one of them now, and by the other another time, as being the onely man that knoweth why he himſelf did ask the Queſtion. But if a man ſhould anſwer, as you would do, that the Proportion of ſix to two is of theſe numbers, a certain Quotient, he would receive but little ſatisfaction. Between the ſaid anſwers to this Queſtion, How much is ſix in reſpect of four? there is this difference. He that anſwereth that it is more by two, compareth not two with four, nor with ſix, for two is the name of a Quantity abſolute. But he that anſwereth it is more by half of four, or by a third of ſix, compareth the difference with one of the differing Quantities. For halfs, and thirds, &c, are names of Quantity compared.

From hence there ariſeth two Species or kinds of (*Ratio*) Proportion, into which the generall word *Proportion* may be divided. The one whereof (namely, that wherein the Difference is not compared with either of the differing Quantities) is called (*Ratio Aritbmetica*) Arithmeticall Proportion; the other (*Ratio Geometrica*) Geometricall Proportion, and (becauſe this latter is onely taken notice of by the name of Proportion) ſimply *Proportion*. Having conſidered this, I defined Proportion, Chap. 11. Art. 3. in this manner, *Ratio eſt Relatio Antecedentis ad Conſequens ſecundum magnitudinem. Proportion is the Relation of the Antecedent to the Conſequent in Magnitude*; having immediately before defined Relatives, Antecedent, and Conſequent in the ſame Article, and by way of explication added, that ſuch Relation was nothing elſe but that one of the Quantities was equall to the other, or exceeded it by ſome Quantity, or was by ſome Quantity exceeded by it. And for exemplification of the ſame, I added further, that the Proportion of three to two, was that three exceeded two by a unity; but ſaid not that the unity, or the difference whatſoever it were, was their proportion, *for Unity*, and *to exceed another by Unity*, is not the ſame thing. This is cleer enough to others. Let us therefore ſee why it is not ſo to you. You ſay I make Proportion to conſiſt in that which remaineth after the leſſer Quantity is ſubſtracted out of the greater; and that you make it to conſiſt in the Quotient, when one number is divided by the other. Wherein you are miſtaken; firſt, in that you ſay I make the Proportion to conſiſt in the Remainder. For I make it to conſiſt in the act of exceeding, or of being exceeded, or of being equall; whereas the Remainder is alwayes an abſolute Quantity, and never a Proportion. To be more or leſs then another number by two, is not the number two; Likewiſe to be equall to two, where the difference is *nothing*, is not that *nothing*? Again, you miſtake in ſaying the Proportion conſiſteth in the Quotient. For divide twenty by five, the Quotient is four. Is it not abſurd to ſay that the Proportion of five to twenty, or of twenty to five, is four? You may ſay the Proportion of five to twenty, is the Proportion of one to four. And ſo ſay I. And you may therefore alſo ſay, that the Proportion of one to four is a meaſure of the Proportion of five to twenty, as being Equall. And ſo ſay I. But that is onely in Geometricall Proportion, and not in Proportion univerſally. For though the *Species* obtain the Denomination of the *Genus*; yet it is not the *Genus*. But as the Quotient giveth us a meaſure of the Proportion of the Dividend to the Diviſor in Geometricall Proportion, ſo alſo the Remainder after Subſtraction is the meaſure of Proportion Arithmeticall.

You

You object in the next place, *That if the Proportion of one Quantity to another be nothing but the exceſs or defect, then, whereſoever the Exceſs or Defect is the ſame, there the Proportion is the ſame.* This you ſay follows in your Logick, and from thence, that the Proportion of three to two, and five to four is the ſame. But is not three to two, and five to four, where the Exceſs is the ſame number, the ſame Proportion Arithmeticall? And is not Arithmeticall Proportion, Proportion? You take here (*Ratio*) Proportion (which is the *Genus*) for that *Species* of it, which is called Geometricall, becauſe uſually this Species has the name of Proportion ſimply. Alſo that the Proportion of three to two, is the ſame with that of nine to ſix; Is it not becauſe the exceſſes are one and three, the ſame portions of three and nine, that is to ſay, the ſame exceſſes comparatively? I wonder you ask me not what is the *Genus* of Arithmeticall, and Geometricall Proportions; and what the *Difference*. The *Genus* is (*Ratio*) Proportion, or Compariſon in Magnitude, and the *Difference* is that one Compariſon is made by the abſolute Quantity, the other by the Comparative Quantity, of the Exceſs or Defect, if there be any. Can any thing be clearer then this? You after come in with *Ignoſce hebitudini* to no purpoſe. I am not ſo inhumane as not to pardon dulneſs, or madneſs; They are not voluntary faults. But when men adventure voluntarily, to talk of that they underſtand not cenſoriouſly and ſcornfully, I may tell them of it.

This difference between the Exceſſes or Defects, as they are ſimply or comparatively reckoned, being thus explained, all the reſt of that you ſay in your objections to this eleventh Chapter (ſaving that Artt. 5. for *Ratio binarii ad quinarium eſt ſuperari Ternario*, as it is in other places, I have put too haſtily *Ratio binarii ad quinarium eſt Ternarius*) will be underſtood by every Reader to be frivolous, and to proceed from the ignorance of what Proportion is.

At the twelfth Chapter you onely note that I ſay, *That the Proportion of Inequality is Quantity, but the Proportion of Equality not Quantity*, and refer that which you have to ſay againſt it to the Chapter following; to which place, I ſhall alſo come in the following Leſſon.

Of the Faults that Occurre in Demonſtration.

To the ſame egregious Profeſſors of the Mathematicks in the Univerſity of *Oxford*.

Leſſon III.

You begin your reprehenſion of my thirteenth Chapter with a Queſtion. Whereas *I* divide Proportion into Arithmeticall, and Geometricall; You ask me what *proportion it is I ſo divide*. *Euclide* divides an Angle into Right, Obtuſe, and Acute. I may ask you as pertinently what Angle it is he ſo divides? Or when you divide *Animal* into *Homo*, and *Brutum*, what Animal that is which you ſo divide? You ſee by this how abſurd your Queſtion is. But you ſay the Definition of Proportion which I make at Chap. 11. Art. 3. namely, that Pro-

portion is the compariſon of two Magnitudes, one to another, agreeth not, neither with Arithmeticall, nor with Geometricall Proportion. I believe you thought ſo then, but having read what I have ſaid in the end of the laſt Leſſon, if you think ſo ſtill, your fault will be too great, to be pardoned eaſily. But why did you think ſo before? Is it not becauſe there was no Definition in *Euclide* of Proportion univerſall, and becauſe he maketh no mention of Proportion Arithmeticall, and becauſe you had not in your minds a ſufficient notion thereof your ſelves to ſupply that Defect? And is not this the cauſe alſo, why you put in this Parentheſis (if Arithmeticall Proportion, ought to be called Proportion)? which is a confeſſion that you know not whether there be ſuch a thing as Arithmeticall Proportion or not; notwithſtanding, that on all occaſions, you ſpeak of Arithmetically Proportionals. Yes, this was it that made yon think that Proportion univerſally, and Proportion Geometricall is the ſame and yet to ſay you cannot tell whether they be the ſame or not. Tis no wonder therefore, if in ſuch confuſion of the underſtanding, you apprehend not that the Proportions of two to five, and nine to twelve, are the ſame; for you are blinded by ſeeing that they are not the ſame Proportions Geometricall. Nor doth it help you that I ſay the Difference is the Proportion, for by Difference you might if you would, have underſtood the act of Differing.

At the ſecond Article you note for a fault in Method, that *after I had uſed the words Antecedent and Conſequent of a Proportion in ſome of the preceedent Chapters, I define them afterwards*.. I do not believe you ſay this againſt your knowledge, but that the eagerneſs of your malice made you overſee. Therefore go back again to the third Article of Chap. 11. Where having defined Correlatives, I add theſe words, *Of which the firſt is called the* Antecedent, *the ſecond the* Conſequent. This is but an overſight, though ſuch as in me, you would not have excuſed.

At the thirteenth Article you find fault with, that I ſay *that the Proportion of Inequality, whether it be of Exceſs, or of Defect, is Quantity, but the Proportion of Equality is not Quanty*. Whether that which you ſay, or that which I ſay be the truth, is a Queſtion worthy of a very ſtrict Examination. The firſt time I heard it argued, was in *Merſennus* his Chamber at *Paris*, at ſuch time, as the firſt volume of his *Cogitata Phyſico-Mathematica* was almoſt printed: In which, becauſe he had not ſaid all he would ſay of Proportion, he was forced to put the reſt into a Generall Preface; which as was his cuſtom, he did read to his Friends, before he ſent it to the Preſs. In that generall Preface under the Title *de Rationibus atq; Proportionibus*, at the Numbers twelve, thirteen, fourteen, he maintaineth againſt *Clavius*, *that the Compoſition of Proportions is* (as of all other things) *a Compoſition of the Parts to make a Totall*; and *that the proportion of equality anſwereth in Quantity, to* non-ens, *or Nothing*; *the proportion of exceſs, to* ens, *or Quantity*; *and the Proportion of Defect, to leſs then Nothing*; *becauſe Equality* (he ſaies) *is a term of middle ſignification, between Exceſs and Defect*. And there alſo he refuteth the Arguments which *Clavius* at the end of the nineth Element of *Euclide* bringeth to the contrary. And though this were approved by divers good Geometricians then preſent, and never gain-ſayed by any ſince, Yet do not I ſay it upon the credit of them, but upon ſufficient grounds. For it hath been demonſtrated by *Eutocius* that *if there be three magnitudes, the proportion of the firſt to the third is compounded of the proportions of the firſt to the ſecond, and of the ſecond to the third*; Which alſo I demonſtrate in this Article. And if there were never ſo many magnitudes ranked, it might be likewiſe demonſtrated, that the Proportion of the firſt to the laſt is compounded of the Proportions of the firſt to the ſecond, and of the ſecond to the third, and of the third to the fourth, and ſo on to the laſt. If therefore we put in order any three numbers, whereof the two laſt be equall, as four, ſeven, ſeven, the Proportion of four the firſt, to ſeven the laſt, will be compounded of the Proportions of four the firſt, to ſeven the ſecond, and of ſeven the ſecond, to ſeven the third. Wherefore the Proportion of ſeven to ſeven (which is of equality) addeth nothing to the Proportion of four the firſt, to ſeven the ſecond; and conſequently the Proportion of ſeven to ſeven hath no Quantity. But that the Proportion of Inequality hath Quantity, I prove it from this, that one Inequality may be greater then another.

But

But for the clearing of this Doctrine (which *Merſennus* cals Intricate) of the compoſition of Proportions, I obſerved, firſt, that any two Quantities, being expoſed to ſenſe, their Proportion was alſo expoſed; which is not Intricate. Again, I obſerved that if beſides the two expoſed Quantities, there were expoſed a third, ſo as the firſt were the leaſt, and the third the greateſt, or the firſt the greateſt, and the third the leaſt, that not only the Proportions of the firſt to the ſecond, but alſo (becauſe the Differences, and the Quantities proceed the ſame way) the Proportion of the firſt to the laſt is expoſed by compoſition, or addition of the Differences; nor is there any intricacy in this. But when the firſt is leſs then the ſecond, and the ſecond greater then the third, or the firſt greater then the ſecond, and the ſecond leſs then the third, ſo that to make the firſt and ſecond equall, if we uſe addition, we muſt to make the ſecond and third equall uſe ſubſtraction, then comes in the intricacy, which cannot be extricated, but by ſuch as know the truth of this Doctrine which I now delivered out of *Merſennus*: namely, That the Proportions of Exceſs, Equality, and Defect, are as *Quantity, not-Quantity, nothing want Quantity*, or as Symbolists mark them o +1 . o . o-1 And upon this ground I thought depended the univerſall truth of this Propoſition, that in any rank of Magnitudes of the ſame kind, the Proportion of the firſt to the laſt, was compounded of all the Proportions (in order) of the intermediate Quantities; the want of the proof thereof Sir *Henry Savile* cals (*Nævus*) a mole in the Body of Geometry. This Propoſition is demonſtrated at the thirteenth Article of this Chapter.

But before we come thither, I muſt examine the Arguments you bring to confute this Propoſition, that the *Proportion of Inequality is Quantity, of Equality not Quantity.*

And firſt, you object that Equality and Inequality are in the ſame Predicament. A pretty Argument to fleſh a young Scholar in the Logick School, that but now begins to learn the Predicaments. But what do you mean by *Æquale*, and *Inequale*? Do you mean *Corpus Æquale*, and *Corpus Inequale*? They are both in the Predicament of Subſtance, neither of them in that of Quantity; Or do you mean *Æqualitas*, and *Inæqualitas*? They are both in the Predicament of Relation, neither of them in that of Quantity, and yet both *Corpus*, and *Inæqualitas*, though neither of them be Quantity, may be *Quanta*, that is, both of them have Quantity, And when men ſay Body is Quantity, or Inequality is Quantity, they are no otherwiſe underſtood, then if they had ſaid *Corpus eſt tantum*, and *Inæqualitas tanta*, not *Tantitas*; that is, Bodies and Inequalities are *ſo much*, not *ſomuchneſs*; and all intelligent men are contented with that expreſſion, and your ſelves uſe it. And the Quantity of Inequality is in the Predicament of Quantity, becauſe the meaſure of it is in that Line by which one Quantity exceeds the other. But when neither exceedeth other, then there is no Line of Exceſs, or Defect by which the Equality can be meaſured, or ſaid to be *ſo much*, or be called Quantity. Philoſophy teacheth us how to range our words; but *Ariſtotles* ranging them in his Predicaments, doth not teach Philoſophy; And therefore no Argument taken from thence, can become a Doctor, and a Profeſſor of Geometry.

To prove that the Proportion of Inequality was Quantity, but the Proportion of Equality, not Quantity, my Argument was this; That *becauſe one Inequality may be greater or leſs then another, but one Equality cannot be greater nor leſs then another, Therefore Inequality hath Quantity, or is* Tanta, *and Equality not.* Here you come in again with your Predicaments, and object, that to be ſuſceptible of *magis* and *minus* belongs not to quantity, but to Quality; but without any proof, as if you took it for an Axiome. But whether true or falſe, you underſtand not in what ſenſe it is true or falſe. 'Tis true, that one Inequality is Inequality, *as well* as another; as one heat is heat *as well* as another; but not *as great*; *Tam*, but not *Tantus*. But ſo it is alſo in the Predicament of Quantity; one Line is as well a Line as another, but not ſo great. All degrees, intenſions, and remiſſions of Quality, are greater or leſs Quantity of force, and meaſured by Lines, Superficies, or Solid Quantity, which are properly in the Predicament of Quantity. You ſee how wiſe a thing it is to argue from the Predicaments of *Ariſtotle*, which you underſtand not. And yet you pretend to be leſs addicted to the authority of *Ariſtotle*, now, then heretofore.

In the next place you say, I may as well conclude from the not susception of *greater* and *less*, that a Right Angle is not Quantity, but an Oblique one is. Very learnedly. As if to be *greater* or *less* could be attributed to a Quantity once determined. Number (that is number indefinitely taken) is susceptible of *greater* and *less*, because one number may be greater then another. And this is a good Argument to prove that Number is Quantity. And do you think the Argument the worse for this, that one six cannot be greater then another six? After all these childish Arguments which you have hitherto urged, can you perswade any man, or your selves, that you are Logicians?

To the fifth and sixth Article you object, first, *that if I had before sufficiently Defined* (Ratio) *Proportion, I needed not again define what is* (eadem Ratio) *the same Proportion*; and ask me *whether when I have defined* man, *I use to define anew what is the* same man? You think when you have the Definition of *Homo*, you have also the Definition of *idem Homo*, when 'tis harder to conceive what *idem* signifies, then what *Homo*. Besides, *idem* hath not the same signification alwayes, and with whatsoever it be joyned; it doth not signifie the same with *Homo*, that it doth with *Ratio*. For with *Homo*, it signifies the same *individual man*, but with *Ratio* it signifies a like, or an equall Proportion. And both (*Ratio*) *Proportion*, and (*idem*) *the same*, being defined, there will still be need of another Definition for (*eadem Ratio*) *the same Proportion*. And this is enough to defend both my self and *Euclide*, against this objection. For *Euclide* also after he had Defined (*Ratio*) *Proportion*, and that sufficiently as he believed, yet he defines *the same Proportion* again apart. I know you did not mean in this place to object any thing against *Euclide*; but you saw not what you were doing. There is within you some speciall cause of Intenebration, which you should do well to look to.

In the next place you say, when I had defined Arithmeticall Proportions to be the same when the difference is the same; it was to be expected I should define Geometricall Proportions to be then the same, when the Antecedents are of their Consequents *Totuple*, or *Tantuple*, that is, equimultiple (for *Tantuplum* signifies nothing.) In plain words, you expected, that as I defined one by substraction, I should define the other by the Quotient in Division. But why should you expect a Definition of the same Proportion by the Quotient? Neither Reason nor the Authority of *Euclide* could move you to expect it. Or why should you say *it was to be expected*? But it seems you have the vanity to place the measure of truth in your own Learning. In Lines incommensurable there may be the same Proportion, when nevertheless there is no Quotient; for setting their Symboles one above another doth not make a Quotient; for Quotient there is none, but in *aliquot parts*. It is therefore impossible to define Proportion universally, by comparing Quotients. This incommensurability of Magnitudes was it that confounded *Euclide* in the framing of his Definition of Proportion at the fifth Element. For when he came to numbers, he defined the *same Proportion* inreprehensibly thus, *Numbers are then Proportional, when the first of the second, and the third of the fourth are equimultiple, or the same part, or the same parts*; and yet there is in this Definition no mention at all of a Quotient. For though it be true that if in dividing two Numbers you make the same Quotient, the Dividends and the Divisors are Proportionall, yet that is not the Definition of the same Proportion, but a Theoreme Demonstrable from it. But this Definition *Euclide* could not accommodate to Proportion in Generall, because of incommensurability.

To supply this want, I thought it necessary to seek out some way, whereby the Proportion of two Lines, Commensurable, or Incommensurable, might be continued perpetually the same. And this I found might be done by the Proportion of two Lines described by some uniform motion, as by an Efficient cause both of the said Lines, and also of their Proportions. Which motions continuing, the Proportions must needs be all the way the same. And therefore I defined those Magnitudes to have the same Geometricall Proportion, *when some cause producing in equall times, Equall Effects, did determine both the Proportions*. This you say needs an *Oedipus* to make it understood. You are (I see) no *Oedipus*; but I do not see any difficulty, neither in the Definition, nor in the Demonstration. That which you call perplexity in the Ex-

Explication, is your prejudice, ariſing from the Symboles in your fancy. For men that pretend no leſs to naturall Phyloſophy, then to Geometry, to find fault with bringing Motion and Time into a Definition, when there is no effect in nature, which is not produced in Time by Motion, is a ſhame. But you ſwim upon other mens bladders in the Superficies of Geometry, without being able to indure diving. Which is no fault of mine; and therefore I ſhall (without your leave) be bold to ſay, I am the firſt that hath made the grounds of Geometry firm and coherent. Whether I have added any thing to the Edifice or not, I leave to be judged by the Readers. You ſee, you that profeſs with the pricking of bladders the letting out of their vapour, how much you are deceived You make them ſwell more then ever.

For the Corollaries that follow this ſixth Article, you ſay they contain nothing new. Which is not true. For the nineth is new, and the Demonſtrations of all the reſt are new, being grounded upon a new Definition of Proportion, and the Corollaries themſelves for want of a good Definition of Proportion, were never before exactly Demonſtrated. For the truth of the ſixth Definition of the fifth Element of *Euclide* cannot be known but by this Definition of mine, becauſe it requires a Triall in all numbers poſſible, that is to ſay, an infinite time of Triall, whether the quimultiples of the firſt and third, and of the ſecond and fourth in all multiplications do together exceed, together come ſhort, and are together equall; which Triall is impoſſible.

In objecting againſt the thirteenth and ſixteenth Article, I obſerve that you bewray together, both the greateſt Ignorance, and the greateſt malice; and 'tis well, for they are ſutable to one another, and fit for one and the ſame man. In the thirteenth Article my Propoſition is this, *If there be three Magnitudes that have Proportion one to another, the Proportions of the firſt to the ſecond, and of the ſecond to the third taken together* (as one Propoſition) *are equall to the Proportion of the firſt to the third.* This Demonſtrated, there is taken away one of thoſe Moles which Sir *Henry Savile* complaineth of in the Body of Geometry. Let us ſee now what you ſay both againſt the Enunciation, and againſt the Demonſtration.

Againſt the Enunciation you object, that *other men would ſay* (not the Proportions of the firſt to the ſecond, and of the ſecond to the third, taken together, &c. but) *the Proportion which is compounded of the* Proportion *of the firſt to the ſecond, and of the ſecond to the third,* &c. Is not the compounding of any two things whatſoever the finding of the ſum of them both, or the taking of them together as one totall. This is that abſurdity of which *Merſennus* in the generall Preface to his *Cogitata Phyſico-Mathematica* hath convinced *Clavius*, who at the end of *Euclides* nineth Element denyeth the compoſition of Proportion to be a Compoſition of Parts to make a Totall; Which therefore he denyed, becauſe he did not obſerve, that the Addition of a Proportion of defect, to a Proportion of Exceſs, was a Subſtraction of Magnitude; and becauſe he underſtood not that to ſay, Compoſition is not the making a whole of Parts, was contradiction; which all, but too learned men would as ſoon as they heard, abhorre. Therefore in ſaying that other men would not ſpeak in that manner, you ſay in effect they would ſpeak abſurdly. You do well to mark what other Geometricians ſay; but you would do better if you could by your own Meditation, upon the things themſelves, examine the truth of what they ſay. But you have no minde (you ſay) to contend about the Phraſe. Let us therefore ſee what it is you contend about.

The Proportion (you ſay) *which is compounded of double and triple Proportion, is not* (as I would have it) *Quintuple, but Sextuple,* as in theſe numbers, ſix, three, one; where the Proportion of ſix to three is double, the Proportion of three to one triple, and the Proportion of ſix to one ſextuple, not quintuple. Tell me (egregious Profeſſors) how is ſix to three double Proportion? Is ſix to three the double of a number, or the double of ſome Proportion? All men know the number ſix is double to the number three, and the number three triple to an unity. But is the Queſtion here of compounding numbers, or of compounding Proportions? *Euclide* at the laſt Propoſition of his nineth Element ſayes indeed, that theſe numbers, one, two, four, eight, are ἐν διπλασίονι ἀναλογία in double Proportion, yet there is no man that underſtands it otherwiſe, then if he had ſaid in Proportion of the ſingle Quantity, to the double Quan-

tity; and after the ſame Rate, if he had ſaid three, nine, twenty-ſeven, &c. had been in triple Proportion, all men would have underſtood it, of the Proportion of any Quantity to its Triple. Your inſtance therefore of ſix, three, one, is here impertinent, there being in them no doubling, no tripling, nor ſextupling of Proportions, but of numbers. You may obſerve alſo that *Euclide* never diſtinguiſheth between double and duplicate, as you do. One word διπλάσιον ſerves him every where for either. Though I confeſs ſome curious Grammarians take διπλάσιον for duplicate in number, and διπλοῦν for double in Quantity; which will not ſerve your turn. Your Geometry is not your own, but you caſe your ſelves with *Euclides*; in which, as I have ſhewed you, there be ſome few great holes; and you by miſunderſtanding him, as in this place, have made them greater, Though the beaſts that think your railing, ſoaring, have for a time admired you; yet now that through theſe holes of your caſe, I have ſhewed them your ears, they will be leſs affrighted. But to exemplifie the compoſition of Proportions, take theſe numbers, thirty-two, eight, one, and then you ſhall ſee that the Proportion of thirty-two to one is the ſum of the Proportions of thirty-two to eight, and of eight to one. For the Proportion of thirty-two to eight is double the Proportion of thirty-two to ſixteen. And the Proportion of eight to one, is triple the Proportion of thirty-two to ſixteen, and the Proportion of thirty-two to one is Quintiple of thirty-two to ſixteen. But double and triple added together maketh quintuple. What can be here denyed?

My Demonſtration conſiſteth of three caſes. The firſt is when both the Proportions are of defect, which is then, when the firſt Quantity is the leaſt; as in theſe three Quantities, A B, A C, A D. The firſt caſe I demonſtrated thus. A B C D / a Let it be ſuppoſed that the point A were moved uniformly through the whole line A D. The Proportions therefore of A B to A C, and of A C to A D, are determined by the difference of the Times in which they are deſcribed. And the Proportion alſo of A B to A D, is that which is determined by the difference of the Times, in which they are deſcribed; but the difference of the Times in which A B and A C are deſcribed, together with the difference of the Times, wherein A C and A D are deſcribed, is the ſame with the difference of the Times, wherein are deſcribed A B and A D. The ſame cauſe therefore which determines both the Proportions of A B to A C, and of A C to A D, determines alſo the Proportion of A B to A D. Wherefore by the Definition of *the ſame Proportion*, Article ſix, the Proportion of A B to A C, together with the Proportion of A C to A D is the ſame with the Proportion of A B to A D.

Conſider now your argumentation againſt it. *Let there be taken* (ſay you) *between A and B the Point a, and then in your own words, I argue thus. The difference of the Times wherein are deſcribed A B and A C, together with the difference of the Times, wherein are deſcribed A C and A D, is the ſame with the difference of the Times, in which are deſcribed a B and a C (namely B D, or B C † C D) wherefore the ſame cauſe which determines the two Proportions of A B to A C, and of A C to A D, determines alſo the Proportion of a B to a D.* Let me ask you here whether you ſuppoſe the Motion from a to B, or from a to D, to have the ſame ſwiftneſs with the motion from A to B, or from A to D. If you do not, then you deny the ſuppoſition. If you do, then B C which is the difference of the Times A B and A C, cannot be the difference of the Times in which are deſcribed a B and a C, except A B and a B are equall. Let any man judge now whether there be any Paralogiſm in *Orontius* that can equall this. And whether all that follows in the reſt of this, and the next two whole Pages, be not all a kind of raving upon the ignorance of what is the meaning of Proportion, which you alſo make more ill-favoured by writing it; not in language, but in *Gamboles*, I mean in the Symboles, which have made you call thoſe demonſtrations ſhort, which put into words ſo many as a true demonſtration requires, would be longer then any of thoſe of *Clavius* upon the twelfth Element of *Euclide*.

To the ſixteenth Article you bring no Argument, but fall into a loud *Oncethmus* (the ſpecial Figure wherewith you grace you Oratory) offended with my unexpected croſſing of the Doctrine

strine you teach, that Proportion consisteth in a Quotient. For that being denyed you, your $\frac{a}{b} - \frac{c}{d} + \frac{e}{f} - \frac{g}{b} + \frac{i}{k}$ comes to nothing, that is, to just as much as it is worth But are not you very simple men, to say that all Mathematicians speak so, when it is not speaking? When did you see any man but your selves publish his Demonstrations by signs not generally received, except it were not with intention to demonstrate, but to teach the use of Signes? Had *Pappus* no Analytiques? Or wanted he the wit to shorten his reckoning by Signes? Or has he not proceeded Analytically in an hundred Problems. (especially in his seventh Book) and never used Symboles? Symboles are poor unhandsome (though necessary) scaffolds of Demonstration; and ought no more to appear in publique, then the most deformed necessary business which you do in your Chambers. *But why* (say you) *is this imitation to the Proportion of the greater to the less?* Ile tell you; because iterating of the Proportion of the less to the greater, is a making of the Proportion less, and the defect greater. And it is absurd to say that the taking of the same Quantity twice should make it less. And thence it is, that in Quantities, which begin with the less, as one, two, four, the Proportion of one to two, is greater then that of one to four, as is Demonstrated by *Euclide* Elem. 5. Prop. 8. and by consequent the Proportion of one to four, is a Proportion of greater littleness then that of one to two. And who is there, that when he knoweth that the respective greatness of four to one, is double to that of the respective greatness of four to two, or of two to one, will not presently acknowledge that the respective greatness of one to two, or two to four is double to the respective greatness of one to four. But this was too deep for such men as take their opinions not from weighing, but from reading.

Lastly, you object against the Corollarie of Art. 28. (which you make absurd enough by rehearsing it thus) *si quantitas aliqua divisa supponatur in partes aliquot æquales numero infinitas.* &c. Do you think that of *partes aliquot*, or of *partes aliquotæ*, it can be said without absurdity, that they are *numero infinitæ*? And then you say I seem to mean, that if of the Quantity A B, there be supposed a part C B, infinitely little, and that between A C and A B be taken two means, one Arithmeticall A E, the other Geometricall A D, the difference between A D, and A E, will be infinitely little. My meaning is, and is sufficiently expressed, that the said means taken every where (not in one place onely) will be the same throughout. And you that say there needed not so much pains to prove it, and think you do it shorter, prove it not at all. For why may not I pretend against your demonstration, that B E the Arithmeticall difference, is greater then B D the Geometricall difference. You bring nothing to prove it, and if you suppose it, you suppose the thing you are to prove. Hitherto you have proceeded in such manner with your *Elenchus*, as that so many objections as you have made, so many false Propositions you have advanced. Which is a peculiar excellence of yours, that for so great a stipend as you receive, you will give place to no man living for the number and grosness of errors you teach your Scholars.

At the fourteenth Chapter your first exception is to the second Article; where I define a plain in this manner. *A plain Superficies is that which is described by a straight Line so moved, as that every Point thereof describe a severall straight Lin*. In which you require, first, that instead of *describe*, I should have said *can describe*. Why do you not require of *Euclide* in the Definition of a Cone, instead of (*Continetur*) *is contained*, he say (*contineri potest*) *can be contained*? If I tell you how one Plain is generated, cannot you apply the same generation to any other Plain? But you object that the Plain of a Circle may be generated by the motion of the *Radius*, whose every point describeth not a straight but a crooked Line, wherein you are deceived; for you cannot draw a Circle (though you can draw the perimeter of a Circle) but in a Plain already generated. For the motion of a straight Line, whose one Point resting, describeth with the other Points severall perimeters of Circles, may as well describe a Conique Superficies, as a Plain. The Question therefore is, how you will in your Definition take in the Plain which must be generated before you begin to describe your Circle, and before you know what Point to make your Center. This objection therefore is to no purpose; and besides, that

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it reflecteth upon the perfect definitions of *Euclide* before the eleventh Element; it cannot make good his Definition (which is nothing worth) of a Plain Superficies, before his firſt Element.

In the next place you reprehend briefly this *Corollarie, that two Plaines cannot incloſe a Solid.* I ſhould indeed have added, *with the baſe on whoſe extreams they inſiſt.* But this is not a fault to be aſhamed of. For any man by his own underſtanding might have mended my expreſſion without departing from my meaning. But from your Doctrine *that a Superficies has no thickneſs,* 'tis impoſſible to include a Solid, with any Number of Plains whatſoever; unleſs you ſay that Solid is included which nothing at all includes.

At the third Article, where I ſay: *of crooked lines, ſome are every where crooked, and ſome have parts not crooked.* You ask me what crooked Line has parts not crooked; and I anſwer, it is that Line which with a ſtraight Line makes a rectilineall Triangle. But this you ſay cannot ſtand with what I ſaid before, namely, that a ſtraight and crooked line cannot be coincident; which is true, nor is there any contradiction; for that part of a crooked line which is ſtraight, may with a ſtraight line be coincident.

To the fourth Article, where I define *the Center of a Circle to be that Point of the* Radius, *which in the deſcription of the Circle is unmoved*; You object as a contradiction, that I had before defined a Point to be the body which is moved in the deſcription of a Line. Fooliſhly, As I have already ſhown at your objection to Chap. 8. Art. 12.

But at the ſixth Article, where I ſay that *crooked, and incongruous Lines touch one another but in one Point*, you make a cavill from this, that *a Circle may touch a Parabola in two Points* Tell me truely, did you read and underſtand theſe words that followed, *a crooked Line cannot be congruent with a ſtraight line, becauſe if it could, one and the ſame line ſhould be both ſtraight and crooked?* If you did, you could not but underſtand the ſenſe of my words to be this; *when two crooked lines which are incongruous, or a crooked and a ſtraight line touch one another, the contact is not in a Line, but only in one Point*; and then your inſtance of a Circle and a Parabola, was a wilfull cavill, not befitting a Doctor. If you either read them not, or underſtood them not, it is your own fault. In the reſt that followeth upon this Article, with your Diagram, there is nothing againſt me, nor any thing of uſe, novelty, ſubtility or learning.

At the ſeventh Article, where I define both an *Angle*, ſimply ſo called, and an *Angle of Contingence*, by their ſeverall generations, namely, that the former is generated *when two ſtraight Lines are coincident, and one of them is moved, and diſtracted from the other by circular motion upon one common Point reſting, &c.* You ask me *to which of theſe kinds of Angle, I refer the Angle made by a ſtraight Line when it cuts a crooked Line.* I anſwer eaſily and truly, to that kind of Angle which is called ſimply an Angle. This you underſtand not. For how (will you ſay) can that Angle which is generated by the divergence of two ſtraight Lines, be other then Rectilineall? O. how can that Angle which is not comprehended by two ſtraight Lines, be other then Curvilineall? I ſee what it is that troubles you, namely, the ſame which made you ſay before, that if the Body which deſcribes a Line be a Point, then there is nothing which is not moved that can be called a Point. So you ſay here, If an Angle be generated by the motion of a ſtraight Line, then no Angle ſo generated can be Curvilineall. Which is as well argued, as if a man ſhould ſay, the Houſe was built by the carriage and motion of Stone and Timber, therefore when the carriage and that motion is ended, it is no more a houſe. Rectilineall and Curvilineall hath nothing to do with the nature of an Angle ſimply ſo called, though it be eſſentiall to an Angle of Contact. The meaſure of an Angle ſimply ſo called is a circumference of a Circle, and the meaſure is alwayes the ſame kind of Quantity with the thing meaſured. The Rectitude or Curvity of the Lines which drawn from the Center intercept the Arch, is accidentary to the Angle, which is the ſame, whether it be drawn by the motion circular of a ſtreight line or of a crooked. The Diameter and the Circumference of a Circle make a right Angle, and the ſame which is made by the Diameter and the Tangent. And becauſe the point of Contact is not (as you think) nothing, but a line unreckoned, and common both to the Tangent, and the

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Circumference, the same Angle computed in the Tangent is Rectilineall, but computed in the Circumference, not Rectilineall, but mixt; or if two Circles cut one another, Curvilineall. For every Chord maketh the same Angle with the Circumference which it maketh with the line that toucheth the Circumference at the end of the Chord. And therefore when I divide an Angle simply so called into Rectilineall, and Curvilineall, I respect no more the generation of it, then when I divide it into Right and Oblique. I then respect the generation, when I divide an Angle into an Angle simply so called, and an Angle of Contact. This that I have now said, if the Reader remember when he reads your objections to this, and to the nineth Article, he will need no more to make him see that you are utterly ignorant of the nature of an Angle, and that if ignorance be mádness, not I, but you are mad; and when an Angle is comprehended between a straight and a crooked Line (if I may compute the same Angle as comprehended between the same straight Line and the Point of Contact) that it is consonant to my definition of a Point by a *Magnitude not considered.* But when you in your treatise *de Angulo Contactûs* Chap. 3. Pag. 6. Lin. 8. have these words, *Though the whole concurrent Lines incline to one another, yet they form no Angle any where but in the very point of concourse*, You, that deny a Point to be any thing, tell me how two nothings can form an Angle; or if the Angle be not formed neither before the concurrent Lines meet, nor in the Point of concourse, how can you apprehend that any Angle can possibly be framed. But I wonder not at this absurdity, because this whole treatise of yours is but one absurdity continued from the beginning to the end; as shall then appear when I come to answer your objections to that which I have briefly and fully said of that Subject in my 14. Chapter.

At the twelfth Article I confess your exception to my universall definition of Parallels to be just, though insolently set down. For it is no fault of ignorance (though it also infect the demonstration next it) but of too much security. The Definition is this: *Parallels are those Lines or Superficies, upon which two straight Lines falling, and whersoever they fall, making equall Angles with them both, are equall*; which is not, as it stands, universally true. But inserting these words *the same way*, and making it stand thus, *Parallel Lines or Superficies are those, upon which two straight Lines falling the same way, and whersoever they fall, making equall Angles, are equall*, it is both true and universall; and the following Consectary with very little change, as you may see in the translation, perspicuously demonstrated. The same fault occurreth once or twice more; and you triumph unreasonably, as if you had given therein a very great proof of your Geometry.

The same was observed also upon this place by one of the prime Geometricians of *Paris*, and noted in a Letter to his friend in these words, *Chap. 14. Art. 12. the Definition of Parallels wanteth somewhat to be supplyed.* And of the Consectary, he says, *it concludeth not, because it is grounded on the Definition of Parallels.* Truely, and severely enough, though without any such words as savour of Arrogance, or of Malice, or of the Clown.

At the thirteenth Article you recite the Demonstration by which I prove the Perimeters of two Circles to be Proportionall to their Semidiameters; and with *Esto, fortasse, recte, omnino*, noddying to the severall parts thereof, you come at Length to my last inference; *Therefore by (Chap. 13. Art. 6.) the Perimeters and Semidiameters of Circles are Proportionall*; which you deny; and therefore deny, because you say it followeth by the same Ratiocination, that *Circles also and Spheres are Proportionall to their Semidiameters. For the same distance* (you say) *of the Perimeter from the Center which determines the magnitude of the Semidiameter, determines also the magnitude both of the Circle, and of the Sphere.* You acknowledge that Perimeters and Semidiameters have the cause of their determination such as in equal times make equall spaces. Suppose now a Sphere generated by the Semidiameters, whilst the Semicircle is turned about. There is but one *Radius* of an infinite number of *Radii*, which describes a great Circle, all the rest describe lesser Circles Parallel to it, in one and the same time of Revolution. Would you have men believe, that describing greater and lesser Circles, is according to the supposition (*temporibus æqualibus æqualia facere*) to make equall spaces in equall times?

times? Or when by the turning about of the Semidiameter is deſcribed the Plain of a Circle, does it (think you) in equall times make the Plains of the interior Circles equall to the plains of the exterior? Or is the *Radius* that deſcribes the inner Circles equall to the *Radius* that deſcribes the exterior? It does not therefore follow from any thing I have ſaid in this demonſtration, that either Spheres, or Plains of Circles, are Proportionall to their *Radii.* And conſequently all that you have ſaid, triumphing in your own Incapacity, is ſaid imprudently by your ſelves to your own diſgrace. They that have applauded you, have reaſon by this time to doubt of all the reſt that follows, and if they can, to diſſemble the opinion they had before of your Geometry. But they ſhall ſee before I have done, that not only your whole *Elenchus*, but alſo your other Books of the *Angle of Contact*, &c. are meer ignorance and gibberiſh.

To the fourteenth Article you object, that (in the ſixth figure) I aſſume *gratis*, that FG, DE, BC, are Proportionall to AF, AD, AB; and you referre it to be judged by the Reader. And to the Reader I referre it alſo. The not exact drawing of the Figure (which is now amended) is it that deceived you. For AF, FD, DB, are equall by conſtruction. Alſo AG, GE, EC, are equall by conſtruction. And FG, DK, BH, KE, HI, IC, are equall by Paralleliſm. And becauſe AF, FG, are as the velocities wherewith they are deſcribed; alſo 2 AF (that is AD) and 2 FG (that is DE) are as the ſame velocities. And finally 3 AF (that is AB) and 3 FG (that is BC) are as the ſame velocities. It is not therefore aſſumed *gratis*, that FG, DE, BC, are Proportionall to AF, AD, AB, but grounded upon the ſixth Article of the thirteenth Chapter; and conſequently your objection is nothing worth. You might better have excepted to the placing of DE, firſt at adventure, and then making AD, two thirds of AB; for that was a fault, though not great enough to trouble a Candid Reader; yet great enough, to be a ground, to a malicious Reader, of a Cavill.

That which you object to the third *Corollarie* of Art. 15. was certainly a dream. There is no aſſuming of an Angle CDE, for an Angle HDE, or BDE, neither in the Demonſtration, nor in any of the Corollaries. It may be you dream't of ſomewhat in the twentieth Article of Chap. 16. But becauſe that Article though once printed, was afterwards left out, as not ſerving to the uſe I had deſigned it for, I cannot gueſs what it is. For I have no Copy of that Article, neither printed nor written, but am very ſure, though it were not uſefull, it was true.

Article the ſixteenth. Here we come to the Controverſie concerning the *Angle of Contact*, *which* (you ſay) *you have handled, in a ſpeciall Treatiſe publiſhed; and that you have clearly demonſtrated in your publick Lectures, that* Peletarius *was in the right. But that I agree not ſufficiently neither with* Peletarius, *nor with* Clavius. I confeſs I agree not in all points with *Peletarius*, nor in all points with *Clavius.* It does not thence follow that I agree not with the Truth. I am not (as you) of any faction, neither in Geometry, nor in Politicks. If I think that you, or *Peletarius*, or *Clavius*, or *Euclide* have erred, or been too obſcure; I ſee no cauſe, for which I ought to diſſemble it. And in this ſame Queſtion, I am of opinion that *Peletarius* did not well in denying the *Angle of Contingence* to be *an Angle.* And that *Clavius* did not well to ſay *the Angle of a Semicircle* was leſs then *a Right-lined Right Angle.* And that *Euclide* did not well to leave it ſo obſcure what he meant by *Inclination* in the Definition of a *Plain Angle*, ſeeing elſe where he attributeth Inclination onely to Acute Angles, and ſcarce any man ever acknowledged Inclination in a ſtraight Line, to any other Line, to which it was perpendicular. But you in this Queſtion of what is Inclination, though you pretend not to depart from *Euclide*, are nevertheleſs more obſcure then he; and alſo are contrary to him. For *Euclide* by Inclination meaneth the Inclination of one Line *to* another; and you underſtand it of the Inclination of one Line *from* another, which is not Inclination, but Declination. For you make two ſtraight Lines when they lye one on another, to lye ἀκλινῶς that is without any Inclination (becauſe it ſerves your turn); not obſerving that it followeth thence that Inclination is a digreſſion of one Line *from* another. This is in your firſt

Argument

Argument in the behalf of *Peletarius* (Pag. 10. Lin. 22.) and destroyshis opinion. For according to *Euclide* the greatest Angle is the greatest Inclination, and an Angle equall to two Right Angles by this ἀκλισία should not be the greatest Inclination, as it is, but the least that can be. But if by the Inclination of two Lines we understand that proceeding of them to a common Point which is caused by their generation, which (I believe) was *Euclides* meaning; then will the *Angle of Contact* be no less an *Angle* then a *rectilineall Angle*, but onely (as *Clavius* truly saies it is) Heterogeneous to it; and the doctrine of *Clavius* more conformable to *Euclide* then that of *Peletarius*. Besides, if it be granted you, that there is no inclination of the Circumference to the Tangent, yet it does not follow that their concourse doth not form some kind of Angle. For *Euclide* defineth there but one of the kinds of a Plain Angle. And then you may as much in vain seek for the Proportion of such an Angle to the Angle of Contact, as seek for the *Focus*, or *Parameter* of the *Parabola of Dives and Lazarus*. Your first argument therefore is nothing worth, except you make good that which in your second Argument you affirm, namely, That all Plain Angles, not excepting the Angle of Contact, are (*Homogeneous*) of the same kind. You prove it well enough of other Curvilineall Angles; but when you should prove the same of an Angle of Contact, you have nothing to say but Pag. 17. Lin. 15. *Unde autem illa quam somniet Heterogenia oriatur, neque potest ille ullatenus ostendere, neque ego vel somniare; whence should arise that diversity of kind, which he dreams of, neither can he at all shew, nor I dream*; as if you knew what he could do if he were to answer you; or all were false which you cannot dream of. So that besides your customary vanity, here is nothing hitherto proved neither for the opinion of *Peletarius*, nor against that of *Clavius*. I have I think sufficiently explicated in the first Lesson, That the Angle of Contact is Quantity, namely, that it is the Quantity of that crookedness or flexion, by which a straight Line is bent into an Arch of a Circle equall to it; and that because the crookedness of one Arch may be greater then the crookedness of another Arch of another Circle equall to it, therefore the Question *Quanta est curvitas*, How much is the crookedness, is pertinent, and to be answered by *Quantity*. And I have also shewn you in the same Lesson, that the Quantity of one Angle of Contact is compared with that of another Angle of Contact, by a Line drawn from the Point of Contact, and intercepted by their Circumferences; and that it cannot be compared by any measure with a Rectilineall Angle.

But let us see how you answer to that which *Clavius* has objected already. *They are Heterogeneous*, sayes he, *because the Angle of Contact, how oft soever multiplyed, can never exceed a Rectilineall Angle.* To answer which you alleadge, *it is no Angle at all; and that therefore it is no Angle at all because the Lines have no Inclination one to another.* How can Lines that have no Inclination one to another, ever come together? But you answer, *at least they have no Inclination in the Point of Contact.* And why have two straight Lines Inclination before they come to touch, more then a straight Line and an Arch of a Circle? And in the Point of Contact it self, how can it be that there is less Inclination of the two Points of a straight Line and an Arch of a Circle, then of the Points of two straight Lines? But the straight Lines you say will cut; Which is nothing to the Question; and yet this also is not so evident, but that it may receive an objection. Suppose two Circles A G B and C F B to touch in B, and have a common Tangent through B. Is not the Line C F B G A a crooked Line? And is it not cut by the common Tangent D B E? What is the Quantity of the two Angles F B E and G B D, seeing you say neither D B G nor E B F is an Angle? 'Tis not therefore the cutting of a crooked Line, and the touching of it, that distinguisheth an Angle simply, from an Angle of Contact. That which makes them differ, and in kind, is, that the one is the Quantity of a *Revolution*, and the other the Quantity of *Flexion*.



In the seventh Chapter of the same Treatise, you think you prove the Angle of Contact, if it be an Angle, and a Rectilineall Angle to

be (*Homogeneous*) of the ſame kind; when you prove nothing but that you underſtand not what you ſay. Thoſe Quantities which can be added together, or ſubſtracted one from another, are of the ſame kind; But an Angle of Contact may be ſubſtracted from a right Angle, and the Remainder will be the Angle of a Semicircle, &c. So you ſay, but prove it not, unleſs you think a man muſt grant you that the Superficies contained between the Tangent and the Arch, which is it you ſubſtract, is the Angle of Contact; and that the Plain of the Semicircle is the Angle of the Semicircle, which is abſurd; though as abſurd as it is, you ſay it directly in your *Elenchus*, Pag. 35. Lin. 14. in theſe words, *When* Euclide *defines a Plain Angle to be the Inclination of two Lines, he meaneth not their aggregate, but that which lyes between them.* It is true, he meaneth not the aggregate of the two Lines; but that he means that which lyes between them, which is nothing elſe but an indeterminate Superficies, is falſe, or *Euclide* was as fooliſh a Geometrician as either of you two.

Again, you would prove the Angle of Contact, if it be an Angle, to be of the ſame kind with a Rectilineall Angle, out of *Eucl.* 3. 16. Where he ſaies, *it is leſs then any acute Angle.* And it follows well, that if it be an Angle, and leſs then any Rectilineall Angle; it is alſo of the ſame kind with it. But to my underſtanding *Euclide* meant no more, but that it was neither greater nor equall; which is as truly ſaid of Heterogeneous, as of Homogeneous Quantities. If he meant otherwiſe, he confirms the opinion of *Clavius* againſt you, or makes the Quantity of an Angle to be a Superficies, and indefinite. But I wonder how you dare venter to determine whether two Quantities be Homogeneous or not, without ſome Definition of Homogeneous (which is a hard word) that men may underſtand what it meaneth.

In your eighth Chapter you have nothing but Sir *H. Saviles* Authority, who had not then reſolved what to hold; but eſteeming the Angle of contact, firſt, as others falſely did, by the Superficies that lyes between the Tangent and the Arch, makes the Angle of Contact, and a Rectilineall Angle Homogeneous; and afterwards, becauſe no multiplication of the Angle of Contact can make it equall to the leaſt Rectilineall Angle, with great ingenuity returneth to his former uncertainty.

In your nineth and tenth Chapters you prove with much ado, that the Angles of like Segments are equall; as if that might not have been taken *gratis* by *Peletarius* without Demonſtration. And yet your Argument contained in the nineth Chapter is not a Demonſtration, but a conjecturall diſcourſe upon the word *Similitude*. And in the eleventh Chapter, wherein you anſwer to an objection, which might be made to your Argument in the precedent Page, taken from the Paralleliſm of two concentrique Circles, though the objection be of no moment, yet you have in the ſame Treatiſe of yours that which is much more fooliſh, which is this, Pag. 38. Lin. 12. *Non enim magnitudo Anguli*, &c. *The magnitude of an Angle is not to be eſtimated by that ſtradling of the legs, which it hath without the Point of concourſe, but by that ſtradling which it hath in the Point of the concourſe it ſelf.* I pray you tell me what ſtradling there is of two coincident Points, eſpecially ſuch Points as you ſay are nothing. When did you ever ſee two nothings ſtraddle?

The Arguments in your twelfth and thirteenth Chapters are grounded all on this untruth, that an Angle is that which is contained between the Lines that make it, that is to ſay, is a Plain Superficies. Which is manifeſtly falſe; becauſe the meaſure of an Angle is an Arch of a Circle, that is to ſay, a Line; which is no meaſure of a Superficies. Beſides this groſs ignorance, your way of Demonſtration by putting N for a great Number of ſides of an æquilaterall Polygon, is not to be admitted. For though you underſtand ſomething by it, you demonſtrate nothing to any Body, but thoſe who underſtand your Symbolique tongue, which is a very narrow Language. If you had demonſtrated it in Iriſh, or Welſh, though I had not read it, yet I ſhould not have blamed you, becauſe you had written to a conſiderable Number of mankind, which now you do not.

In your laſt Chapters you defend *Vitellio* without need; for there is no doubt but that whatſoever crooked Line be touched by a ſtraight Line, the Angle of Contingence will neither add

any

any thing to, nor take any thing from a Rectilineall Right Angle; but that it is becauſe the Angle of Contact is no Angle, or no Quantity, is not true. For it is therefore an Angle, becauſe an Angle of Contact; and therefore Quantity, becauſe one Angle of Contact may be greater then another; and therefore Heterogeneall, becauſe the meaſure of an Angle of Contact cannot (*congruere*) be applyed to the meaſure of a Rectilineall Angle, as they think it may, who affirm with you that the Nature of an Angle conſiſteth in that which is contained between the Lines that comprehend it, viz. in a plain Superficies. And thus you ſee in how few Lines, and without Brachygraphie, your Treatiſe of the Angle of Contingence is diſcovered for the greateſt part to be falſe, and for the reſt, nothing but a detection of ſome errors of *Clavius* grounded on the ſame falſe Principles with your own. To return now from your Treatiſe of the Angle of Contact back again to your *Elenchus*.

The fault you find at Art. 18. is, that I underſtand not that *Euclide* makes a *Plain Angle* to be that which is contained between the two Lines that form it. 'Tis true, that I do not underſtand that *Euclide* was ſo abſurd, as to think the nature of an Angle to conſiſt in Superficies; but I underſtand that you have not had the wit to underſtand *Euclide*.

The nineteenth Article of mine in this fourteenth Chapter is this; *All reſpect, or variety of Poſition of two Lines, ſeemeth to be comprehended in four kinds.* For they are either *Parallel*; or, (*being if need be produced*) *make an Angle*; or, (if drawn out faire enough) *Touch*; or laſtly, they are *Aſymptotes*. In which you are firſt offended with the word *It ſeems*. But I allow you that never erre, to be more peremptory then I am. For to me it ſeemed, I ſay again ſeemed, that ſuch a Phraſe, in caſe I ſhould leave out ſomething in the enumeration of the ſeverall kinds of Poſition, would ſave me from being cenſured for untruth. And yet your inſtance of two ſtraight Lines in divers Plains, does not make my enumeration inſufficient. For thoſe Lines though not Parallels, nor cutting both the Plaines, yet being moved Parallelly from one Plain to another, will fall into one or other of the kinds of poſition by me enumerated; and conſequently are as much that poſition, as two ſtraight Lines in the ſame Plain not parallel, make the ſame Angle, though not produced till they meet, which they would make if they were ſo produced. For you have no where proved, nor can prove, that two ſuch Lines do not make an Angle. It is not the actuall concurrence of the Lines, but the Arch of a Circle, drawn upon that point for Center, in which they would meet, if they were produced, and intercepted between them, that conſtitutes the Angle.

Alſo your objection concerning Aſymptotes *in generall*, is abſurd. You would have me add, that *their diſtance ſhall at laſt be leſs then any diſtance that can be aſſigned*; and ſo make the definition of the *Genus* the ſame with that of the *Species*. But becauſe you are not Profeſſors of Logick, it is not neceſſary for me to follow your councell. In like manner, if we underſtand one Line to be moved towards another alwayes parallely to its ſelf, which is, though not actually, yet potentially the ſame poſition; all the reſt of your inſtances will come to nothing.

At the two and twentieth Article you object to me the uſe of the word *Figure*, before I had defined it: wherein alſo you do abſurdly; for I have no where before made ſuch uſe of the word *Figure*, as to argue any thing from it; and therefore your objection is juſt as wiſe as if you had found fault with putting the word Figure in the Titles of the Chapters placed before the Book. If you had known the nature of Demonſtration, you had not objected this.

You add further, that by my Definition of *Figure*, a ſolid Sphere, and a Sphere made hollow within, is the ſame Figure; but you ſay not why, nor can you derive any ſuch thing from my definition. That which deceived your ſhallowneſs, is, that you take thoſe Points that are in the concave Superficies of a hollowed Sphere, not to be contiguous to any thing without it, becauſe that whole concave Superficies is within the whole Sphere. Laſtly, for the fault you find, with the definition of *like Figures in like poſitions*; I confeſs there wants the ſame word which was wanting in the Definition of Parallels, namely, *ad eaſdem partes (the ſame way)* which ſhould have been added in the end of the definition of like Figures, &c. and may eaſily be ſupplyed by any ſtudent of Geometry, that is not otherwiſe a fool.

At the fifteenth Chapter Art 1. Numb. 6. you object as a contradiction, that *I make Motion to be the meaſure of Time, and yet in other places do uſually meaſure Motion and the affections thereof by Time.* If your thoughts were your own, and not taken rashly out of Books, you could not but (with all men elſe that ſee Time meaſured by Clocks, Dyals, Hour-glaſſes, and the like) have conceived ſufficiently, that there cannot be of Time any other meaſure beſides Motion; and that the moſt univerſall meaſure of Motion, is a Line deſcribed by ſome other Motion. Which Line being once expoſed to ſenſe, and the motion whereby it was deſcribed ſufficiently explicated, will ſerve to meaſure all other Motions and their Time; for Time and Motion (Time being but the mentall Image or remembrance of the motion) have but one and the ſame dimenſion, which is a Line. But you that would have me meaſure *ſwiftneſs* and *ſlowneſs* by longer and ſhorter motion, what do you mean by *longer* and *ſhorter motion*? Is *longer* and *ſhorter*, in the motion, or in the Duration of the motion, which is Time? Or is the Motion, or the Duration of the motion that which is expoſed, or deſigned by a Line? Geometricians ſay often, *let the Line A B, be the Time*; but never ſay, *let the Line A B be the Motion.* There is no unlearned man that underſtandeth not what is Time, and Motion, and Meaſure; and yet you that undertake to teach it (moſt egregious Profeſſors) underſtand it not.

At the ſecond Article you bring another Argument (which it ſeems in its proper place, you had forgotten) to prove that a Point is not Quantity not conſidered, but abſolutely Nothing; which is this, *That if a Point be not nothing, then the whole is greater then its two halfs.* How does that follow? Is it impoſſible when a Line is divided into two halfs that the middle Point ſhould be divided into two halfs alſo, being Quantity?

At the ſeventh Article, I have ſufficiently demonſtrated, that all Motion is infinitely propagated, as far as ſpace is filled with Body. You alleadge no fault in the demonſtration, but object from ſenſe, *that the skipping of a Flea, is not propagated to the Indies.* If I ask you how you know it, you may wonder perhaps; but anſwer you cannot. Are you Philoſophers or Geometricians, or Logicians, more then are the ſimpleſt of rurall people? Or are you not rather leſs, by as much as he that ſtandeth ſtill in ignorance, is nearer to knowledge, then he that runneth from it by erroneous learning?

And laſtly, what an abſurd objection is it which you make to the eighth Article, where I ſay that *when two Bodies of equall magnitude fall upon a third Body, that which falls with greater velocity, imprints the greater motion*? You object, *that not ſo much the magnitude is to be conſidered as the weight*; as if the weight made no difference in the velocity, when notwithſtanding weight is nothing elſe but motion downward? Tell me, when a weighty body thrown upwards worketh on the Body it meeteth with, do you not then think it worketh the more for the greatneſs, and the leſs for the weight?

Of

Of the Faults that Occurre in Demonſtration.

To the ſame egregious Profeſſors of the Mathematicks in the Univerſity of *Oxford*.

Lesson IIII.

OF twenty Articles which you ſay (of nineteen which I ſay) make the ſixteenth Chapter; you except but three, and confidently affirm the reſt are falſe. On the contrary, except three or four faults, ſuch as any Geometrician may ſee proceed not from ignonorance of the Subject, or from want of the Art of Demonſtration, (and ſuch as any man might have mended of himſelf) but from ſecurity; I affirm that they are all true, and truly Demonſtrated; and that all your objections proceed from meer ignorance of the Mathematiques.

The firſt fault you find is this, that I expreſs not, (*Art.* 1.) what *Impetus* it is, which I would have to be multiplyed into the Time.

The laſt Article of my thirteenth Chapter was this, *If there be a Number of Quantities propounded, howſoever equall or unequall to one another; and there be another Quantity which ſo often taken as there be Quantities propounded, is equall to their whole ſum; that Quantity I call the mean Arithmeticall of them all.* Which Definition I did there inſert to ſerve me in the explication of thoſe Propoſitions of which the ſixteenth Chapter conſiſteth, but did not uſe it here as I intended. My firſt Propoſition therefore as it ſtandeth yet in the Latine, being this, *The velocity of any Body moved during any Time, is ſo much as is the product of the* Impetus *in one Point of Time, multiplyed into the whole Time*; to a man that hath not skill enough to ſupply what is wanting, is not intelligible. Therefore I have cauſed it in the Engliſh to go thus, *The velocity of any Body in whatſoever Time moved, hath its Quantity determined by the ſum of all the ſeverall* (Impetus) *Quickneſſes, which it hath in the ſeverall Points of the Time of the Bodies motion,* And added, *that all the* Impetus *together taken through the whole Time is the ſame thing with the Mean* Impetus (which Mean is defined Chapter 13. Art. 29.) *multiplyed into the whole Time.* To this firſt Article, as it is uncorrected in the Latine, you object, *That meaning by* Impetus *ſome middle* Impetus, *and aſſigning none, I determine nothing.* And 'tis true. But if you had been Geometricians ſufficient to be Profeſſors, you would have ſhewed your skill much better, by making it appear that this middle *Impetus* could be none but that, which being taken ſo often, as there be Points in the Line of Time, would be equall to the ſum of all the ſeverall *Impetus* taken in the Points of Time reſpectively; which you could not do.

To the *Corollary*, you ask firſt how *Impetus* can be ordinately applyed to a Line; Abſurdly. For does not *Archimedes* ſometimes ſay, and with him many other excellent Geometricians, *let ſuch a Line be the Time?* And do they not mean, that that Line, or the motion over it, is

the

the meaſure of the Time? And may not alſo a Line ſerve to meaſure the ſwiftneſs of a Motion? *You thought* (you ſay) *onely Lines ought to be ſaid to be ordinately applyed to Lines.* Which I eaſily believe; for I ſee you underſtand not that a Line, though it be not the Time it ſelf, may be the quantity of a Time. You thought alſo all you have ſaid in your *Elenchus*, in your Doctrine of the *Angle of Contact*, in your *Arithmetica Infinitorum*, and in your Coniques is true; and yet it is almoſt all proved falſe, and the reſt nothing worth.

Secondly, you object, that *I deſign a Parallelogram by one onely ſide.* It was indeed a great overſight, and argueth ſomewhat againſt the man, but nothing againſt his Art. For he is not worthy to be thought a Geometrician that cannot ſupply ſuch a fault as that, and correct his Book himſelf. Though you could not do it, yet another from beyond Sea took notice of the ſame fault in this manuer, *He maketh a Parallelogram of but one ſide*; it ſhould be thus, *Vel denique per Parallelogrammum cujus unum latus eſt medium proportionale inter Impetum maximum (ſive ultimò acquiſitum) & impetûs ejuſdem maximi ſemiſſem; alterum vero latus, medium proportionale, inter totum tempus, & ejuſdem totius temporis ſemiſſem.* Which I therefore repeat, that you may learn good manners; and know, that they who reprehend, ought alſo, when they can, to add to their reprehenſion the correction.

At the ſecond Article, you are pleaſed to adviſe me, inſtead of *In omni motu uniformi*, to put in *In omnibus motibus uniformibus.* You have a ſtrange opinion of your own Judgement, to think you know to what end another man uſeth any word, better then himſelf. My intention was onely to conſider motions uniform, and motions from reſt uniformly, or regularly accelerated, that I might thereby compute the lengths of crooked Lines, ſuch as are deſcribed by any of thoſe motions. And therefore it was enough to prove this Theoreme to be true in all uniform or uniformly accelerated *Motion*, not *Motions*; though it be true alſo in the Plurall. It ſeems you think a man muſt write all he knows, whether it conduce, or not, to his intended purpoſe. But that you may know that I was not, (as you think) ignorant how far it might be extended, you may read it Demonſtrated at the ſame Article in the Engliſh univerſally. Againſt the demonſtration it ſelf you run to another Article, namely, the thirteenth, which is this Probleme, *The length being given, which is paſſed over in a given Time by uniform Motion, to finde e length which ſhall be paſſed over by Motion uniformely accelerated in the ſame Time, ſo as that the* Impetus *laſt acquired be equall to the Time.* Which you recite imperfectly, thereby to make it ſeem that ſuch a Length is not determined. Whether you did this out of ignorance, or on purpoſe, thinking it a piece of wit, as your pretended myſterie which goes immediately before, I cannot tell, for in neither place can any wit be eſpied by any but your ſelves. To imagine Motions with their Times and Wayes, is a new buſineſs, and requires a ſteddy brain, and a man that can conſtantly read his own thoughts, without being diverted by the noiſe of words. The want of this ability, made you ſtumble and fall unhandſomely in the very firſt place, (that is in Chap. 13. Art. 13.) where you venture to reckon both Motion and Time at once; and hath made you in this Chapter to ſtumble in the like manner at every ſtep you go. As for example, when I ſay, *as the product of the Time, and Impetus, to the product of the Time and Impetus, ſo the Space to the Space when the Motion is Vniform*, you come in with *nay, rather as the Time to the Time*; as if the Parallelograms A I, and A H, were not alſo as the Times A B, and A F. Thus it is, when men venture upon ways they never had been in before, without a guide.

In the *Corollary*, you are offended with the permutation of the Proportion of Times and Lines, becauſe you think, you that have ſcarce one right thought of the Principles of Geometry, that Line, and Time are Heterogeneous Quantities. I know Time and Line are of divers natures; and more, that neither of them *is Quantity*. Yet they may be both of them *Quanta*, that is, they may *have Quantity*; but that their Quantities are Heterogeneous is falſe. For they are compared and meaſured both of them by ſtraight lines. And to this there is nothing contrary in the place cited by you out of *Clavius*; or if there were, 'twere not to be valued. And to your queſtion what is the Proportion of an *Hour* to an *Ell*; I anſwer, it is the ſame

Proportion

Proportion that *two Hours* have to two *Ells*. You see your Question is not so subtile as you thought it: By and by you confess that in Times and Lines there is *Quid Homogeneum* (this *Quid* is an infallible sign of not fully understanding what you say) which is false it you take it of the Lines themselves; though if you take it of their Quantities, it is true without a *Quid*. Lastly, you tell me how I might have expressed my self so as it might have been true. But because your expressions please me not, I have not followed your advice.

To the third Article, which is this, *In motu uniformiter à quiete accelerato, &c. In motion uniformly accelerated from Rest, that is, when the* Impetus *increaseth in Proportion to the Times, The Length run over in one Time is to the Length run over in another Time, as the Product of the* Impetus *multiplyed by the Time, to the Product of the* Impetus *multiplyed by the Time*; you object, *that the Lengths run over, are in that Proportion which the* Impetus *hath, to the* Impetus; *not that which the* Impetus *hath to the Time, because* Impetus *to Time has no Proportion, as being Heterogeneous.* First, when you say the *Impetus*, do you mean some one *Impetus* designed by some one of the unequall straight Lines Parallel to the Base B I? That is manifestly false. You mean the aggregate of all those unequall Parallels. But that is the same thing with the Time multiplyed into the mean *Impetus*. And so you say the same that I do. Again, I ask where it is that I say or dream that the Lengths run over are in the Proportion of the *Impetus* to the Times? Is it you or I that dream? And for your Heterogeneity of the Quantities of Time and of Swiftness, I have already in divers places shewed you your error. Again, Why do you make B I represent the Lengths run over, which I make to be represented by D E, a Line taken at pleasure; and you also a few Lines before make the same B I to design the greatest acquired *Impetus*? These are things which shew that you are puzzled and intangled with the unusuall speculation of Time and Motion, and yet are thrust on with Pride and Spite to adventure upon the examination of this Chapter.

Secondly, you grant the Demonstration to be good, supposing I meane it (as I seeme to speak) of one and the same Motion. But why doe I not meane it of one and the same Motion, when I say not in *Motions*, but in *Motion* uniform? *Because* (say you) *in that which follows, I draw it also to different Motions.* You should have given at least one Instance of it; but there is no such matter. And yet the Proposition, is in that case also true; though then it must not be Demonstrated by the similitude of Triangles, as in the case present. And therefore the objections you make from different *Impetus* acquired in the same time, and from other cases which you mention, are nothing worth.

At the fourth Article, you allow the Demonstration all the way (except the faults of the third, which I have already proved to be none) till I come to say, *that because the Proportion of F K to B I is double to the Proportion of A F to A B, therefore the Proportion of A B to A F is double to the Proportion of B I to F K.* This you deny, and wonder at, as strange (for it is indeed strange to you) and in many places you exclaim against it as extream Ignorance in Geometry. In this place you only say, *No such matter; for though one Proportion be double to another, yet it does not follow that the Converse is the double of the Converse.* So that this is the issue to which the Question is reduced, whether you have any or no Geometry. I say, if there be three Quantities in continuall Proportion, and the first be the least, the Proportion of the first to the second is double to the Proportion of the first to the third; and you deny it. The reason of our dissent consisteth in this, that you think the doubling of a Proportion to be the doubling of the Quantity of the Proportion, as well in Proportions of Defect, as in Proportions of Excess; and I think that the doubling of a Proportion of Defect, is the doubling of the defect of the Quantity of the same. As for example in these three numbers, 1, 2, 4, which are in continuall Proportion, I say the Quantity of the Proportion of one to two, is double the Quantity of the Proportion of one to four. And the Quantity of the Proportion of one to four, is half the Quantity of the Proportion of one to two. And yet deny not but that the Quantity of the Defect in the Proportion of one to two is doubled in the Proportion of one to four. But because the doubling of defect makes greater defect, it

maketh the Quantity of the Proportion less. And as for the part which I hold in this Question, first, there is thus much demonstrated by *Euclide*, El. 5. Prop. 8; that the Proportion of one to two is greater then the Proportion of one to four, though how much it is greater be not there Demonstrated. Secondly, I have Demonstrated (Chap. 13. Art. 16.) That it is twice as great, that is to say, (to a man that speaks English) double. The introducing of *duplicate*, *triplicate*, &c. instead of *double*, *triple*, &c. (though now they be words well understood by such as understand what Proportion is) proceeded at first from such as durst not for fear of absurdity, call the half of any thing double to the whole, though it be manifest that the half of any defect is a double Quantity to the whole defect; for want added to want maketh greater want, that is, a less positive Quantity. This difference between *double*, and *duplicate*, lighting upon weak understandings, has put men out of the way of true reasoning in very many Questions of Geometry. *Euclide* never used but one word both for *double* and *duplicate*. It is the same fault when men call half a Quantity *Subduplicate*, and a third part *Subtriplicate* of the whole, with intention (as in this case) to make them pass for words of signification different from the *half* and the *third part*. Besides, from my Definition of Proportion (which is clear, and easie to be understood by all men, but such as have read the Geometry of others unluckily) I can Demonstrate the same evidently and briefly thus. My Definition is this, *Proportion is the Quantity of one Magnitude taken comparatively to another*. Let there be therefore three Quantities, 1, 2, 4, in continuall Proportion. Seeing therefore the Quantity of four in respect of one, is twice as great as the Quantity of the same four in respect of 2, it followeth manifestly that the Quantity of 1 in respect of 4, is twice as little as the Quantity of the same 1 in respect of 2; and consequently the Quantity of one in respect of two, is twice as great as the Quantity of the same one in respect of four; which is the thing I maintain in this Question. Would not a man that imployes his time at Bowles, choose rather to have the advantage given him of three in nine, then of one in nine? And why, but that three is a greater Quantity in respect of nine, then is one? Which is as much as to say, three to nine hath a greater Proportion then one to nine; as is Demonstrated by *Euclide*, El. 5.8. Is it not therefore (you that profess Mathematiques, and Theology, and practise the depression of the truth in both) well owl'd of you, to teach the contrary? But where you say *that the Point K* (in the second Figure of the Table belonging to this 16. Chapter) *is not in the Parabolicall Line whose Diameter is A B, and Base B I, but in the Parabolicall Line of the Complement of my Semiparabola (as I may learn from the twenty-third Proposition of your* Arithmetica Infinitorum*) whose Diameter is A C, and Base I C.* What Line is that? Is it the same Line with that of my Semiparabola, or not the same? If the same, why find you fault? If not the same, you ought to have made a Semiparabola on the Diameter A C, and Base I C, and following my Construction made it appear that K is not in the Line wherein I say it is; which you have not done, nor could do.

Then again, running on in the same blindness of Passion, you pretend I make the Proportion of B I to F K double to that of A B to A F, and then confute it; when you knew I made the Proportion of F K to B I, double to that of F N, to B I, that is, of A F to A B; and this was it you should have confuted. That which followeth is but a Triumphing in your own Ignorance, wherein you also say, *That all that I afterwards build upon this Doctrine is false.* You see whether it be like to prove so or not. As for your *Arithmetica Infinitorum*, I shall then read to you a piece of a Lesson on it when I come to your objections against the next Chapter. In the mean Time let me tell you, it is not likely you should be great Geometricians, that know not what is Quantity, nor Measure, nor Straight, nor Angle, nor Homogeneous, nor Heterogeneous, nor Proportion, as I have already made appear in this and the former Lessons.

To the first *Corollary* of this fourth Article your exception I confess is just, and (which I wonder at) without any incivility. But this argues not Ignorance, but Security. For who is there that ever read any thing in the Coniques, that knows not that the parts of a Parabola cut

cut off by Lines Parallel to the Baſe, are in Triplicate Proportion to their Baſes? But having hitherto deſigned the Time by the Diameter, and the *Impetus* by the Baſe; and in the next Chapter (where I was to calculate the Proportion of the Parabola, to the Parallelogram) intending to deſign the Time by the Baſe, I miſtook and put the Diameter again for the Time; which any man but you might as eaſily have corrected as reprehended.

To the ſecond *Corollarie*, which is this, *That the Lengths run over in equall Times by Motion ſo accelerated, as that the* Impetus *increaſe in double Proportion to their Times, are as the differences of the Cubick numbers beginning at unity, that is, as ſeven, nineteen, thirty-ſeven*, &c. You ſay 'tis falſe. But why? *Becauſe* (ſay you) *portions of the Parabola of equall altitude taken from the beginning are not as thoſe numbers ſeven, nineteen, thirty-ſeven*, &c. Does this think you, contradict any thing in this Propoſition of mine? Yes, becauſe (you think) the lengths gone over in equall Times, are the ſame with the parts of the Diameter cut off from the Vertex, and proportionall to the numbers one, two, three, &c. Whereas the lengths run over, are as the aggregates of their velocities, that is, as the parts of the Parabola it ſelf, that is, as the Cubes of their Baſes, that is, as the numbers one, eight, twenty-ſeven, ſixty-four, &c. and conſequently the lengths run over in equall Times, are as the differences of thoſe Cubick Numbers one, eight, twenty-ſeven, ſixty-four, whoſe differences are ſeven, nineteen, thirty-ſeven, &c. The cauſe of your miſtake was, that you cannot yet, nor perhaps ever will contemplate Time and Motion (which requireth a ſteddy brain) without confuſion.

The third *Corollary*, you alſo ſay is falſe, *Whether it be meant of Motion uniformly accelerated* (as the words are) *or* (*as perhaps*, you ſay, *I meant it*) *of ſuch Motion as is accelerated in double Proportion to the Time*. You need not ſay perhaps I meant it. The words of the Propoſition are enough to make the meaning of the *Corollary* underſtood. But ſo alſo you ſay it is falſe. Me thinks you ſhould have offered ſome little proof to make it ſeem ſo. You think your Authority will carry it. But on the contrary I believe rather that they that ſhall ſee how your other objections hitherto have ſped, will the rather think it true, becauſe you think it falſe. The Demonſtration as it is, is evident enough; and therefore I ſaw no cauſe to change a word of it.

To the fifth Article you object nothing, but that it dependeth on this Propoſition (Chap. 13. Art. 16.) *That when three Quantities are in continuall Proportion, and the firſt is the leaſt, as in theſe numbers, four, ſix, nine. The Proportion of the firſt to the ſecond is double to the Proportion of the ſame firſt to the laſt.* Which is there demonſtrated, and in the former Leſſons ſo amply explicated, as no man can make any further doubt of the truth of it. And you will, I doubt not aſſent unto it. But in what eſtate of mind will you be then? A man of a tender forehead after ſo much inſolence, and ſo much contumelious language grounded upon arrogance and ignorance, would hardly indure to out-live it. In this vanity of yours, you ask me whether I be angry, or bluſh, or can endure to hear you. I have ſome reaſon to be angry; for what man can be ſo patient as not to be moved with ſo many injuries? And I have ſome reaſon to bluſh, conſidering the opinion men will have beyond Sea, (when they ſhall ſee this in Latine) of the Geometry taught in *Oxford*. But to read the worſt you can ſay againſt me, I can indure, as eaſily at leaſt, as to read any thing you have written in your Treatiſes of the *Angle of Contact*, of the *Conique-ſections*, or your *Arithmetica Infinitorum*.

The ſixth, ſeventh, eighth Articles, you ſay are ſound. True. But never the more to be thought ſo for your approbation, but the leſs; becauſe you are not fit, neither to reprehend, nor praiſe; and becauſe all that you have hitherto condemned as falſe, hath been proved true. Then you ſhew me how you could demonſtrate the ſixth and ſeventh Articles a ſhorter way. But though there be your Symboles, yet no man is obliged to take them for demonſtration. And though they be granted to be dumb Demonſtrations, yet when they are taught to ſpeak as they ought to do, they will be longer Demonſtrations then theſe of mine.

To the nineth Article, which is this, *If a Body be moved by two Movents at once, concurring in what Angle ſoever, of which, one is moved uniformly, the other, with Motion uni-*

formly accelerated from Reſt, till it acquire an Impetus *equall to that of the uniform motion, the line in which the Body is carried, ſhall be the crooked line of a Semiparabola.* You lift up your voice again, and ask *what Latitude? what Diameter? what Inclination* of the Diameter to the Ordinate Lines? If your Founder ſhould ſee this, or the like objections of yours, he would think his money ill beſtowed. When I ſay *in what Angle ſoever*, you ask, *in what Angle*? When I ſay *two Movents, one uniform, the other uniformly accelerated, make the Body deſcribe a Semiparabolicall line*; you ask, *which is the Diameter*; as not knowing that the accelerated motion deſcribes the Diameter, and the other a Parallel to the Baſe. And when I ſay *the two Movents meet in a Point, from which Point both the Motions begin, and one of them from Reſt*, you ask me, *what is the Altitude*? As if that Point where the Motion begins from Reſt were not the Vertex; or that the Vertex and Baſe being given, you had not wit enough to ſee that the Altitude of the Parabola is determined? When *Galileo's* Propoſition, which is he ſame with this of mine, ſuppoſed no more but a Body moved by theſe two Motions, to prove the Line deſcribed to be the crooked Line of a Semiparabola, I never thought of asking him what Altitude, nor what Diameter, nor what Angle, nor what Baſe had his Parabola. And when Archimedes ſaid, let the Line A B be the Time, I ſhould never have ſaid to him, *do you think Time to be a Line*, as you ask me whether I think *Impetus* can be the Baſe of a Parabola. And why, but becauſe I am not ſo egregious a Mathematician, as you are. In this giddineſs of yours, cauſed by looking upon this intricate buſineſs of Motion, and of Time, and the concourſe of Motion uniform, and uniformly accelerated, you rave upon the numbers 1, 4, 9, 16, &c. without reference to any thing that I had ſaid; inſomuch as any one that had ſeen how much you have been deceived in them before in your ſcurvy Book of *Arithmetica Infinitorum*, would preſently conclude, that this objection was nothing elſe but a fit of the ſame madneſs which poſſeſt you there.

My tenth Article is like my nineth; and your objections to it are the ſame which are to the former. Therefore you muſt take for anſwer juſt the ſame which I have given to your objection there.

To the eleventh, you ſay firſt, you have done it better at the ſixty-fourth Article of your *Arithmetica Infinitorum*. But what you have done there, ſhall be examined when I come to the Defence of my next Chapter. And whereas I direct the Reader for the finding of the Proportions of the Complements of thoſe Figures to the Figures themſelves, to the Table of Art. 3. Chap. 17. you ſay that if the encreaſe of the *Spaces*, were to the encreaſe of the Times, as one to two, then the complement ſhould be to the Parallelogram as one to three, and ſay you find not $\frac{1}{3}$ in the Table. Did you not ſee that the Table is onely of thoſe Figures which are deſcribed by the concourſe of a Motion uniform with a motion accelerated? You had no reaſon therefore to look for $\frac{1}{3}$ in that Table; for your caſe is of Motion uniform concurring with Motion retarded, becauſe you make not the Proportions of the Spaces to the Proportions of the Times as two to one, but the contrary; ſo that your objection ariſeth from want of obſerving what you read. But *I may learn* (you ſay) *theſe, and greater Matters then theſe, in your twenty-third and ſixty-fourth Propoſitions of your* Arithmetica Infinitorum. This which you ſay here is a great Abſurdity; but if you mean I ſhall finde greater there, I will not ſay againſt you. This $\frac{1}{3}$ you looked for, belongs to the Complements of the Figures calculated in that Table; which becauſe you are not able to find out of your ſelves, I will direct you to them. Your caſe is of $\frac{1}{3}$ for the Complement of a Parabola. Take the Denominator of the Fraction which belongs to the Parabola, namely three, and for Numerator take the Numerator of the Fraction which belongs to the Triangle, namely one, and you have the Fraction ſought. And in like manner for the

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Complement of any other Figuer. As for example, of the second Parabolaster, whose Fraction hath for Denominator five, take the Numerator of the Fraction of the same Triangle which is one, and you have $\frac{1}{5}$ for the Fraction sought for; and so of the rest, taking alwayes one for the Numerator.

The twelfth Article, which you say is miserably false, I have left standing unaltered. For not comprehending the sense of the Proposition, you make a Figure of your own, and fight against your own fancied Motions, different from mine. Other Geometricians that understand the construction better, find no fault. And if you had in your own fifth Figure drawn a Line through N Parallel to A E, and upon that Line supposed your accelerated Motion, you would quickly have seen that in the Time AE, the Body moved from Rest in A, would have fallen short of the Diagonall A D; and that all your extravagant pursuing of your own mistake had been absurd.

My thirteenth Article you say is ridiculous. But why? *The* Impetus *last acquired cannot (you say) be equall to a Time.* But the Quantity of the *Impetus* may be equall to the Quantity of a Time, seeing they are both measured by Line. And when they are measured by the same described Line, each of their Quantities is equall to that same Line, and consequently to one another. But when I meet with this kind of objection again, since I have so often already shewn it to be frivolous, and no less to be objected against all the Antients that ever Demonstrated any thing by Motion, then against me, I purpose to neglect it.

Secondly, you object *That Motion uniformly accelerated does no more determine Swiftness, then Motion uniform.* True; you needed not have used sixteen Lines to set down that. But suppose I add (as I do) so as the last acquired *Impetus* be equall to the Time. *But that* (you say) *is not sense*; Which is the objection I am to neglect. But (you say again) supposing it sense, this limitation helps me nothing. Why? *Because* (you say) *a Parabola may be described upon a Base given, and yet have any Altitude, or any Diameter one will.* Who doubts it? But how follows it from thence, that when a Parabolicall Line is described by two Motions, one uniform, the other uniformly accelerated from Rest, that the determining of the Base does not also determine the whole Parabola? But fifthly (you say) *that this equality of the* Impetus *to the Time does not determine the Base.* Why not? *Because* (you say) *it is an error proceeding from this, that I understand not what is* Ratio subduplicata. I look't for this. I have shewn and inculcated sufficiently before, that the error is on your side; and therefore must tell you, that this objection, and also a great part of the rest of your errors in Geometry proceedeth from this, that you know not what Proportion is. But see how wisely you argue about this duplication of Proportion. For thus you say *verbatim. Stay a little. What Proportion has duplicate Proportion to single Proportion? Is it alwayes the same? I think not. For example.* *Duplicate Proportion* $\frac{4}{1} = \frac{2}{1}$ in $\frac{2}{1}$ *is double to the single* $\frac{2}{1}$. *Duplicate Proportion* $\frac{9}{1} = \frac{3}{1}$ in $\frac{3}{1}$ *is triple to its single* $\frac{3}{1}$. Let any man, even of them that are most ready in your Symboles, say in your behalf (if he be not ashamed) that the Proportion of nine to one is triple to the Proportion of three to one, as you do.

In the fourteenth, fifteenth, and sixteenth Article, you bid me repeat your objections to the thirteenth. I have done it; and find that what you have objected to the thirteenth, may as well be objected to these; and consequently, that my answer there will also serve me here. Therefore (if you can endure it) read the same answer over again.

But you have not yet done (you say) with these Articles. Therefore (after you had for a while spoken perplextly; conjecturing not without just cause, that I could not understand you) you say that to the end I may the better perceive your meaning, I should take the example following. *Let a Movent (in the first Figure of this Chapter) be moved uniformly, in the Time A B, with the continuall* Impetus *A C, or* B I, *whose whole velocity shall therefore be the*

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Parallelogram A C I B. And another Movent be uniformly accelerated, ſo as in the Time A B it acquire the ſame Impetus *B I. Now as the whole velocity, is to the whole velocity, ſo is the length run over, to the length run over.* All this I acknowledge to be according to my ſenſe, ſaving that your putting your word *Movens* inſtead of my word *Mobile* hath corrupted this Article. For in the firſt Article, I meddle not with Motion by Concourſe, wherein only I have to do with two Movents to make one Motion; but in this I do, wherein my word is not *Movens* but *Mobile*; by which it is eaſie to perceive you underſtand not this Propoſition. Then you proceed, *But the length run over by that accelerated Motion is greater then the length run over by that uniform Motion.* Where do I ſay that? You anſwer, *in the nineth and thirteenth Article, in making A B (in the fifth Figure) greater then A C; and A H (in the eighth Figure) greater then A B; and conſequently, the Triangle A B I, greater then the Parallelogram A C I B.* That conſequently is without conſequence; for it importeth nothing at all in this Demonſtration, whether A B, or A C in the fifth Figure be the greater. Beſides, I ſpeak there of the Concourſe of two Movents that deſcribe the Parabolicall Line A G D; where the increaſing *Impetus*, (becauſe it increaſeth as the Times) will be deſigned by the Ordinate Lines in the Parabola A G D B. And if both the Motions in A B and A C were uniform, the Aggregate of the *Impetus* would be deſigned by the Triangle A B D, which is leſs then the Parallelogram A C D B. But you thought that the Motion made by A C uniformly, is the ſame with the Motion made uniformly in the ſame Time by the Motions in A B, and A C, concurring. So likewiſe in the eighth Figure, there is nothing hinders A H from being greater then A B, unleſs I had ſaid that A B had been deſcribed in the Time A C with the whole *Impetus* A C maintained entire; of which there is nothing in the Propoſition, nor would at all have been pertinent to it. Therefore all this new undertaking of the thirteenth, fourteenth, fifteenth and ſixteenth Articles, is to as little purpoſe as your former objections. But I perceive that theſe new and hard Speculations, though they turn the edge of your wit, turn not the edge of your malice.

At the ſeventeenth Article, you ſhew again the ſame confuſion. Return to the eighth Figure, *If in a Time given a Body run over two lengths, one with uniform, the other with accelerated Motion*; as for example, if in the ſame Time A C a Body run over the Line A B with uniform Motion, and the Line A H with Motion accelerated; *and again in a part of that Time it run over a part of the length A H, with uniform Motion, and another part of the ſame with Motion accelerated*; as for example, in the Time A M it run over with uniform Motion the Line A I, and with Motion accelerated the Line A B. *I ſay the exceſs of the whole A H above the part A B, is to the exceſs of the whole A B above the part A I, as the whole A H, to the whole A B.* But firſt you will ſay, that theſe words *as the whole A H to the whole A B*, are left out in the Propoſition. But you acknowledge that it was my meaning; and you ſee it is expreſſed before I come to the Demonſtration. And therefore it was abſurdly done to reprehend it. Let us therefore paſs to the Demonſtration. Draw I K Parallel to A C, and make up the Parallelogram A I K M. And ſuppoſing firſt the acceleration to be uniform, divide I K in the midſt at N; and between I N, and I K, take a mean Proportionall I L. *And the ſtraight Line A L, drawn and produced, ſhall cut the Line* B D *in F, and the Line C G in G* (which Lines C G, and B D, as alſo H G and B F, are determined (though you could not carry it ſo long in memory) by the Demonſtration of the thirteen Article.) *For ſeeing A B is deſcribed by Motion uniformly accelerated, and A I by Motion uniform in the ſame Time A M; and I L is a mean Proportionall between I N (the half of I K) and I K; therefore by the Demonſtration of the thirteenth Article; A I is a mean Proportionall between A B and the half of* A B, *namly* A O. *Again, becauſe A B is deſcribed by uniform Motion, and* A H *by Motion uniformly accelerated, both of them in the ſame Time* A C, B F *is a mean Proportionall between* B D *and half* B D, *namely* B E; *Therefore by the Demonſtration of the ſame thirteenth Article, the ſtraight Line* A L F *produced will fall on G; and the Line* A H *will be to the Line* A B, *as the Line* A B *to the Line* A I. *And conſequently as* A H *to* A B,

ſo

ſo HB *to* BI; *which was to be Demonſtrated.* And by the like Demonſtration the ſame may be proved, where the acceleration is in any other Proportion that can be aſſigned in Numbers, ſaving that whereas this Demonſtration dependeth on the conſtruction of the thirteenth Article, if the Motion had been accelerated in double Proportion to the Times, it would have depended on the fourteenth, where the Lines are determined. Which determinations being not repeated, but declared before in the thirteenth Article, to which this Diagram belongeth, you take no notice of, but go back to a Figure belonging to another Article where there was no uſe of theſe Determinations. But becauſe I ſee that the words of the Propoſition, are as of four Motions, and not of two Motions made by twice two Movents, I muſt pardon them that have not rightly underſtood my meaning; and I have now made the Propoſition according to the Demonſtration. Which being done, all that you have ſaid in very neer two leaves of your *Elenchus* comes to nothing; and the fault you find comes to no more then a too much truſting to the skill and diligence of the Reader. And whereas after you had ſufficiently troubled your ſelf upon this occaſion, you add, *that if Sir H. Savile had read my Geometry, he had never given that cenſure of* Joſeph Scaliger, *in his Lectures upon* Euclide, *that he was the the worſt Geometrician of all Mortall men, not excepting ſo much as* Orontius, *but that praiſe ſhould have been kept for me*; You ſee by this time, at leaſt others do, how little I ought to value that opinion; and that though I be the leaſt of Geometricians, yet my Geometry is to yours as 1 to 0. I recite theſe words of yours, to let the world ſee your indiſcretion in mentioning ſo needleſly that paſſage of your Founder. It is well known that *Joſeph Scaliger* deſerved as well of the ſtate of Learning, as any man before or ſince him; and that though he failed in his Ratiocination concerning the Quadrature of the Circle, yet there appears in that very failing ſo much knowledge of Geometry, that Sir *H. Savile* could not but ſee that there were mortall men very many that had leſs; and conſequently he knew that that cenſure of his in a rigid ſenſe (without the Licenſe of an *Hyperbole*) was unjuſt. But who is there that will approve of ſuch *Hyperboles* to the diſhonour of any but of unworthy perſons, or think *Joſeph Scaliger* unworthy of honour from Learned men? Beſides, it was not Sir *H. Savile* that confuted that falſe Quadrature, but *Clavius*. What honour was it then for him to triumph in the victory of another? When a beaſt is ſlain by a Lion, is it not eaſie for any of the Fowles of the Air to ſettle upon, and peck him? Laſtly, though it were a great error in *Scaliger*, yet it was not ſo great a fault as the leaſt Sin; and I believe that a publique contumely done to any worthy perſon after his death, is not the leaſt of Sins. Judge therefore whether you have not done indiſcreetly, in reviving the onely fault, perhaps that any man living can lay to your Founders charge; and yet this error of *Scaligers* was no greater then one of your own of the like nature, in making the true Spirall of Archimedes equall to half the circumference of the Circle of the firſt revolution; and then thinking to cover your fault by calling it afterwards an Aggregate of Arches of Circles (which is no Spirall at all of any kind) you do not repair but double the abſurdity. What would Sir *Henry Savile* have ſaid to this?

The eighteenth Article is this, *In any Parallelogram, if the two ſides that contain the Angle be moved to their oppoſite ſides, the one uniformly, the other uniformly accelerated; the ſide that is moved uniformly, by its concourſe through all its Longitude, hath the ſame effect which it would have if the other Motion were alſo uniform, and the Line deſcribed, were a mean Proportionall between the whole Length, and the half of the ſame.*

To the Propoſition you object firſt, *that it is all one whether the other motion be uniform or not, becauſe the effect of each of their Motions, is but to carry the Body to the oppoſite ſide.* But do you think that whatſoever be the Motions, the Body ſhall be carryed by their concourſe alwayes to the ſame Point of the oppoſite ſide? If not, then the effect is not all one when a Motion is made by the concourſe of two Motions uniform and accelerated, and when it is made by the concourſe of two uniform or of two accelerated Motions.

Secondly, you ſay that theſe words, *and the Line deſcribed were a mean Proportionall between*

tween the whole Length, and the half of the ſame, have no ſenſe, or that you are deceived. True. For you are deceived; or rather you have not underſtanding enough diſtinctly to conceive variety of Motions though diſtinctly expreſſed. For when a Line is gone over with Motion uniformly accelerated, you cannot underſtand how a mean Proportionall can be taken between it and its half; or if you can, you cannot conceive that that mean can be gone over with uniform motion in the ſame Time that the whole Line was run over by motion uniformly accelerated. Yet theſe are things conceivable, and your want of underſtanding muſt be made my fault.

My Demonſtration is this, *In the* Parallelogram A B C D, (*Fig.* 11.) *Let the ſide* A B *be conceived to be moved uniformly till it lye in* C D; *and let the Time of that Motion be* A C, *or* B D. *And in the ſame Time let it be conceived that* A C *is moved with uniform acceleration, till it lye in* B D. To which you object, *that then the acceleration laſt acquired muſt be far greater then that wherewith* A B *is moved uniformly; elſe it ſhall never come to the place you would have it in the ſame Time.* What proof bring you for this? None here. Where then? No where that I remember. On the contrary I have proved (Art. 9. of this Chapter) that the Line deſcribed by the concourſe of thoſe two Motions, namely, uniform from A B to C D, and uniformly accelerated from A C to B D, is the crooked Line of the Semiparabola A H D. And though I had not, yet it is well known that the ſame is demonſtrated by *Galileo*. And ſeeing it is manifeſt that in what Proportion the Motion is accelerated in the Line A B, in the ſame Proportion the *Impetus* beginning from Reſt in A is encreaſed in the ſame Times (which Impetus is deſigned all the way by the ordinate Lines of the Semi-Parabola) the greateſt *Impetus* acquired muſt needs be the Baſe of the Semiparabola, namely B D, equal to A C which deſigns the whole Time. I cannot therefore imagine what ſhould make you ſay without proof, that the greateſt acquired *Impetus* is greater then that which is deſigned by the Baſe B D. Next you ſay, you ſee not to what end I divide A B in the middle at E. No wonder; for you have ſeen nothing all the way. Others would ſee it is neceſſary for the Demonſtration; as alſo that the Point F is not to be taken arbitrarily; and likewiſe that the thirteenth Article (which you admit not for proof) is ſufficiently demonſtrated, and your objections to it anſwered. By the way you adviſe me, where I ſay *percurſam eodem motu uniformi, cum Impetu ubiq;* &c. to blot out *cum*; becauſe the *Impetus* is not a *companion* in the way, but the *cauſe*. Pardon me in that I cannot take your Learned Counſell; for the word *motu uniformi* is the Ablative of the *Cauſe*, and *Impetu* the Ablative of the *Manner*. But to come again to your objections, you ſay I make *a greater ſpace run over in the ſame Time by the ſlower Motion then by the ſwifter.* How does that appear? *becauſe there is no doubt, but the ſwiftneſs is greater where the greateſt* Impetus *is alwaies maintained, then where it is attained to in the ſame Time from Reſt.* True, But that is when they are conſidered aſunder without concourſe, but not then when by the concourſe they debilitate one another, and deſcribe a third Line different from both the Lines, which they would deſcribe ſingly. In this place I compare their effects as contributing to the deſcription of the Parabolicall Line A H D. What the effects of their ſeverall Motions are, when they are conſidered aſunder, is ſufficiently ſhewn before in the firſt Article. You ſhould firſt have gotten into your mindes the perfect and diſtinct Ideas of all the Motions mentioned in this Chapter, and then have ventured upon the cenſure of them, but not before. And then you would have ſeen that the Body moved from A, deſcribeth not the Line A C, nor the Line A B, but a third, namely the Semiparabolicall Line A H D.

Again, where I ſay, *Wherefore if the whole A* B *be uniformly moved to C D, in the ſame Time wherein* A *C is moved uniformly to F G*; you aſk me *whether with the ſame Impetus or not.* How is it poſſible that in the ſame time two unequall Lengths ſhould be paſſed over with the ſame *Impetus*? *But why* (ſay you) *do you not tell us with what* Impetus A C *comes to F G*? What need is there of that, when all men know that in uniform Motion and the ſame Time, *Impetus* is to *Impetus*, as Length to Length? Which to have expreſſed had not been pertinent to the Demonſtration. That which follows in the Demonſtration *rurſus ſuppono*

pono quod latus A C, &c. to these words, *ut ostensum est, Art.* 12, You confute with saying you have proved that Article to be false. But you may see now, if you please, at the same place that I have proved your objections to be frivolous.

After this you run on without any Argument against the rest of the Demonstration, shewing nothing all the way, but that the variety and concourse of Motions, the Speculations whereof you have not been used to, have made you giddy.

To the nineteenth Article you apply the same objection which you made to the eighteenth. Which having been answered, it appeares that from the very beginning of your *Elenchus* to this place all your objections (except such as are made to three or four mistakes of small importance in setting down my mind,) are meer Paralogisms, and such as are less pardonable then any Paralogism in *Orontius*, both because the Subject as less difficult is more easily mastered, and because the same faults are more shamefully committed by a Reprehender then by any other man.

I had once added to these nineteen Articles a twentieth, which was this, *If from a Point in the Circumference there be drawn a Chord, and a Tangent equall to it; the Angle which they make shall be double to the aggregate of all the Angles made by the chords of all the equal Arches into which the Arc given canpossibly be divided.* Which Proposition is true, and I did when I writ it think I might have use of it. But be it, or the Demonstration of it true or false, seeing it was not published by me, it is somewhat barbarous to charge me with the faults thereof. No Doctor of Humanity but would have thought it a poor and wretched malice, publiquely to examine and censure papers of Geometry never published, by what means soever they came into his hands. I must confess that in these words, *in such kind of Progression Arithmeticall* (that is, which begins with 0) *the sum of all the Numbers taken together, is equall to half the Number that is made by multiplying the greatest into the least*, there is a great error; for by this account these Numbers, 0, 1, 2, 3, 4, taken together should be equall to nothing. I should have said they are equall to that Number which is made by multiplying half the greatest into the Number of the Terms. There was therefore, if those words were mine (for truly I have no Copy of them, nor have had since the Book was Printed, and I have no great reason, as any man may see, to trust your Faith) a great error in the writing, but not an erroneous opinion in the writer. The Demonstration so corrected is true. And the Angles that have the Proportions of the Numbers 1, 2, 3, 4, are in the Table of your *Elenchus*, Fig. 12. the Angles G A D, H D E, I E F, K F B. And if the Divisions were infinite, so that the first were not to be reckoned but as a cypher, the Angle C A B would be double to them all together. This mistake of mine, and the finding that I had made no use of it in the whole book, was the cause why I thought fit to leave it quite out. But your Professorships, could not forbear to take occasion thereby, to commend your zeal against Leviathan to your Doctorships of Divinity, by censuring it.

Of the Faults that Occurre in Demonſtration.

To the ſame egregious Profeſſors of the Mathematicks in the Univerſity of *Oxford.*

Lesson V.

AT the ſeventeenth Chapter, your firſt exception is to the Definition of Proportionall Proportions, which is this, *Four Proportions are then Proportionall, when the firſt is to the ſecond, as the third to the fourth.* The Reader will hardly believe that your exception is in earneſt. You ſay I mean not by Proportionality the *Quantity of the Proportions.* Yes I do, Therefore I ſay again, that *four Proportions are then Proportionall, when the Quantity of the firſt Proportion, is to the Quantity of the ſecond Proportion, as the Quantity of the third Proportion, to the Quantity of the fourth Proportion.* Is not my meaning now plainly enough expreſſed? Or is it not the ſame Definition with the former? But what do I mean (you will ſay) by the Quantity of a Proportion? I mean the determined greatneſs of it, that is, for example, in theſe Numbers, the Quantity of the Proportion of two to three, is the ſame with the Quantity of the Proportion of four to ſix, or ſix to nine; and again, the Quanrity of the Proportion of ſix to four, is the ſame with the Quantity of the Proportion of nine to ſix, or of three to two. But now what do you mean by the Quantity of a Proportion? You mean that two and three, are the Quantities of the Proportion of two to three (for ſo *Euclide* calls them) and that ſix and four are the Quantities of the Proportion of ſix to four, which is the ſame with the Proportion of three to two. And by this Rule, one and the ſame Proportion ſhall have an infinite Number of Quantities; and conſequently the Quantity of a Proportion can never be determined. I call one Proportion double to another, when one is equall to twice the other; as the Proportion of four to one, is double to the Proportion of two to one. You call that Proportion double where one Number, Line, or Quantity abſolute is double to the other; ſo that with you the Proportion of two to one is a double Proportion. It is eaſie to underſtand how the Number two is double to one, but to what I pray you, is double the Proportion of two to one, or of one to two? Is not every double Proportion double to ſome Proportion? See whether this Geometry of yours can be taken by any man of ſound mind for ſenſe. *But 'tis known* (you ſay) *that in Proportions double is one thing, and duplicate another*; So that it ſeems to you, that in talking of Proportion men are allowed to ſpeak ſenſleſsly. *'Tis known,* you ſay. To whom? It is indeed in uſe at this day to call *double duplicate,* and *triple triplicate.* And it is well enough; for they are words that ſignifie the ſame thing, But that they differ (in what ſubject ſoever) I never heard till now. I am ſure that *Euclide* whom you have undertaken to expound, maketh no ſuch difference. And even there where he putteth theſe Numbers, one, two, four, eight, &c. for Numbers in *double* Proportion (which is the laſt Propoſition of the nineth Element) he meaneth not that one to

two,

two, or two to one, is a *double* Proportion, but that every Number in that Progression is *double* to the number next before it; and yet he does not call it *Analogia dupla*, but *Duplicata*. This distinction in Proportions between *double* and *duplicate*, proceeded long after from want of knowledge that the Proportion of one to two is *double* to the Proportion of one to four; and this from ignorance of the different nature of Proportions of *Excess*, and Proportions of *Defect*. And you that have nothing but by tradition saw not the absurdities that did hang thereon.

In the second Article I make E K, (Fig. 1.) the third part of L K, which you say is false; and consequently the Proposition undemonstrated. And thus you prove it false. *Let A C be to G C, or G K to G L, as eight to one* (*for seeing the Point G is taken arbitrarily, we may place it where we will, &c.*) and upon this placing of G arbitrarily, you prove well enough that E K is not a third part of L K. But you did not then observe, that I make *the Altitude* A G, *less then any Quantity given*, and by consequence E K to differ from a third part by a less difference then any Quantity that can be given. Therefore as yet the Demonstration proceedeth well enough. But perceiving your oversight, you thought fit (though before, you thought this confutation sufficient) to endeavour to confute it another way; but with much more evidence of ignorance. For when I come to say, *the Proportion therefore between* A C *and* G C *is triple* (*in Arithmeticall Proportion*) *to the Proportion between G K and G E, &c.* you say *the Proportion of* A C *to* G C, *is the Proportion of Identity, as also that of G K to G E.* But why? Does my construction make it so? Do not I make G C less then A C, though with less Difference then any Quantity that can be assigned? And then where I say *therefore E K is the third part of L K*, you come in (by Parenthesis) with (*or a fourth, or a fifth* &c.) Upon what ground? Because you think it will pass for current, without proof, that a Point is nothing. Which if it do, Geometry also shall pass for nothing, as having no ground nor beginning but in nothing. But I have already in a former Lesson sufficiently shew'd you the consequence of that opinion. To which I may add, that it destroys the method of *Indivisibles*, invented by *Bonaventura*; and upon which, not well understood, you have grounded all your scurvy book of *Arithmetica Infinitorum*; where your Indivisibles have nothing to do, but as they are supposed to have Quantity, that is to say, to be *Divisibles*. You allow, it seems, your own nothings to be somethings, and yet will not allow my somethings to be considered as nothing. The rest of your objections having no other ground then this, *that a Point is nothing*, my whole Demonstration standeth firm; and so do the Demonstrations of all such Geometricians, Ancient and Modern, as have inferred any thing in the manner following, viz. *If it be not greater nor less, then it is equall. But it is neither greater nor less. Therefore, &c. If it be greater, say by how much. By so much. 'Tis not greater by so much. Therefore it is not greater. If it be less, say by how much, &c.* Which being good Demonstrations are together with mine overthrown by the nothingness of your *Point*, or rather of your unstanding; upon which you nevertheless have the vanity of advising me what to do, if I demonstrate the same again; meaning I should come to your false, impossible, and absurd Method of *Arithmetica Infinitorum*, worthy to be gilded, I do not mean with Gold.

And for your Question, why I set the Base of my Figure upwards, you may be sure it was not because I was afraid to say, that the Proportions of the ordinate Lines beginning at the Vertex were triplicate, or otherwise multiplicate of the Proportions of the intercepted parts of the Diameter. For I never doubted to call double duplicate, nor triple triplicate, &c. or if I had, I should have avoided it afterwards at the tenth Article of the same Chapter. But because when I went about to compare the Proportions of the ordinate Lines with those of their contiguous Diameters, the first thing I considered in them was in what manner the Base grew less and less till it vanished into a Point. And though the base had been placed below, it had not therefore required any change in the Demonstration. But I was the more apt to place the Base uppermost, because the Motion began at the Base, and ended at the Vertex. To proceed which way I pleased was in my own choice; and it is of grace that I give you any account of it at all.

To the third Article together with its Table, you ſay, *it falls in the ruine of the ſecond; and that the ſame is to be underſtood of the ſixth, ſeventh, eighth and nineth.* For confutation whereof I need to ſay no more, but that they all ſtand good by the confutation of your objections to the ſecond.

To the fourth Article you ſay, *the deſcription of thoſe curvilineall Figures is caſie.* True, to ſome men; and now that I have ſhewed you the way, 'tis eaſie enough for you alſo. For the way you propound is wholly tranſcribed out of the Figure of the ſecond Article, which Article you had before rejected. For ſeeing the Lines H F, G E, A B, &c. are equall to the Lines C Q, C O, C D; and the Lines Q F, O E, D B, equall to the Lines C H, C G, C A; the Proportion of D B to O E, will be triple (that is, triplicate) to the Proportion of C O to G E; and the Proportion of D B to Q F, triple to the Proportion of C D to C Q; and conſequently, becauſe the Complement B D C F E B is made by the decreaſe of A C in triple Proportion to that of the decreaſe of C D, it will be (by the ſecond Article) a third part of the Figure A B E F C A. So that it comes all to one paſs, whether we take triple Proportion in decreaſing to make the Complement, or triple Proportion in increaſing to make the Figure; for the Proportion of H F to B A, is triple to the Proportion of C H to C A. Wherefore you have done no more but what you have ſeen firſt done, ſaving that from your conſtruction you prove not the Figure to be triple to the Complement; perhaps becauſe you have proved the contrary in your *Arithmetica Infinitorum.* But your way differs from mine, in that you call the Proportion ſubtriplicate, which I call triplicate; as if the divers naming of the ſame thing made it differ from its ſelf. You might as well have ſaid briefly, the Propoſition is true, but ill proved, becauſe I call the Proportion of one to two triple, or triplicate of that of one to eight; which you ſay is falſe, and hath infected the fourth, fifth, nineth, tenth, eleventh, thirteenth, fourteenth, fifteenth, ſixteenth, ſeventeenth and nineteenth Articles of the ſixteenth Chapter. But I ſay, and you know now, that it is true; and that all thoſe Articles are Demonſtrated.

Laſtly you add, *Tu vero, in preſente Articulo, &c. id eſt, you bid find as many mean Proportionals as one will, between two given Lines; as if that could not be done by the Geometry of Plains, &c.* You might have left out *Tu vero* to ſeek an *Ego quidem.* But tell me, do you think you can find two mean Proportionals (which is leſs then as many as one will) by the Geometry of Plaines? We ſhall ſee anon how you go about it. I never ſaid it was impoſſible, and if you look upon the places cited by you more attentively, you will find your ſelf miſtaken. But I ſay, the way to do it has not been yet found out, and therefore it may prove a ſolid Probleme for any thing you know.

The fifth Article you reject, becauſe it citeth the Corollarie of the twenty-eighth Article of the thirteenth Chapter, where there is never a word to that purpoſe. But there is in the twenty-ſixth Article; which was my own fault, though you knew not but it might have been the Printers.

To the tenth you object for almoſt three leaves together, againſt theſe words of mine, *Becauſe* (in the ſixth Figure) *B F is to B E in triplicate Proportion of C D to F E, therefore inverting, F E is to C D in triplicate Proportion of B F to B C.* This you objected then. But now that I have taught you ſo much Geometry, as to know *that of three Quantities, beginning at the leaſt, if the third be to the firſt in triplicate Proportion of the ſecond to the firſt, alſo by converſion the firſt to the ſecond ſhall be in triplicate Proportion of the firſt to the third;* if it were to do again, you would not object it.

My eleventh Article you would allow for demonſtrated, if my ſecond had been Demonſtrated, upon which it dependeth. Therefore ſeeing your objections to that Article are ſufficiently anſwered, this Article alſo is to be allowed.

The twelfth alſo is allowed upon the ſame reaſon. What falſities you ſhall finde in ſuch following Propoſitions as depend upon the ſame ſecond Article, we ſhall then ſee when I come to the places where you object againſt them.

To

To the thirteenth Article you object, *That the same Demonstration may be as well applyed to a Portion of any* Conoeides *Parabolicall, Hyperbolicall, Elliptical, or any other, as to the Portion of a Sphere.* By the truth of this let any man judge of your and my Geometry. Your Comparison of the Sphere and *Conoeides*, so far holds good, as to prove that the Superficies of the *Conoeides* is greater then the Superficies of the Cone described by the subtense of the Parabolicall, Hyperbolicall, or Elliptical Line. But when I come to say that *The cause of the excess of the Superficies of the Portion of the Sphere above the Superficies of the Cone, consists in the Angle D A B, and the cause of the excess of the Circle made upon the Tangent A D above the Superficies of the same Cone, consists in the magnitude of the same Angle D A B,* how will you apply this to your *Conoeides*? For suppose that the crooked Line A B (in the seventh Figure) were not an Arch of a Circle, do you think that the Angles which it maketh with the Subtense A B, at the Points A and B must needs be equall? Or if they be not, does the excess of the Superficies of the Circle upon A D above the Superficies of the Cone, or the excess of the Superficies of the Portion of the *Conoeides* above the Superficies of the same Cone consist in the Angle D A B, or rather in the magnitude of the two unequall Angles D A B, and A B A? You should have drawn some other crooked Line, and made Tangents to it through A and B, and you would presently have seen your error. See how you can answer this; for if this Demonstration of mine stand firm, I may be bold to say, though the same be well Demonstrated by *Archimedes*, that this way of mine is more naturall, as proceeding immediately from the naturall efficient causes of the effect contained in the conclusion; and besides, more brief and more easie to be followed by the fancy of the Reader.

To the fourteenth Article you say that I *commit a Circle in that I require in the fourth Article the finding of two mean Proportionals, and come not till now to shew how it is to be done.* Nor now neither. But in the mean time you commit two mistakes in saying so. The place cited by you in the fourth Article is (in the Latine) Pag. 149. Lin. 5. (in the English) Pag. 188. Lin. 3. Let any Reader judge whether that be a requiring it, or a supposing it to be done; this is your first mistake. The second is, that in this place the Proposition it self, which is, *If these Deficient Figures could be described in a Parallelogram exquisitely, there might be found thereby between any two Lines given, as many mean Proportionals as one would,* is a *Theoreme*, upon supposition of these crooked Lines exquisitely drawn; but you take it for a *Probleme.*

And proceeding in that error, you undertake the invention of two mean Proportionals, using therein my first Figure, which is of the same construction with the eighth that belongeth to this fourteenth Article. Your construction is, *Let there be taken in the Diameter C A (Figure 1.) the two given Lines, or two others Proportionall to them, as C H, C G, and their ordinate Lines H F, G E (which by construction are in subtriplicate Proportion of the intercepted Diameters.) These Lines will shew the Proportions which these four Proportionals are to have.* But how will you find the Length of H F or G E, the ordinate Lines? Will you not do it by so drawing the crooked Line C F E as it may pass through both the Points F and E? You may make it pass through one of them, but to make it pass through the other, you must finde two mean Proportionals between G K and G L, or between H I and H P; Which you cannot do, unless the crooked Line be exactly drawn; which it cannot be by the Geometry of Plaines. Go shew this Demonstration of yours to *Orontius*, and see what he will say to it.

I am now come to an end of your objections to the seventeenth Chapter, where you have an *Epiphonema* not to be passed over in silence. But because you pretend to the Demonstration of some of these Propositions by another Method in your *Arithmetica Infinitorum*, I shall first try whether you be able to defend those Demonstrations as well as I have done these of mine by the Method of Motion.

The first Proposition of your *Arithmetica Infinitorum* is this *Lemma*: *In a Series* (or Row) *of Quantities Arithmetically Proportionall, beginning at a Point or Cypher as* 0, 1, 2, 3, 4, *&c. to finde the Proportion of the Aggregate of them all, to the Aggregate*

of

of so many times the greatest, as there are Terms. This is to be done by multiplying the greatest into half the Number of the Terms. The Demonstration is easie. But how do you demonstrate the same? *The most simple way* (say you) *of finding this and some other* Problemes, *is to do the thing it self a little way, and to observe and compare the appearing Proportions, and then by* Induction, *to conclude it universally.* Egregious Logicians and Geometricians, that think an *Induction* without a *Numeration* of all the particulars sufficient to infer a Conclusion universall, and fit to be received for a Geometricall Demonstration! But why do you limit it to the naturall consequution of the Numbers, 0, 1, 2, 3, 4, &c? Is it not also true in these Numbers, 0, 2, 4, 6, &c. or in these, 0, 7, 14, 21, &c? Or in any Numbers where the Difference of nothing and the first Number is equall to the difference between the first and second, and between the second and third &c? Again, are not these Quantities 1, 3, 5, 7, &c. in continuall Proportion Arithmeticall? And if you put before them a Cypher thus, 0, 1, 3, 5, 7, do you think that the sum of them is equall to the half of five times seven? Therefore though your *Lemma* be true, and by me (Chap. 13. Art. 5.) demonstrated; yet you did not know why it is true; which also appears most evidently in the first Proposition of your *Conique-sections.* Where first you have this, *That a Parallelogram whose Altitude is* infinitely little, *that is to say,* none, is scarce *any thing else but a Line.* Is this the Language of Geometry? How do you determine this word *scarce*? The least Altitude, is Somewhat or Nothing. If Somewhat, then the first character of your Arithmeticall Progression must not be a cypher; and consequently the first eighteen Propositions of this your *Arithmetica Infinitorum* are all naught. If Nothing, then your whole figure is without Altitude, and consequently your Understanding naught. Again, in the same Proposition, you say thus, *We will sometimes call those Parallelograms rather by the name of Lines then of Parallelograms, at least, when there is no* consideration *of a determinate* Altitude; *But where there is a* consideration *of a determinate* Altitude *(which will happen sometimes) there that little* Altitude *shall be so far* considered, *as that being infinitely multiplyed it may be equall to the* Altitude *of the whole Figure.* See here in what a confusion you are when you resist the truth. When you *consider* no determinate Altitude (that is, *no Quantity* of Altitude) then you say your Parallelogram shall be called a *Line.* But when the Altitude is determined (that is, when it is *Quantity*) then you will call it a Parallelogram. Is not this the very same doctrine which you so much wonder at and reprehend in me, in your objections to my eighth Chapter, and your word *considered* used as I used it? 'Tis very ugly in one that so bitterly reprehendeth a doctrine in another, to be driven upon the same himself by the force of truth when he thinks not on't. Again, seeing you admit in any case, those *infinitely little* altitudes to be quantity, what need you this limitation of yours, *so far forth as that by multiplication they may be made equall to the Altitude of the whole figure*? May not the half, the third, the fourth, or the fifth part, &c. be made equall to the whole by multiplication? Why could you not have said plainly, *so far forth as that every one of* those infinitely little *Altitudes be not only* something *but an* aliquot part *of the whol*? So you will have an *infinitely little* Altitude, that is to say *a Point, to be both nothing and something and an aliquot part.* And all this proceeds from not understanding the ground of your Profession.

Well, the *Lemma* is true. Let us see the *Theoremes* you draw from it. The first is (Pag. 3.) *that a Triangle to a Parallelogram of equall Base and Altitude is as one to two.* The conclusion is true, but how know you that? *Because* (say you) *the Triangle consists as it were (as it were,* is no Phrase of a Geometrician) *of an Infinite Number of straight Parallel Lines.* Does it so? Then by your own doctrine, which is, that *Lines have no breadth,* the Altitude of your Triangle consisteth of an infinite Number of *no Altitudes,* that is of an infinite Number of Nothings, and consequently the *Area* of your Triangle has no Quantity. If you say that by the Parallels you mean infinitely little Parallelograms, you are never the better; for if infinitely little, either they are nothing, or if somewhat, yet seeing that no two sides of a Triangle are Parallel, those Parallels cannot be Parallelograms. I see they may be counted for

Parallelo-

Parallelograms by not conſidering the Quantity of their Altitudes in the Demonſtration. But you are barred of that Plea, by your ſpightfull arguing againſt it in your *Elenchus*. Therefore this third Propoſition, and with it the fourth is undemonſtrated.

Your fifth Propoſition is, *The Spirall Line is equall to half the Circle of the firſt Revolution.* But what Spirall Line? We ſhall underſtand that by your conſtruction, which is this, *The ſtraight Line M A*, (in your Figure which I have placed at the end of the fifth Leſſon) *turned round* (*the Point M remaining unmoved*) *is ſuppoſed to deſcribe with its Point A the Circle A O A, whilſt ſome Point* (*in the ſame M A whilſt it goes about*) *is ſuppoſed to be moved uniformly from M to A deſcribing the Spirall Line.* This therefore is the Spirall Line of *Archimedes*; and your Propoſition affirms it to be equall to the half of the Circle A O A; which you perceived not long after to be falſe. But thinking it had been true, you go about to prove it, *by inſcribing in the Circle an infinite multitude of equall Angles, and conſequently an infinite Number of Sectors, whoſe Arches will therefore be in Arithmeticall Proportion*; Which is true. *And the Aggregate of thoſe Arches equall to half the Circumference A O A*; Which is true alſo. And thence you conclude *that the Spirall Line is equall to half the Circumference of the Circle A O A*; Which is falſe. For the Aggregate of that infinite Number of infinitely little Arches, is not the Spirall Line made by your conſtruction, ſeeing by your conſtruction the Line you make is manifeſtly the Spiral of Archimedes; whereas no Number (though infinite) of Arches of Circles (how little ſoever) is any kind of Spirall at all; and though you call it a Spirall, that is but a patch to cover your fault, and deceiveth no man but your ſelf. Beſides, you ſaw not how abſurd it was (for you that hold a Point to be abſolutely nothing) to make an infinite Number of equall Angles (the Radius increaſing as the Number of Angles increaſeth) and then to ſay *that the Arches of the Sectors whoſe Angles they are, are as* 0, 1, 2, 3, 4, *&c.* For you make the firſt Angle 0, and all the reſt equall to it; and ſo make 0, 0, 0, 0, 0, &c. to be the ſame Progreſſion with 0, 1, 2, 3, 4, &c. The influence of this abſurdity reacheth to the end of the eighteenth Propoſition. So many are therefore falſe, or nothing worth. And you needed not to wonder that the Doctrine contained in them was omitted by Archimedes, who never was ſo ſenſeleſs as to think a Spirall Line was compounded of Arches of Circles.

Your nineteenth Propoſition is this other *Lemma, In a Series* (or a Row) *of Quantities, beginning from a Point, or Cypher, and proceeding according to the order of the ſquare Numbers, as* 0, 1, 4, 9, 16, *&c. to finde what Proportion the whole Series hath to ſo many times the greateſt.* And you conclude *the Proportion to be that of* 1 *to* 3. Which is falſe, as you ſhall preſently ſee. First, let the Series of Squares with the prefixed Cypher, and under every one of them, the greateſt 4 be $\frac{0.1.4.}{4.4.4}$. And you have for the ſum of the Squares 5, and for thrice the greateſt 12, the third part whereof is 4. But 5 is greater then 4, by 1, that is, by one twelfth of 12; which Quantity is ſomewhat, let it be called A. Again, let the Row of Squares be lengthened one term further, and the greateſt ſet under every one of them as $\frac{0.1.4.9}{9.9.9.9}$. The ſum of the Squares is 14, and the ſum of four times the greateſt is 36, whereof the third part is 12. But 14 is greater then 12 by two unities, that is, by two twelfths of 12, that is, by 2 A. The difference therefore between the ſum of the Squares, and the ſum of ſo many times the greateſt Square is greater, when the cypher is followed by three Squares, then when by but two. Again, let the Row have five terms as in theſe Numbers $\frac{0.1.4.9.16}{16.16.16.16.16}$ with the greateſt five times ſubſcribed, and the ſum of the Squares will be 30, the ſum of all the greateſt will be 80. The third part whereof is $26\frac{2}{3}$. But 30 is greater then

$26\frac{1}{3}$ by $3\frac{1}{3}$, that is, by three twelfths of twelve and $\frac{1}{3}$ of a twelfth, that is, by $3\frac{1}{3}$ A. Likewise in the Series continued to six places with the greatest six times subscribed, as $\frac{0.1.4.9.16.25.}{25.25.25.25.25.25.}$ the sum of the Squares is 55, and the sum of the greatest six times taken is 150, the third part where of is 50. But 55 is greater then 50 by 5, that is, by five twelfths of 12, that is, by 5 A. And so continually as the Row groweth longer, the excess also of the aggregate of the Squares above the third part of the aggregate of so many times the greatest Square, groweth greater. And consequently if the Number of the Squares were infinite, their sum would be so far from being equal to the third part of the aggregate of the greatest as often taken, as that it would be greater then it by a Quantity greater then any that can be given or named.

That which deceived you was partly this, that you think (as you do in your *Elenchus*) that these Fractions $\frac{1}{12}\ \frac{1}{18}\ \frac{1}{24}\ \frac{1}{30}\ \frac{1}{36}$ &c. are Proportions, as if $\frac{1}{12}$ were the Proportion of one to twelve; and consequently $\frac{2}{12}$ double the Proportion of one to twelve; which is as unintelligible as School-Divinity; and I assure you, far from the meaning of Mr. *Oughtred* in the sixth Chapter of his *Clavis Mathematicæ*, where he sayes that $4\frac{3}{7}$ is the Proportion of 31 to 7; for his meaning is, that the Proportion of $4\frac{3}{7}$ to one, is the Proportion of 31 to 7; whereas if he meant as you do, then $8\frac{6}{7}$ should be double the Proportion of 31 to 7. Partly also becaufe you think (as in the end of the twentieth Propofition) that if the Proportion of the Numerators of these Fractions $\frac{1}{12}\ \frac{1}{18}\ \frac{1}{24}\ \frac{1}{30}\ \frac{1}{36}$ to their Denominators decrease eternally, they shall so vanish at last as to leave the Proportion of the sum of all the Squares to the sum of the greatest so often taken, (that is, an infinite Number of times) as one to three, or the sum of the greatest to the sum of the increasing Squares, as three to one; for which there is no more reason then for four to one, or five to one, or any other such Proportion. For if the Proportions come eternally nearer and nearer to the subtriple, they must needs also come nearer and nearer to subquadruple; and you may as well conclude thence that the upper Quantities shall be to the Lower Quantities as one to four, or as one to five, &c. as conclude they are as one to three. You can see without admonition, what effect this false ground of yours will produce in the whole structure of your *Arithmetica Infinitorum*; and how it makes all that you have said unto the end of your thirty-eighth Propofition, undemonstrated, and much of it false.

The thirty-nineth is this other *Lemma*, *In a Series of Quantities beginning with a Point or Cypher, and proceeding according to the Series of the Cubique Numbers, as* 0. 1. 8. 27. 64, &c. *to finde the Proportion of the sum of the Cubes to the sum of the greatest* Cube, *so many times taken as there be Terms*, And you conclude that *they have the Proportion of* 1 *to* 4; which is false.

Let the first Series be of three terms subscribed with the greatest $\frac{0.1.8}{8.8.8}$; the sum of the Cubes is nine; the sum of all the greatest is 24; a quarter whereof is 6. But 9 is greater then 6 by three unities. An unity is something. Let it be therefore A. Therefore the Row of Cubes is greater then a quarter of three times eight, by three A. Again, let the Series have four terms, as $\frac{0.1.8.27}{27.27.27.27}$; the sum of the Cubes is 36; a quarter of the

sum

sum of all the greatest is twenty-seven. But thirty-six is greater then twenty-seven by nine that is, by 9 A. The excels therefore of the sum of the Cubes above the fourth part of the sum of all the greatest, is increased by the increase of the Number of terms. Again, let the terms be five as $\frac{0.1.8.27.64}{64.64.64.64.64}$ the sum of the Cubes is one hundred; the sum of all the greatest three hundred and twenty; a quarter whereof is eighty. But one hundred is greater then eighty by twenty, that is, by 20 A. So you see that this *Lemma* also is false. And yet there is grounded upon it all that which you have of comparing *Parabolas* and *Paraboloeides* with the Parallelograms wherein they are accommodated. And therefore though it be true, that the *Parabola* is $\frac{2}{3}$, and the Cubicall *Paraboloeides* $\frac{3}{4}$ of their Parallelograms respectively, yet it is more then you were certain of when you referred (me for the learning of Geometry) to this Book of yours. Besides, any man may perceive that without these two *Lemmas* (which are mingled with all your compounded Series with their excesses) there is nothing demonstrated ro the end of your Book. Which to prosecute particularly, were but a vain expence of time. Truly were it not that I must defend my reputation, I should not have shewed the world how little there is of sound Doctrine in any of your Books. For when I think how dejected you will be for the future, and how the grief of so much time irrecoverably lost, together with the conscience of taking so great a stipend, for mis-teaching the young men of the University, & the consideration of how much your friends wil be ashamed of you, will accompany you for the rest of your life, I have more compassion for you then you have deserved. Your Treatise of the *Angle of Contact*, I have before confuted in a very few leaves. And for that of your *Conique Sections*, it is so covered over with the scab of Symboles that I had not the patience to examine whether it be well or ill demonstrated.

Yet I observed thus much, that you find a Tangent to a Point given in the Section, by a Diameter given; and in the next Chapter after, you teach the finding of a Diameter, which is not artificially done.

I observe also, that you call the *Parameter* an Imaginary Line, as if the place thereof were less determined then the Diameter it self; and then you take a mean Proportionall between the intercepted Diameter, and its contiguous ordinate Line to find it. And tis true, you find it: But the Parameter has a determined Quantity to be found without taking a mean Proportional. For the Diameter and half the Section being given, draw a Tangent through the Vertex, and dividing the Angle in the midst which is made by the Diameter and Tangent, the Line that so divideth the Angle, will cut the crooked Line. From the intersection draw a Line (if it be a Parabola) Parallel to the Diameter, and that Line shall cut off in the Tangent from the Vertex the Parameter sought. But if the Section be an Ellipsis, or an Hyperbole, you may use the same Method, saving that the Line drawn from the intersection must not be Parallel, but must pass through the end of the transverse Diameter, and then also it shall cut off a part of the Tangent, which measured from the Vertex is the Parameter. So that there is no more reason to call the Parameter an Imaginary Line then the Diameter.

Lastly, I observe that in all this your new Method of *Coniques* you shew not how to find the *Burning Points*, which writers call the *Foci* and *Umbilici* of the Section, which are of all other things belonging to the *Coniques* most usefull in Philosophy. Why therefore were they not as worthy of your pains as the rest, for the rest also have already been demonstrated by others? You know the *Focus* of the *Parabola* is in the Axis distant from the Vertex a quarter of the Parameter. Know also that the *Focus* of an *Hyperbole*, is in the Axis, distant from the Vertex, as much as the Hypotenusall of a rectangled Triangle, whose one side is half the *transverse Axis*, the other side half the mean Proportionall between the whole *transverse Axis* and the *Parameter*, is greater then half the *transverse Axis*.

The cause why you have performed nothing in any of your Books (saving that in your *Elenchus* you have spied a few negligences of mine, which I need not be ashamed of) is this, that

you underſtood not what is *Quantity*, *Line*, *Superficies*, *Angle*, and *Proportion*; without which you cannot have the Science of any one Propoſition in Geometry. From this one and firſt Definition of *Euclide*, *a Point is that whereof there is no part*, underſtood by *Sextus Empiricus*, as you underſtand it, that is to ſay, miſ-underſtood, *Sextus Empiricus* hath utterly deſtroyed moſt of the reſt, and Demonſtrated, that in Geometry there is no Science, and by that means you have betrayed the moſt evident of the Sciences to the *Sceptiques*. But as I underſtand it for *that whereof no part is reckoned*, his Arguments have no force at all, and Geometry is redeemed. If a Line have no Latitude, how ſhall a *Cylinder* rowling on a Plain, which it toucheth not but in a Line, deſcribe a Superficies? How can you affirm that any of thoſe things can be without Quantity, whereof the one may be greater or leſs then the other? But in the common Contact of divers Circles the externall Circle maketh with the common Tangent a leſs Angle of Contact then the internall. Why then is it not Quantity? An Angle is made by the concourſe of two Lines from ſeverall Regions, concurring (by their generation) in one and the ſame Point. How then can you ſay the Angle of Contact is no Angle? One meaſure cannot be applicable at once to the Angle of Contact, and Angle of Converſion. How then can you infer, if they be both Angles, that they muſt be Homogeneous? Proportion is the Relation of two Quantities. How then can a Quotient or Fraction, which is Quantity abſolute, be a Proportion? But to come at laſt to your *Epiphonema*, wherein, though I have perfectly demonſtrated all thoſe Propoſitions concerning the Proportion of Parabolaſters to their Parallelograms, and you have demonſtrated none of them (as you cannot now but plainly ſee) but committed moſt groſs Paralogiſms, How could you be ſo tranſported with pride, as inſolently to compare the ſetting of them forth as mine, to the Act of him that ſteals a horſe, and comes to the gallows for it. You have read, I think, of the gallows ſet up by *Haman*. Remember therefore alſo who was hanged upon it.

After your dejection I ſhall comfort you a little, a very little, with this, that whereas this 18 Chapter containeth two Problemes, one, *the finding of a ſtraight Line equall to the crooked Line of a Semiparabola*, The other, *the finding of ſtraight Lines equall to the crooked Lines of the Parabolaſters in the table of the third Article of the 17 Chapter*; You have truly demonſtrated that they are both falſe; and another hath alſo Demonſtrated the ſame another way. Nevertheleſs the fault was not in my method, but in a miſtake of one Line for another, and ſuch as was not hard to correct; and is now ſo corrected in the Engliſh as you ſhall not be able (if you can ſufficiently imagine Motions) to reprehend. The fault was this, That in the Triangles which have the ſame Baſe and Altitude with the Parabola and Parabolaſter, I take for deſignation of the mean uniform *Impetus*, a mean Proportionall (in the firſt Figure) between the whole Diameter and its half, and (in the ſecond Figure) a mean Proportionall between the whole Diameter and its third part; which was manifeſtly falſe, and contrary to what I had ſhewn in the 16 Chapter. Whereas I ought to have taken the half of the Baſe, as now I have done, and thereby exhibited the Straight Lines equall to thoſe crooked Lines, as I undertook to do. Which error therefore proceeded not from want of skill, but from want of care; and what I promiſed (as bold as you ſay the promiſe was,) I have now performed.

The reſt of your exceptions to this Chapter, are to theſe words in the end, *there be ſome that ſay, that though there be equality between a ſtraight and crooked Line, yet now*, (they ſay) *after the fall of* Adam, *it cannot be found without the eſpeciall help of divine grace*. And you ſay you think there be none that ſay ſo. I am not bound to tell you who they are. Nevertheleſs, that other men may ſee the Spirit of an ambitious part of the Clergy, I will tell you where I read it. It is in the *Prolegomena* of *Lalovera* (a Jeſuite) to his Quadrature of the Circle, pag. 13 & 14, in theſe words, *Quamvis circuli tetragoniſmus ſit φύσει poſſibilis, an tamen etiam ſit μεθ' ἡμᾶς, hoc eſt, poſt Adæ lapſum homo ejus ſcientiam abſq; ſpeciali divinæ gratiæ auxilio, poſſit comparare, jure merito inquirunt Theologi, pronunciantq; hanc veritatem tanta eſſe caligine involutam ut illam videre nemo poſsit, niſi Ignorantiæ ex primi*

mi parentis prævaricatione propagatas tenebras indebitus divinæ lucis radius dissipet; quod verissimum esse sentio. Wherein I observed that he (supposing he had found that Quadrature) would have us believe it was not by the ordinary and Naturall help of God (whereby one man reasoneth, judgeth and remembreth better then another) but by a Special (which must be a Supernaturall) help of God, that he hath given to him of the order of *Iesus* above others that have attempted the same in vain. Insinuating thereby, as handsomely as he could, a Speciall love of God towards the *Jesuites*. But you taking no notice of the word *Speciall*, would have men think I held, that humane Sciences might be acquired without any help of God. And thereupon proceed in a great deal of ill language to the end of your objections to this Chapter. But I shall take notice of your Manners for altogether in my next Lesson.

At the nineteenth Chapter you see not (you say) the Method. Like enough. In this Chapter I consider not the Cause of Reflection, which consisteth in the resistance of Bodies naturall; but I consider the consequences, arising from the supposition of the equality of the Angle of Reflection, to that of Incidence; leaving the causes both of Reflection, and of Refraction to be handled together in the 24 Chapter. Which Method (think what you will) I still think best.

Secondly, you say I define not here, but many Chapters after, what an Angle of Incidence, and what an Angle of Reflection is. Had you not been more hasty then diligent Readers, you had found that those Definitions of the Angle of Incidence, and of Reflection were here set down in the first Article, and not deferred to the 24. Let not therefore your own oversight be any more brought in for an objection.

Thirdly, you say there is no great difficulty in the business of this Chapter. It may be so, now 'tis down; but before it was done, I doubt not but you that are a Professor would have done the same, as well as you have done that of the *Angle of Contact*, or the business of your *Arithmetica Infinitorum*. But what a novice in Geometry would have done I cannot tell.

To the third, fourth, and fifth Article, you object a want of Determination; and shew it by instance, as to the third Article. But what those Determinations should be, you determine not, because you could not. The words in the third Article, are first these, *If there fall two straight Lines Parallel, &c.* which is too generall. It should be, *if there fall the same way two straight Lines Parallel*, &c. Next these, *their Reflected Lines produced inwards shall make an Angle*, &c. This also is too generall. I should have said *their Reflected Lines produced inwards, if they meet within, shall make an Angle*, &c. Which done, both this Article and the 4 and 5 are fully demonstrated. And without it, an intelligent Reader had been satisfied, supplying the want himself by the construction.

To the eight, you object onely the two great Length, and labour of it, because you can do it a shorter way. Perhaps so now, as being easie to shorten many of the Demonstrations both of *Euclide*, and other the best Geometricians that are or have been. And this is all you had to say to my 19 Chapter. Before I proceed, I must put you in mind that these words of yours, *Adducis malleum, ut occidas muscam*, are not good Latine, *Malleum affers, Malleum adhibes, Malleo uteris*, are good. When you speak of *bringing* Bodies animate, *Ducere* and *Adducere* are good, for there *to bring*, is *to guide* or *lead*. And of Bodies inanimate *Adducere* is good for *Attrahere*, which is to draw to. But when you bring a hammer, will you say *Adduco Malleum, I lead a hammer*? A man may lead another man, and a ninny may be said to lead another ninny, but not a hammer. Nevertheless, I should not have thought fit to reprehend this fault upon this occasion in an English man, nor to take notice of it, but that I finde you in some places nibling (but causelessly) at my Latine.

Concerning the twentieth Chapter, before I answer to the objections against the Propositions themselves, I must answer to the exception you first take to these words of mine, *Quæ de dimensione Circuli & Angulorum pronuntiata sunt tanquam exactè inventa, accipiat Lector tanquam dicta Problematice.* To which you say thus, *we are wont in Geometry to call some Pro-*

poſitions Theoremes, others Problems, *&c. of which a Theoreme is that wherein ſome aſſertion is propou nded to be proved, a Probleme that wherein ſomething is commanded to be done.* Do you mean *to be done*, and not proved? By your favour, a Probleme in all ancient writers ſignifies no more but a Propoſition uttered, to the end to have it by them to whom it is uttered, examined whether it be true or not true, faiſable or not faiſable; and differs not amongſt Geometricians from a Theoreme, but in the manner of Propounding. For this Propoſition, *To make an equilaterall Triangle*, ſo propounded they call a Problem. But if propounded thus, *If upon the ends of a ſtraight Line given be deſcribed two Circles, whoſe* Radius *is the ſame ſtraight Line, and there be drawn from the interſection of the Circles to their two Centers, two ſtraight Lines, there will be made an equilateral Triangle*, then they call it a Theoreme; and yet the Propoſition is the ſame. Therefore theſe words *Accipiat Lector tanquam dicta Problematice*, ſignifie plainly this, that I would have the Reader, take for propounded to him to examine, whether from my conſtruction the Quadrature of the Circle can be truly inferred or not; and this is not to bid him (as you interpret it) to ſquare the Circle. And if you believe that *Problematice* ſignifies *probably*, you have been very negligent in obſerving the ſenſe of the antient Greek Philoſophers in the word *Probleme*. Therefore your *Solemus in Geometria*, &c. is nothing to the purpoſe; nor had it been though you had ſpoken more properly, and ſaid *Solent*, leaving out your ſelves.

My firſt Article hath this Title, *from a Falſe ſuppoſition, a falſe Quadrature of the Circle.* Seeing therefore you were reſolved to ſhew where I erred, you ſhould have proved either that the Suppoſition was true, and the Concluſion falſely inferred, or contrarily that though the Suppoſition be falſe, yet the Concluſion is true; for elſe you object nothing to my Geometry, but only to my Judgement, in thinking fit to publiſh it; which nevertheleſs you cannot juſtly do, ſeeing it was likely to give occaſion to ingenuous men (they practiſe of it being ſo accurate to ſenſe) to inquire wherein the Fallacy did conſiſt. And for the Probleme as it was firſt printed, but never publiſhed, and conſequently ought to have paſſed for a private paper ſtoln out of my ſtudy, your publique objecting againſt it, (in the opinion of all men that have converſed ſo much with honeſt company as to know what belongs to civill converſation,) was ſufficiently barbarous in Divines And ſeeing you knew I had rejected that Propoſition, it was but a poor Ambition to take wing as you thought to do, like Beetles from my egeſtions. But let that be as it will, you will think ſtrange now I ſhould reſume, and make good (at leaſt againſt your objection) that very ſame Propoſition. So much of the Figure as is needfull you will finde noted with the ſame letters, and placed at the end of this 5 Leſſon. Wherein let B I, be an Arch not greater then the *Radius* of the Circle, and divided into four equall parts, in L, N, O. Draw S N, the Sine of the Arch B N, and produce it to T, ſo as S T be double to S N, that is, equall to the Chord B I. Draw likewiſe *a* L, the Sine of the Arch B L, and produce it to *c*, ſo as *a c* be quadruple to *a* L, that is, equall to the two Chords B N, N I. Upon the Center N with the *Radius* N I, draw the Arch I *d*, cutting B U the Tangent in *d*. Then will B N produced cut the Arch I *d*, in the midſt at *o*. In the Line B S produced take S *b*, equall to B S; then draw and produce *b* N, and it will fall on the Point *d*. And B *d*, S T, will be equall; and *d* T joyned and produced will fall upon *o*, the midſt of the Arch I *d*. Joyn I T, and produce it to the Tangent B U in U. I ſay, that the ſtraight Line I T U ſhall paſs through *c*. For ſeeing B S, S *b*, are equall, and the Angle at S a right Angle, the ſtraight Lines B N, and *b* N, are alſo equall, and the Triangles B N *b*, *d* N *o* like and equall; and the Lines *d* T, T *o* equall. Draw *o i* Paralld to *d* U, cutting I U in *i*; and the Triangles *d* T *v*, *o* T *i* will alſo be like and equal. Produce S T to the Arch *d o* I in *e*, and produce it further to *f*, ſo that the Line *e f* be equall to T *e*; and then S *f* will be equall to *a c*. Therefore *f c* joyned will be Parallel to B S. In *c f* produced take *f g* equall to *c f*; and draw *g m* Parallel to *d* U, cutting I U in *m*, and *d o* in *n*; and let the interſection of the two Lines *a c* and *d o* be in *r*; which being done, the Triangles *m n* T, *r c* T will be like and equall. Therefore *m n* and *r c* are equall; and conſequently the ſtraight Line I *m* T U ſhall paſs through *c*. Dividing therefore *a c* in the midſt at *t*, and S N

in

in the midst at *l*, and joyning *t* N, L *l*, the Lines L *l*, *t* N, and *c* T produced with all meet in one and the same Point of B S produced; suppose at *q*. Therefore the Point *q* being given by the two known Points T and I, the Lines drawn from *q* through equall parts of the Sine of the Arch B I, for example through the Points P, Q, R, of the Sine M I, shall cut off equall Arches, as B L, L N, N O, O I. And this is enough to make good that Probleme, as to your objection.

The straight Line therefore B U for any thing you have said is proved equall to the Arch B I, and the division of any Angle given into any proportion given, the Quadrature of any Sector, and the Construction of any equilaterall Polygon is also given. And though in this also I should have erred, yet it cannot be denyed but that I have used a more natural, a more Geometrical, and a more perspicuous method in the search of this so difficult a Probleme, then you have done in your *Arithmetica Infinitorum* For though it be true that the aggregate of all the mean Proportionals between the Radius together with an infinitely little part of the same, and the Radius wanting an infinitely little part of the same; and again, between the Radius, together with two infinitely little parts, and the Radius wanting two infinitely little parts, and so on eternally will be equall to the Quadrant (a thing which every mean Geometrician knew before) yet it was absurd to think those Means could be calculated in Numbers by *Interpoling* of a Symbole; especially when you make that Symbole to stand for a Numbet neither true nor surd; as if there were a number that could neither be uttered in words, nor not be uttered in words. For what else is *surd*, but that which cannot be spoken?

To the fifth Article (though your discourse be long) you object but two things. One is, that *Whereas the Spirall of* Archimedes *is made of two Motions, one straight, the other circular, both uniform, I taking the Motion compounded of them both for one of those that are compounded, conclude falsely, that the generation of the Spirall is like to the generation of the Parabola.* What heed you use to take in your reprehensions, appears most manifestly in this objection. For I say in that demonstration of mine that *the velocity of the point A in describing the Spirall, encreaseth continually in Proportion to the Times.* For seeing it goes on uniformly in the Semidiameter, it is impossible it should not pass into greater and greater Circles proportionally to the Times; and consequently it must have a swifter and swifter Motion circular, to be compounded with the uniform Motion in every Point of the Radius as it turneth about. This objection therefore is nothing but an effect of a Will (without cause) to contradict.

The other objection is that *Granting all to be true hitherto, yet because it depends upon the finding of a straight Line equall to a Parabolicall Line in the* 18 *Chapter where I was deceived, I am also deceived here.* True. But because in the 18 Chapter of this English Edition I have found a straight Line equall to a Parabolicall Line, I have also found a straight Line equall to the Spirall Line of *Archimedes*. I must here put you in minde that by these words in your objections to the fifth Article at your Number 2, *Quatenus verum est, &c. we have demonstrated Prop.* 10, 11, 13. *Arithmet. Infinit.* you make it appear that you thought your Spirall (made of Arches or Circles) was the true Spirall of Archimedes; which is fully as absurd as the Quadrature of *Joseph Scaliger*, whose Geometry you so much despise.

To the sixth Article which is a Digression concerning the Analytiques of Geometricians, you deny *that the Efficient cause of the Construction ought to be contained in the Demonstration.* As if any Probleme could be known to be truly done, otherwise then by knowing first how, that is to say, by what Efficient Cause, and in what manner it is to be done. Whatsoever is done without that knowledge, cannot be demonstrated to be done; as you see in your computation of the Parabola, and *Paraboloeides*, in your *Arithmetica Infinitorum.*

And whereas I said that *The ends of all straight Lines drawn from a straight Line, and passing through one and the same Point, if their parts be* Proportionall, *shall be in a straight Line*; is true and accurate; as also *If they begin in the Circumference of a Circle, they shall*

alſo be in the Circumference of another Circle. And ſo is this, *If the Proportion be duplicate, they ſhall be in a Parabola.* All this I ſay is true and accurately ſpoken. But this was no place for the demonſtration of it. Others have done it. And I perceive by that you put in by Parentheſis (*intelligis credo inter duas Peripherias concentricas*) that you underſtand not what I mean.

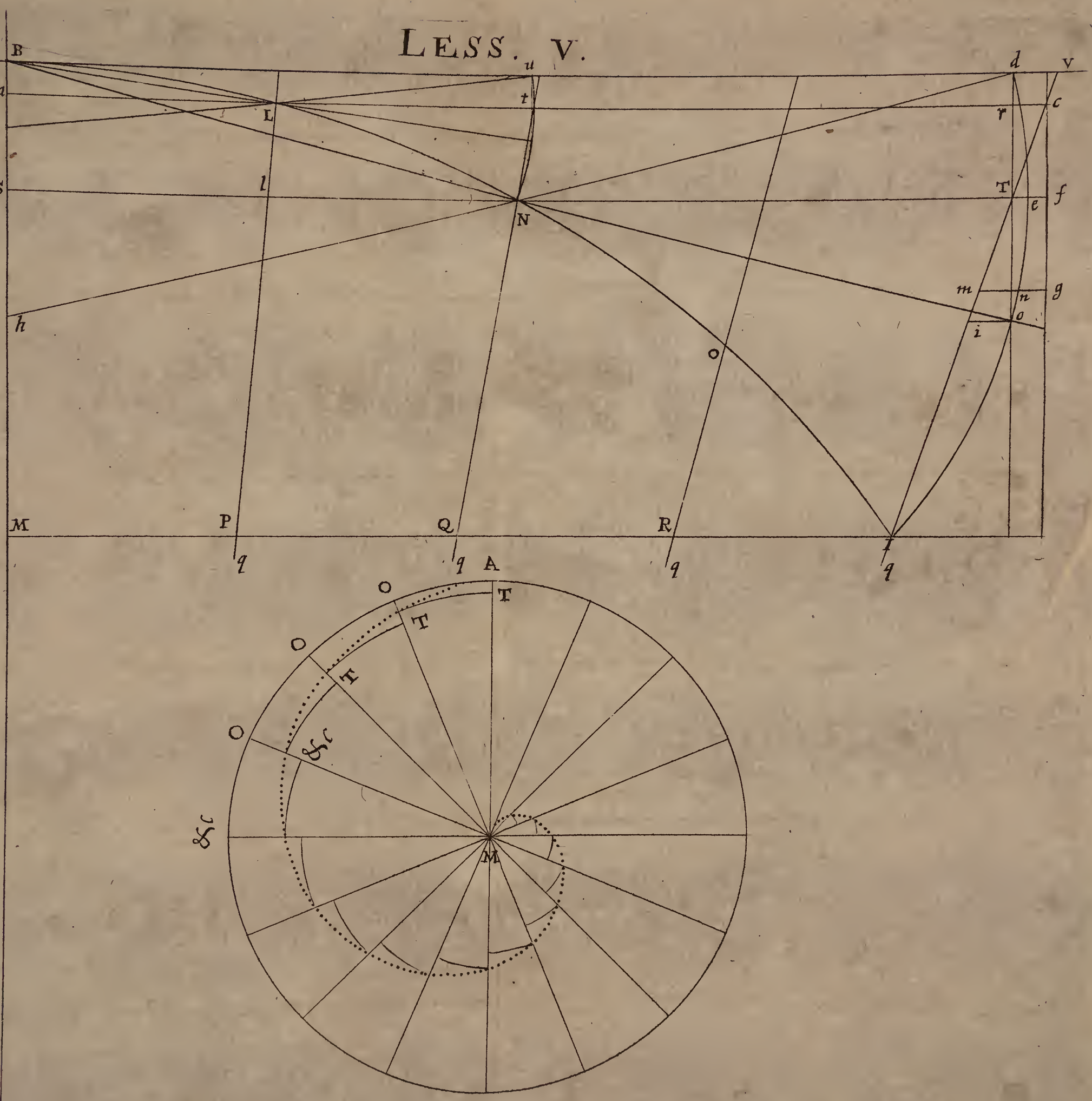
Hitherto reach your objections to my Geometry. For the reſt of your *Book*, it containeth nothing but a collection of lies, wherewith you do what you can, to extenuate as vulgar, and diſgrace as falſe, that which followeth, and to which you have made no ſpeciall objection.

I ſhall therefore only add in this place concerning your *Analytica per Poteſtates*, that it is no Art. For the Rule, both in Mr. *Oughtred*, and in *Des Cartes* is this, *When a Probleme or Queſtion is propounded, ſuppoſe the thing required done, and then uſing a fit ratiocination, put A or ſome other vowell for the magnitude ſought.* How is a man the better for this Rule without another rule, *How to know when the ratiocination is fit?* There may therefore be in this kind of Analyſis more or leſs naturall prudence, according as the Analyſt is more or leſs wiſe, or as one man in chuſing of the unknown Quantity with which he will begin, or in chuſing the way of the conſequences which he will draw from the *Hypotheſis*, may have better luck then another. But this is nothing to Art. A man may ſometimes ſpend a whole day in deriving of conſequences in vain, and perhaps another time ſolve the ſame Probleme in a few minutes.

I ſhall alſo add, that Symboles though they ſhorten the writing, yet they do not make the Reader underſtand it ſooner then if it were written in words. For the conception of the Lines and Figures (without which a man learneth nothing) muſt proceed from words, either ſpoken or thought upon. So that there is a double labour of the mind, one to reduce your Symboles to words (which are alſo Symboles) another to attend to the Ideas which they ſignifie. Beſides, if you but conſider how none of the Antients ever uſed any of them in their publiſhed demonſtrations of Geometry, nor in their Books of Arithmetique, more then for the *Rootes* and *Poteſtates* themſelves; and how bad ſucceſs you have had your ſelf in the unskilfull uſing of them, you will not, I think, for the future be ſo much in love with them as to demonſtrate by them that firſt part you promiſe of your *Opera Mathematica*. In which if you make not amends for that which you have already publiſhed, you will much diſgrace thoſe Mathematicians you addreſs your Epiſtles to, or otherwiſe have commended; as alſo the Univerſities (as to this kinde of Learning) in the ſight of learned men beyond Sea. And thus having examined your panier of Mathematiques, and finding in it no knowledge neither of Quantity, nor of meaſure, nor of Proportion, nor of Time, nor of Motion, nor of any thing, but only of certain Characters, as if a Hen had been ſcraping there; I take out my hand again, to put it in to your other panier of Theology, and good Manners. In the mean time I will truſt the objections made by you the Aſtronomer (wherein there is neither cloſe reaſoning, nor good ſtile, nor ſharpneſs of wit, to impoſe upon any man) to the diſcretion of all ſorts of Readers.

Of

LESS. V.



Of Manners.

To the same egregious Professors of the Mathematicks in the University of *Oxford.*

Lesson VI.

HAving in the precedent Lessons maintained the Truth of my Geometry, and sufficiently made appear, that your objections against it are but so many errors of your own, proceeding from misunderstanding of the Porpositions you have read in *Euclide*, and other Masters of Geometry; I leave it to your consideration to whom belong (according to your own sentence) the unhandsome attributes you so often give me upon supposition, that you your selves are in the right, and I mistaken; and come now to purge my self of those greater accusations which concern my Manners. It cannot be expected there should be much Science of any kinde in a man that wanteth Judgement; nor Judgement in a man that knoweth not the Manners due to a publique disputation in writing; wherein the scope of either party ought to be no other then the examination and manifestation of the truth. For whatsoever is added of contumely, either directly, or *scommatically*, is want of Charity, and uncivil; unless it be done by way of Reddition from him that is first provoked to it. I say unless it be by way of Reddition; for so was the Judgement given by the Emperor *Vespasian* in a quarrell between a Senator and a Knight of *Rome* which had given him ill language. For when the Knight had proved, that the first ill language proceeded from the Senator, the Emperor acquitted him in these words *Maledici senatoribus non oportere; remaledicere, fas & civile esse.* Nevertheless, now a dayes uncivill words are commonly and bitterly used by all that write in matter of Controversie, especially in Divinity, excepting now and then such writers as have been more then ordinarily well bred, and have observed, how hainous, and hazardous a thing such contumely is amongst some sorts of men, whether that which is said in disgrace be true or false. For evill words by all men of understanding are taken for a defiance, and a challenge to open war. But that you should have observed so much, who are yet in your mothers belly, was not a thing to be much expected.

The faults in Manners you lay to my charge, are these, 1. *Self conceit.* 2. *That I will be very angry with all men that do not presently submit to my Dictates.* 3. *That I had my Dove strive concerning Vision, out of papers which I had in my hands of Mr.* Warners. 4. *That I have injured the Universities.* 5. *That I am an Enemy to Religion.* These are great faults; but such as I cannot yet confess. And therefore I must as well as I can, seek out the grounds upon which you build your Accusation. Which grounds (seeing you are not acquainted with my conversation) must be either in my published writings, or reported to you by honest men, and without suspition of interest in reporting it. As for my *self-conceit* and *ostentation*, you shall finde no such matter in my writings That which your alleadge from thence is first that in the Epistle Dedicatory I say of my Book *de Corpore*, *Though it be little, yet it is full; and if good may go for great, great enough.* When a man presenting a gift great or small to his betters,

betters, adorneth it the beſt he can to make it the more acceptable; he that thinks this to be *Oſtentation*, and *ſelf-conceit*, is little verſed in the common actions of humane life. And in the ſame Epiſtle where I ſay of Civill Philoſophy, *it is no antienter then my* Book *de Cive*, theſe words are added, *I ſay it provoked, and that my detractors may ſee they loſe their labour.* But that which is truly ſaid, and upon provocation, is not boaſting, but defence. A ſhort ſum of that Book of mine, now publiquely in French, done by a Gentleman I never ſaw, carrieth the Title of *Ethiques demonſtrated.* The Book it ſelf tranſlated into French hath not onely a great teſtimony from the Tranſlator *Serberius*, but alſo from *Gaſſendus*, and *Merſennus*, who being both of the Roman Religion had no cauſe to praiſe it, or the Divines of *England* have no cauſe to finde fault with it. Beſides, you know that the Doctrine therein contained is generally received by all but thoſe of the Clergy, who think their intereſt concerned in being made ſubordinate to the Civil Power; whoſe teſtimonies therefore are invalide. Why therefore if I commend it alſo againſt them that diſpraiſe it publiquely, do you call it boaſting? *You have heard* (you ſay) *that I had promiſed the Quadrature of the Circle*, &c. You heard then that which was not true. I have been asked ſometimes, by ſuch as ſaw the Figure before me, what I was doing, and I was not afraid to ſay I was ſeeking for the ſolution of that Probleme; but not that I had done it. And afterwards being asked of the ſucceſs, I have ſaid, I thought it done. This is not boaſting; and yet it was enough, when told again, to make a fool believe 'twas boaſting. But you the Aſtronomer in the Epiſtle before your Philoſophicall Eſſay, ſay *you had a great expectation of my Philoſophicall, and Mathematicall works, before they were publiſhed.* It may be ſo. Is that my fault? can a man raiſe a great expectation of himſelf by boaſting? If he could, neither of you would be long before you raiſed it of your ſelves; ſaving that what you have already publiſhed, has made it now too late. For I verily believe there was never ſeen worſe reaſoning then in that Philoſophicall Eſſay; which any judicious Reader would believe proceeded from a Prævaricator, rather then from a man that believed himſelf; nor worſe Principles, then thoſe in your Books of Geometry. The expectation of that which ſhould be written by me, was raiſed partly by the *Cogitata Phyſico-Mathematica of Merſennus*, wherein I am often named with honour; and partly by others with whom I then converſed in *Paris*, without any oſtentation. That no man has a great expectation of any thing that ſhall proceed from either of you two, I am content to let it be your praiſe.

Another Argument of my *ſelf-conceit*, you take from my contempt of the writers of Metaphyſiques and School-Divinity. If that be a ſign of ſelf-conceit, I muſt confeſs I am guilty; and if your Geometry had then been publiſhed, I had contemned that as much. But yet I cannot ſee the conſequence (unleſs you lend me your better Logique) from *deſpiſing inſignificant and abſurd language* to *ſelf-conceit.*

And again, in your *Vindiciæ Academiarum*, you put for boaſting, that in my Leviathan Pag. 180. I would have *that Book by entire Soveraignty impoſed upon the Univerſities*; and in my Review Pag. 395. That I ſay of my Leviathan, *I think it may be profitably printed, and more profitably taught in the Univerſity.* The cauſe of my writing that Book, was the conſideration of what the Miniſters before, and in the beginning of the Civill War, by their preaching and writing did contribute thereunto. Which I ſaw not onely to tend to the Abatement of the then Civill Power, but alſo to the gaining of as much thereof as they could (as did afterwards more plainly appear) unto themſelves. I ſaw alſo that thoſe Miniſters, and many other Gentlemen who were of their Opinion, brought their Doctrines againſt the Civill Power from their Studies in the Univerſities. Seeing therefore that ſo much as could be contributed to the Peace of our Country, and the ſettlement of Soveraign Power without any Army, muſt proceed from Teaching; I had reaſon to wiſh, that Civill Doctrine were truly taught in the Univerſities. And if I had not thought that mine was ſuch, I had never written it. And having written it, if I had not recommended it to ſuch as had the Power to cauſe it to be taught,

I had written it to no purpose. To me therefore that never did write any thing in Philosophy to show my Wit, but (as I thought at least) to benefit some part or other of mankind, it was very necessary to commend my Doctrine to such men as should have the Power and Right to Regulate the Universities. I say *my Doctrine*; I say not my *Leviathan*. For wiser men may so digest the same Doctrine as to fit it better for a publique teaching. But as it is, I believe it hath framed the minds of a thousand Gentlemen to a conscientious obedience to present Government, which otherwise would have wavered in that Point. This therefore was no vanting, but a necessary part of the business I took in hand. You ought also to have considered, that this was said in the close of that part of my Book which concerneth Policy meerly Civill. Which part if you the Astronomer, that now think the Doctrine unworthy to be taught, were pleased once to honour with praises printed before it, you are not very constant, nor ingenuous. But whether you did so or not, I am not certain, though it was told me for certain. If it were not you, it was some body else whose Judgement has as much weight at least as yours.

And for any thing you have to say from your own knowledge, I remember not that ever I saw either of your faces. Yet you the Professor of Geometry go about obliquely to make me believe that *Vindex* hath discoursed with me, once at least, though I remember it not. I suppose it therefore true; But this I am sure is false, that either he or any man living did ever hear me brag of my Science, or praise my self, but when my defence required it. Perhaps some of our Philosophers that were at *Paris* at the same time, and acquainted with the same Learned men that I was acquainted with, might take for bragging the maintaining of my opinions, and the not yielding to the reasons alleadged against them. If that be ostentation, they tell you the truth. But you that are so wise should have considered, that even such men as profess Philosophy are carried away with the passions of Emulation and Envy (the sole ground of this your accusation) as well as other men, and instanced in your selves. And this is sufficient to shake off your aspersions of *Ostentation* and *Self-conceit*. For if I added, that my acquaintance know that I am naturally of modest rather then of boasting speech, you will not believe it, because you distinguish not between that which is said upon provocation, and that which is said without provocation, from vain glory.

The next accusation is, *That I will be very angry with all men that do not presently submit to my dictates; and that for advancing the reputation of my own skill, I care not what unworthy reflections I cast on others.* This is in the Epistle placed before the *Vindiciæ Academiarum*, subscribed by N S, as the Plain Song for H D in the rest of the Book to Descant upon. I know well enough the Authors names; and am sorry that N S has lent his name to be abused to so ill a purpose. But how does this appear? What Argument, what witness is there of it? You offer none; nor am I conscious of any. I begin to suspect, since you the Professor of Geometry have in your objections to the 20 Chapter these words concerning *Vindex*, *Oularis ille testis de quo hic agitur, erat, ni fallor, ille ipse*; That *Vindex* himself, in other company, has bestow'd a visit on me. Seeing you will have me believe it, let it be so; and (as it is likely) not long after my return into *England*. At which time (for the reputation, it seems, I had gotten by my boasting) divers persons that professed to love Philosophy and Mathematiques, came to see me; and some of them to let me see them, and hear and applaud what they applauded in themselves. I see now it hath happened to me with *Vindex*, as it happened to Dr. *Harvey* with *Moranus*. *Moranus* a Jesuite came out of *Flanders* hither, especially (as he sayes) to see what learned men in Divinity, Ethiques, Physiques, and Geometry were here yet alive, to the end that by discoursing with them in these Sciences, he might correct either his own, or their errors. Amongst others he was brought (he sayes) to that most civill and renowned old man Dr. *Harvey*. That's very well. And in good earnest if he had made good use of the Time which was very patiently afforded him, he might have learned of him (or of no man living) very much knowledge concerning the Circulation of the blood, the Generation of living Creatures, and many other

other difficult Points of naturall Philoſophy. And if he had had any thing in him but common and childiſh learning, he could have ſhewed it no where more to his advantage, then before him that was ſo great a Judge of ſuch matters. But what did he? That pretious time (which was but little, becauſe he was to depart again preſently for *Flanders*) he beſtowed wholly in venting his own childiſh Opinions, not ſuffering the Doctor ſcarce to ſpeak; loſing thereby the benefit he came for, and diſcovering that he came not to hear what others could ſay, but to ſhow to others how learned he was himſelf already. Why elſe did he take ſo little time, and ſo miſpend it? Or why returned he not again? But when he had talked away his time, and found (though patiently and civilly heard) he was not much admired, he took occaſion (writing againſt me) to be revenged of Dr. *Harvey*, by ſleighting his learning publiquely; and tels me that his learning was onely Experiments, which he ſayes I ſay have no more certainty then Civil Hiſtories. Which is falſe. My words are, *Ante hos nihil certi in Phyſicâ erat præter Experimenta cuiq; ſua, & Hiſtorias Naturales, ſi tamen & hæ dicendæ certæ ſint, quæ Civilibus Hiſtoriis certiores non ſunt.* Where I except expreſsly from uncertainty the Experiments that every man maketh to himſelf. But you ſee the nere cut, by which vain Glory joyned with Ignorance paſſeth quickly over to Envy and Contumely.

Thus it ſeems by your own confeſſion I was uſed by *Vindex*. He comes with ſome of my acquaintance in a Viſit. What he ſaid I know not, but if he diſcourſed then, as in his Philoſophicall Eſſay he writeth, I will be bold to ſay of my ſelf, I was ſo far from morolity, or (to uſe his Phraſe) from being *tetricall*, as I may very well have a good opinion of my own patience. And if there paſſed between us the diſcourſe you mention in your *Elenchus*, Pag. 116. it was an incivility in him ſo great, that without great civility I could not have abſtained from bidding him be gone. That which paſſed between us, you ſay was this. *I complained that whereas* I *made* Senſe, *nothing but a perception of Motion in the Organ, nevertheleſs the Philoſophy Schools through all Europe led by the Text of* Ariſtotle, *teach another Doctrin, namely, that* Senſation *is performed by* Species. This is a little miſtaken. For I do glory, not complain, that whereas all the Univerſities of *Europe* hold Senſation to proceede from *Species*, I hold it to be a perception of Motion in the Organ. The anſwer of *Vindex*, you ſay, was, *That the other Hypotheſis, whereby Senſe was explicated by the Principles of Motion, was commonly admitted here before my Book came out*, as having been *ſufficiently delivered by* Des Cartes, Gaſſendus *and* Sir Kenelme Digby *before I had publiſhed any thing in this kinde.* This then, it ſeems, was it that made me angry. Truly I remember not any angry word that ever I uttered in all my life to any man that came to ſee me, though ſome of them have troubled me with very impertinent diſcourſe; and with thoſe that argued with me, how impertinently ſoever, I always thought it more civility to be ſomewhat earneſt in the defence of my opinion, then by obſtinate and affected ſilence to let them ſee I contemned them, or hearkned not to what they ſaid. If I were earneſt in making good, that the manner of Senſation by ſuch Motion as I had explicated in my Leviathan, is in none of the Authors by him named, it was not Anger, but a care of not offending him, with any ſigne of the contempt which his diſcourſe deſerved. But it was Incivility in him to make uſe of a Viſit, which all men take for a profeſſion of Friendſhip, to tell me that that which I had already publiſhed for my own, was found before by *Des Cartes*, *Gaſſendus*, and Sir *Kenelme Digby*. But let any man read *Des Cartes*, he ſhall finde, that he attributeth no *Motion* at all to the object of ſenſe, but an *inclination* to action, which *inclination* no man can imagine what it meaneth. And for *Gaſſendus*, and S. *Kenelme Digby*, it is manifeſt by their writings, that their opinions are not different from that of *Epicurus*, which is very different from mine. Or if theſe two, or any of thoſe I converſed with at *Paris*, had prevented me in publiſhing my own Doctrine, yet ſince it was there known, and declared for mine by *Merſennus* in the Preface to his *Balliſtica* (of which the three firſt leaves are imployed wholly in the ſetting forth of my opinion concerning ſenſe, and the reſt of the faculties of the Soul) they ought not therefore to be ſaid to have found it out before me. And conſequently this anſwer

which

which you say was given me by *Vindex* was nothing else but Untruth and Envy; and (because it was done by way of Visit) Incivility. But you have not alleadged, nor can alleadge any words of mine, from which can be drawn that I am so angry as you say I am with those that submit not to my Dictates. Though the discipline of the University be never so good; yet certainly this behavour of yours and his are no good Arguments to make it thought so. But you the Professor of Geometry, that out of my words spoken against *Vindex* in my 20 Chapter, argue my angry humour; do just as well, as when (in your *Arithmetica Infinitorum*) from the continuall increase of the excess of the row of Squares above the third part of the aggregate of the greatest, you conclude they shall at last be equall to it. For though you knew that *Vindex* had given me first the worst words that possibly can be given, yet you would have that return of mine to be a Demonstration of an angry humour; not then knowing what I told you even now in the beginning of this Lesson, of the sentence given by *Vespasian*. But to this Point I shall speak again hereafter.

Your third Accusation is, *That I had my doctrine of Vision, which I pretend to be my own, out of papers which I had a long time in my hands of Mr.* Warners. I never had sight of any of Mr. *Warn.* papers in all my life but that of *Vision by Refraction* (which by his approbation I carryed with me to *Paris*, and caused it to be printed under his own name, at the end of *Mersennus* his *Cogitata Physico-Mathematica*, which you may have there seen) and another Treatise of the Proportions of Alloy in Gold and Silver coine; which is nothing to the present purpose. In all my conversation with him, I never heard him speak of any thing he had written, or was writing *de Penicillo optico*. And it was from me that he first heard it mentioned that Light and Colour were but Fancy. Which he imbraced presently as a truth, and told me it would remove a rub he was then come to in the discovery of the place of the Image. If after my going hence he made any use of it (though he had it from me, and not I from him) it was well done. But wheresoever you finde my Principles, make use of them, if you can, to demonstrate all the Symptomes of Vision; and I will do (or rather have done and mean to publish) the same; and let it be judged by that, whether those Principles be of mine, or other mens invention. I give you time enough, and this advantage besides, that much of my *Optiques* hath been privately read by others. For I never refuted to lend my papers to my friends, as knowing it to be a thing of no prejudice to the advancement of Philosophy, though it be (as I have found it since) some prejudice to the advancement of my own reputation in those Sciences; which reputation I have alwayes postposed to the common benefit of the studious.

You say further (you the Geometrician) that I had the Proposition of the Spirall Line equall to a Paraboli call Line from Mr. *Robervall*. True. And if I had remembred it, I would have taken also his demonstration, though if I had publisht his, I would have suppressed mine. I was comparing in my thoughts those two Lines, Spirall and Paraboli call, by the Motions wherewith they were described; and considering those Motions as uniform, and the Lines from the Center to the Circumference, not to be little Parallelograms, but little Sectors, I saw that to compound the true Motion of that Point which described the Spirall, I must have one Line equall to half the Perimeter, the other equall to half the Diameter. But of all this I had not one word written. But being with *Mersennus* and Mr. *Robervall* in the Cloister of the Convent, I drew a Figure on the wall, and Mr. *Robervall* perceiving the deduction I made, told me that since the Motions which make the Paraboli call Line, are one uniform, the other accelerated, the Motions that make the Spirall must be so also; Which I presently acknowledged; and he the next day, from this very method brought to *Mersennus* the demonstration of their equality. And this is the story mentioned by *Mersennus*, Prop. 25. Corol. 2. of his *Hydraulica*; Which I know not who hath most magnanimously interpreted to you in my disgrace.

The fourth accusation is, *That I have injured the Universities*. Wherein? First, *In that I would have the Doctrine of my* Leviathan *by entire Soveraignty be imposed on them.* You often upbraid me with thinking well of my own Doctrine; I agree by consequence, that I thought this Doctrine good, I desired not therefore that any thing should be imposed upon them

but what (at least in my opinion) was good both for the Common-wealth and them. Nay more I would have the State make use of them to uphold the civil Power, as the Pope did to uphold the Ecclesiasticall. Is it not absurdly done to call this an Injury? But to question (you will say) whether the Civill Doctrine there taught, be such as it ought to be, or not, is a disgrace to the Universities. If that be certain, it is certain also that those Sermons and Books, which have been preached and published, both against the former and the present Government, directly or obliquely, were not made by such Ministers and others as had their breeding in the Universities; though all men know the contrary. But the Doctrine which I would have to be taught there, what is it? It is this, *That all men that live in a Common-wealth, and receive protection of their lives and fortunes from the Supreme Governour thereof, are reciprocally bound as far as they are able, and shall be required, to protect that Governour.* Is it, think you, an unreasonable thing to impose the teaching of such Doctrine upon the Universities? Or will you say they taught it before, when you know that so many men which came from the Universities to preach to the People, and so many others that were not Ministers did stir the People up to resist the then Supreme Civill Power? And was it not truly therefore said, that the Universities receiving their Discipline from the Authority of the Pope, were the Shops, and Operatories of the Clergy? Though the Competition of the Papall and Civill power be taken now away, yet the Competition between the Ecclesiasticall and the Civill power hath manifestly enough appeared very lately. But neither is this an upbraiding of an University (which is a Corporation or Body Artificial) but of particular men that desire to uphold the Authority of a Church, as of a distinct thing from the Common-wealth. How would you have exclaimed, if instead of recommending my *Leviathan* to be taught in the Universities, I had recommended the erecting of a New and Lay-University, wherein Lay-men should have the reading of *Physiques*, *Mathematicks*, *Morall Philosophy*, and *Politicks*, as the Clegy have now the sole teaching of Divinity? Yet the thing would be profitable, and tend much to the Polishing of mans nature, without much publique charge. There will need but one House, and the endowment of a few Professions. And to make some learn the better, it would do very well that none should come thither sent by their Parents, as to a Trade to get their living by, but that it should be a place for such ingenuous men, as being free to dispose of their own time, love Truth for it self. In the mean time Divinity may go on in *Oxford* and *Cambridge* to furnish the Pulpit with men to cry down the Civil Power, if they continue to do as they did. If I had (I say) made such a Motion in my Leviathan, though it would have offended the Divines, yet it had been no injury. But 'tis an injury (you will say) to deny in generall the utility of the Antient Schooles, and to deny that we have received from them our Geometry. True, if I had not spoken distinctly of the Schools of Philosophy, and said expresly, that the Geometricians passed not then under the name of Philosophers; and that in the School of *Plato* (the best of the Antient Philosophers) none were received that were not already in some measure Geometricians. *Euclide* taught Geometry; but I never heard of a Sect of Philosophers, called *Euclidians*, or *Alexandrians*, or ranged with any of the other Sects, as *Peripatetiques*, *Stoiques*, *Academiques*, *Epicureans*, *Pyrrhonians*, &c. But what is this to the Universities of Christendome? Or why are we beholding for Geometry to our Universities, more then to *Gresham* Colledge, or to private men in *London*, *Paris*, and other places, which never taught or learned it in a publique School? For even those men that living in our Universities have most advanced the Mathematiques, attained their knowledge by other means then that of publique Lectures, where few Auditors, and those of unequall proficiency cannot make benefit by one and the same Lesson. And the true use of publique Professors, especially in the Mathematiques, being to resolve the Doubts, and Problems (as far as they can) of such as come unto them with desire to be informed.

That the Universities *now* are not regulated by the Pope, but by the Civill Power, is true, and well. But where say I the contrary? And thus much for the first injurie.

Ano-

Another (you say) is this, that in my *Leviathan* Pag. 380. I say, *The principall Schooles were ordained for the three Professions of Roman Religion, Roman Law, and the Art of Medicine.* Thirdly, that I say. *Philosophy had no otherwise place there then as a hand-maid to Roman Religion,* Fourthly, *since the authority of* Aristotle *was received there, that Study is not properly Philosophy but Aristotelity.* Fifthly, *That for Geometry, till of late times it had no place there at all.* As for the second, it is too evident to be denyed; the Fellowships having been all ordained for those Professions; and (saving the Change of Religion) being so yet. Nor hath this any Reflection upon the Universities, either as they now are, or as they then were, seeing it was not in their own power to endow themselves, or to receive other Laws and Discipline then their Founder, and the State were pleased to ordain. For the third, it is also evident. For all men know that none but of the Roman Religion had any Stipend or preferment in any University, where that Religion was established; No, nor for a great while, in their Common-wealths; but were every where persecuted as Heretiques. But you will say the words in my *Leviathan* are not, Philosophy *had no place*, but *hath no place.* Are you not ashamed to lay to my charge a mistake of the word *hath* for *had*? Which was either a mistake of the Printer, or if it were so in the Copy, it could be no other then the mistake of a letter in the writing, unless you think you can make men believe that after fifty years being acquainted with what was publiquely professt and practised in *Oxford* and *Cambridge*, I knew not what Religion they were of. This taking of advantage from the mistake of a word, or of a letter, I finde also in the *Elenchus*, where for *prætendit se scire*, there is *prætendit scire*, which you the Geometrician sufficiently mumble, mistaking it I think for an Anglicisme, not for a fault in the impression.

To the fourth, you pretend, that men are not now so tied to *Aristotle* as not to *enjoy a liberty of Philosophising, though it were otherwise when I was conversant in Magdalen Hall.* Was it so then? Then am I absolved, unless you can shew some publique Act of the University made since that time to alter it. For it is not enough to name some few particular ingenuous men that usurpe that Liberty in their private discourses, or (with connivence) in their publike disputations. And your Doctrine, that even here you avow, of *Abstracted Essences, Immateriall Substances*, and of *Nunc-stans*; and your improper language in using the word (not as mine, for I have it no where) *Successive Eternity*; as also your Doctrine of *Condensation*, and your arguing from naturall reason the incomprehensible Mysteries of Religion, and your Malicious Writing, are very shrewd signes, that you your selves are none of those which you say do *freely Philosophise*, but that both your Philosophy and your Language are under the Servitude, not of the Roman Religion, but of the Ambition of some other Doctors, that seek, as the Romish Clergy did, to draw all humane learning to the upholding of their Power Ecclesiasticall. Hitherto therefore there is no injury done to the Universities. For the fifth, you grant it, namely, *That till of late there was no allowance for the teaching of Geometry.* But least you should be thought to grant me any thing, you say, you the Astronomer, *Geometry hath* now *so much place in the* Universities, *that when Mr.* Hobs *shall have published his Philosophicall and Geometricall pieces, you assure your self you shall be able to finde a great number in the* University, who *will understand as much or more of them, then he desireth they should*, &c. But though this be true of the *now*, yet it maketh nothing against my *then*. I know well enough that Sir *Henry Saviles* Lectures were founded and endowed since. Did I deny *then*, that there were in *Oxford* many good Geometricians? But I deny *now*, that either of you is of the Number. For my Philosophicall and Geometricall pieces, are published, and you have understood onely so much in them, as all men will easily see by your objections to them, and by your own published Geometry, that neither of you understand any thing either in Philosophy or in Geometry. And yet you would have those Books of yours to stand for an Argument, and to be an *Index* of the Philosophy and Geometry to be found in the Universities. Which is a greater injury and disgrace to them, then any words of mine though interpreted by your selves.

Your

Your last and greatest accusation, or rather railing (for an Accusation should contain, whether true or false, some particular fact, or certain words, out of which it might seem at least to be inferred) is, that I am an enemy to Religion. Your words are, *It is said that Mr* Hobs *is no otherwise an Enemy to the Roman Religion, saving onely as it hath the name of Religion.* This is said by *Vindex*: You the Geometrician in your Epistle Dedicatory say thus, *With what pride and imperiousness he tramples on all things both Humane and Divine, uttering fearfull and horrible words of God,* (peace) *of Sin, of the Holy Scripture, of all Incorporeal substances in generall, of the Immortall soul of man, and of the rest of the weighty points of Religion* (down) *it is not so much to be doubted as lamented.* And at the end of your objections to the 18 Chapter, *Perhaps you take the whole History of the Fall of* Adam *for a Fable, which is no wonder, when you say the Rules of honouring and worshipping of God are to be taken from the Laws.* Down I say; you bark now at the Supreme Legislative Power. Therefore it is not I, but the Laws which must rate you off. But do not many other men as well as you read my *Leviathan*, and my other Books? And yet they all finde not such enmity in them against Religion. Take heed of calling them all Atheists that have read and approved my *Leviathan*. Do you think I can be an Atheist and not know it? Or knowing it durst have offered my Atheism to the Press? Or do you think him an Atheist, or a contemner of the Holy Scripture, that sayeth nothing of the Deity, but what he proveth by the Scripture? You that take so hainously that I would have the Rules of Gods worship in a Christian Common-wealth taken from the Laws, tell me, from whom you would have them taken? From your selves? Why so, more then from me? From the Bishops? Right, if the Supreme Power of the Common-wealth will have it so; If not, why from them rather then from me? From a Consistory of Presbyters by themselves, or joyned with Lay-Elders, whom they may sway as they please? Good, If the Supreme Governour of the Common-wealth will have it so. If not, why from them, rather then from me, or from any man else? They are wiser and learneder then I. It may be so; but it has not yet appeared. Howsoever, let that be granted. Is there any man so very a fool as to subject himself to the Rules of other men in those things which so neerly concern himself, for the Title they assume of being wise and learned, unless they also have the sword which must protect them. But it seems you understand the sword as comprehended. If so, do you not then receive the Rules of Gods worship from the Civill Power? Yes doubtless; and you would expect, if your Consistory had that sword, that no man should dare to exercise or teach any Rules concerning Gods worship which were not by you allowed. See therefore how much you have been transported by your malice towards me, to injure the Civill Power by which you live. If you were not despised, you would in some places and times, where and when the Laws are more severely executed, be shipt away for this your madness to *America*, I would say, to *Anticyra*. What luck have I, when this, of the Laws being the Rules of Gods publique worship, was by me said and applyed to the Vindication of the Church of *England* from the Power of the Roman Clergy, it should be followed with such a storm from the Ministers Presbyterian and Episcopall of the Church of *England*? Again, for those other Points, namely, that I approve not of Incorporall Bodies, nor of other Immortality of the soul, then that which the Scripture calleth Eternall Life, I do but as the Scripture leads me. To the Texts whereof by me alleadged, you should either have answered, or else forborne to revile me for the conclusions I derived from them. Lastly, what an absurd question is it to ask me whether it be in the Power of the Magistrate, whether the world be eternall or not? It were fit you knew it is in the Power of the Supreme Magistrate to make a Law for the punishment of them that shall pronounce publiquely of that question any thing contrary to that which the Law hath once pronounced. The truth is, you are content that the Papall power be cut off, and declaimed against as much as any man will; but the Ecclesiasticall Power which of late was aimed at by the Clergy here, being a part thereof, every violence done to the Papall Power is sensible to them yet; like that which I have heard say of a man, whose leg being cut off for prevention of a Gangrene that began in his Toe, would nevertheless complain of a pain in his Toe, when his leg was cut off.

Thus

Thus much in my defence; which I believe if you had foreseen, this Accusation of yours had been left out. I come now to examine (though it be done in part already) what Manners those are which I finde every where in your Writings.

And first, How came it into your minds that a man can be an Atheist, I mean an Atheist in his Conscience? I know that *David* confesseth of himself, upon sight of the Prosperity of the wicked, that his feet had almost slipt, that is, that he had slipt into a short doubtfulness of the Divine Providence. And if any thing else can cause a man to slip in the same kinde, it is the seeing such as you (who though you write nothing, but what is dictated to each of you by a Doctor of Divinity) to break the greatest of Gods Commandement (which is Charity) in every Line before his face. And though such forgettings of God be somewhat more then short doubtings, and sudden transportations incident to humane Passion, yet I do not for that cause think you Atheists, and enemies of Religion, but only ignorant and imprudent Christians. But how, I say, could you think me an Atheist, unless it were because finding your doubts of the Deity more frequent then other men do, you are thereby the apter to fall upon that kinde of reproach? Wherein you are like women of poor & evil Education when they scold; amongst whom the readiest disgracefull word is *Whore*. Why not Thief, or any other ill name, but because when they remember themselves, they think that reproach the likeliest to be true?

Secondly, tell me what crime it was which the Latines called by the name of *Scelus*? You think not (unless you be Stoiques) that all Crimes are equall. *Scelus* was never used but for a Crime of greatest mischief, as the taking away of Life and Honour; and besides, basely acted, as by some clandestine way, or by such a way as might be covered with a Lye. But when you insinuate in a writing published that I am an Atheist, you make your selves Authors to the multitude, and do all you can to stir them up to attempt upon my life; and if it succeed, then to sneak out of it by leaving the fault on them that are but actors. This is to indeavour great mischief basely; and therefore *Scelus*. Again, to deprive a man of the honour he hath merited, is no little wickedness; and this you endeavour to do by publishing falsely that I challenge as my own the inventions of other men. This is therefore *Scelus* Publiquely to tel all the world that I wil be angry with all men that do not presently submit to my Dictates; to deprive me of the friendship of all the world. Great dammage, and a lie, and yours. For to publish any untruth of another man to his disgrace, on hear-say from his enemy, is the same fault as if he published it on his own credit. If I should say I have heard that Dr. *Wallis* was esteemed at *Oxford* for a simple fellow, and much inferiour to his fellow-professor Dr. *Ward* (as indeed I have heard, but do not believe it) though this be no great disgrace to Dr. *Wallis*, yet he would think I did him injury. Therefore publique Accusation upon hear-say is *Scelus*. And whosoevet does any of these things does *Scelerate*. But you the Professors of the Mathematiques at *Oxford*, by the advice of two Doctors of Divinity have dealt thus with me. Therefore you have done (I say not foolishly, though no wickedness be without folly, but *Scelerate*, ὅπερ ἔδει δεῖξαι.

Thirdly, it is ill Manners, in reprehending truth, to send a man in a boasting way to your own errors; as you the Professor of Geometry have often sent me to your two Tractates of the *Angle of Contact*, and *Arithmetica Infinitorum*.

Fourthly, it is ill Manners, to diminish the just reputation of worthy men after they be dead, as you the Professor of Geometry have done in the case of *Joseph Scaliger*.

Fifthly, when I had in my *Leviathan* suffered the Clergy of the Church of *England* to escape, you did imprudently in bringing any of them in again. An *Ulysses* upon so light an occation would not have ventured to return again into the Cave of *Polyphemus*.

Lastly, how ill does such Levity and Scurrility, which both of you have shewn so often in your writings, become the gravity and sanctity requisite to the calling of the Ministery? They are too many to be repeated. Do but consider you the Geometrician how unhandsome it is to play upon my name, when both yours and mine are plebeian names; though from *Willis* by *Wallis*, you go from yours in *Wallisius*. The jest of using at every word *Mi Hobbi*, is lost to them beyond Sea. But this is not so ill as some of the rest. I will write out one of them as it is in

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the fourth Page of your *Elenchus*; *Whence it appears that your* Empusa *was of the number of those Fairies which you call in English* Hob-goblins. *The word is made of ἐν and πὺς; and thence comes the childrens play called the play of* Empusa, *Anglicé* (hitherto in Latine all but *Hob-goblins*, then follows) **Fox, Fox,** *come out of your hole* (in English, then in Latine again,) *in which the boy that is called the* **Fox,** *holds up one foot, and jumps with the other, which in English is to hop.* When a stranger shall read this, and hoping to finde therein some witty conceit, shall with much adoe have gotten it interpreted and explained to him, what will he think of our Doctors of Divinity at *Oxford*, that will take so much pains as to go out of the language they set forth in, for so ridiculous a purpose? You will say it is a pretty *Paranomasia*. How you call it there I know not, but it is commonly called here a *Clinch*; and such a one as is too insipide for a boy of twelve years old, and very unfit for the sanctity of a Minister, and gravity of a Doctor of Divinity. But I pray you tell me where it was you read the word *Empusa* for the boys play you speak of, or for any other play amongst the Greeks. In this (as you have done throughout all your other writings) you presume too much upon your first cogitations. There be a hundred other scoffing passages, and ill-favoured attributes given me in both your writings, which the Reader will observe without my pointing to them, as easily as you would have him; and which perhaps some young Students, finding them full of Gall, will mistake for Salt. Therefore to disabuse those youngmen, and to the end they may not admire such kind of wit, I have here and there been a little sharper with you then else I would have been. If you think I did not spare you, but that I had not wit enough to give you as scornfull names as you give me, are you content I should try? Yes (you the Geometrician will say) give me what names you please, so you call me not *Arithmetica Infinitorum* (I will not.) Nor *Angle of Contact*, nor *Arch-Spirall*. Nor *Quotient*. (I will not.) But I here dismiss you both together. So go your wayes, you *uncivill Ecclesiastiques, Inhumane Divines, Dedoctors of Morality, Unasinous Colleagues, Egregious pair of* Issachars, *most wretched Vindices* and *Indices Academiarum*; and remember *Vespasian's* Law, that it is uncivill to give ill language first, but civill and lawfull to return it. But much more remember the Law of God, to obey your Soveraigns in all things; and not only not to derogate from them, but also to pray for them, and as far as you can to maintain their Authority, and therein your own protection. And (do you hear?) take heed of speaking your minde so cleerly in answering my Leviathan, as I have done in writing it. You should do best not to meddle with it at all, because it is undertaken, and in part published already, and will be better performed, from Term to Term, by one *Christopher Pike*.